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相对论性非完整系统的 Lagrange 对称性与守恒量*

张斌[1†] 方建会[1] 张东爱[2] 徐瑞莉[1] 刘学锋[1]
(1. 中国石油大学(华东)理学院,青岛 266580)(2. 甘肃省白银市会宁县大沟中学,白银 730721)

摘要 本文研究相对论性非完整系统的 Lagrange 对称性,给出相对论性非完整系统 Lagrange 对称性的判据,得到相对论性非完整系统 Lagrange 对称性导致的守恒量及其存在条件,最后举例说明结果的应用.

关键词 相对论, 非完整系统, Lagrange 对称性, 守恒量

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引 言

对称性原理是物理学中更高层次的法则,对称性原理是近代分析力学的重要分支之一. 分析力学中的近代对称性主要有 Noether 对称性,Lie 对称性和 Mei 对称性. 近年来,关于约束力学系统三种对称性及其导致守恒量的研究取得了一系列重要成果[1-7,38]. 寻求新的对称性是分析力学对称性理论研究的难点和重点[8]. Lagrange 对称性是有别于三大对称性的一种新型对称性,20 世纪六七十年代 Currie 等对不同自由度[9,10] Lagrange 函数等价问题的研究是人们对 Lagrange 对称性的最早探索,上世纪 70 年代末到 90 年代,Lutzky 等对力学系统的 Lagrange 函数等价问题做了一系列的研究[11-14],后来将这种 Lagrange 函数等价关系称为 Lagrange 对称性[14,15],Lagrange 对称性现已被推广到 Hamilton 等系统[15-26].

20 世纪八九十年代,罗绍凯、方建会等将相对论效应引入约束力学系统进行研究[27,28],建立了相对论力学系统的一系列运动微分方程和变分原理[29-32],并且研究了相对论力学系统的对称性理论[33-37]. 本文结合相对论性非完整力学系统方程的特点,探讨该系统的 Lagrange 对称性理论,给出相对论性非完整系统 Lagrange 对称性的判据、得到 Lagrange 对称性导致守恒量的条件及守恒量的形式.

1 力学系统的运动微分方程

设力学系统由 N 个静止质量分别为 $m_{0i}(i=1,\cdots,N)$ 的质点组成,设系统受有 g 个理想 Chetaev 型非完整约束

$$f_\beta(t,\boldsymbol{q},\dot{\boldsymbol{q}})=0 \quad (\beta=1,\cdots,g) \tag{1}$$

约束方程 f_β 和变分 δ_s 满足 Appell - Chetaev 条件

$$\frac{\partial f_\beta}{\partial \dot{q}}\delta q_s=0 \quad (\beta=1,\cdots,g) \tag{2}$$

则系统的运动微分方程为

$$\frac{d}{dt}\frac{\partial L^*}{\partial \dot{q}_s}-\frac{\partial L^*}{\partial q_s}=Q_s^*+\Lambda_s^* \tag{3}$$

Q_s^* 为广义非势力,Λ_s^* 为广义约束反力,L^* 为系统的相对性 Lagrange 函数,即

$$\Lambda_s^*=\lambda_\beta\frac{\partial f_\beta}{\partial \dot{q}_s},\quad L^*=T^*-V \tag{4}$$

V 为系统的广义势能,T^* 为系统相对论性广义动能

$$T^*=\sum_{i=1}^{N}m_{0i}c^2\left(1-\sqrt{1-\dot{r}_i^2/c^2}\right) \tag{5}$$

其中 c 为光速.

2 系统的 Lagrange 对称性

给定系统(3)的两组动力学函数 L^*,Q_r^*,Λ_r^* 和 $\bar{L}^*,\bar{Q}_r^*,\bar{\Lambda}_r^*$,定义 L_r^* 和 $\bar{L}_r^*$ 分别为

$$L_r^*=\frac{\partial^2 L^*}{\partial \dot{q}_r\partial \dot{q}_k}\ddot{q}_k+\frac{\partial^2 L^*}{\partial \dot{q}_r\partial q_k}\dot{q}_k+\frac{\partial^2 L^*}{\partial \dot{q}_r\partial t}-\frac{\partial L^*}{\partial q_r}-Q_r^*-$$

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† 通讯作者 E-mail:teamup@yeah.net

$$\Lambda_r^* = W_{rk}\ddot{q}_k + \frac{\partial^2 L^*}{\partial\dot{q}_r \partial q_k}\dot{q}_k + \frac{\partial^2 L^*}{\partial\dot{q}_r \partial t} - \frac{\partial L^*}{\partial q_r} - Q_r^* - \Lambda_r^* \tag{6}$$

$$\bar{L}_r^* = \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_r \partial\dot{q}_k}\ddot{q}_k + \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_r \partial q_k}\dot{q}_k + \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_r \partial t} - \frac{\partial \bar{L}^*}{\partial q_r} - \bar{Q}_r^* - \bar{\Lambda}_r^* = W_{rk}\ddot{q}_k + \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_r \partial q_k}\dot{q}_k + \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_r \partial t} - \frac{\partial \bar{L}^*}{\partial q_r} - \bar{Q}_r^* - \bar{\Lambda}_r^* \tag{7}$$

其中

$$W_{rk} = \frac{\partial L^*}{\partial\dot{q}_r \partial\dot{q}_k},\ \bar{W}_{rk} = \frac{\partial \bar{L}^*}{\partial\dot{q}_r \partial\dot{q}_k} \tag{8}$$

定义 对受有 g 个约束方程(1)的系统(3),如果由动力学函数 L^*,$\boldsymbol{Q}^*$ 和 $\boldsymbol{\Lambda}^*$ 确定的

$$L_r^* = 0, \tag{9}$$

的每一个解都满足由动力学函数 $\bar{L}^*$,$\bar{\boldsymbol{Q}}^*$ 和 $\bar{\boldsymbol{\Lambda}}^*$ 确定的

$$\bar{L}_r^* = 0, \tag{10}$$

反之亦然,则表明系统具有 Lagrange 对称性.

由式(7)和(10)得

$$\ddot{q}_k = \bar{U}^{kr}\left(\bar{Q}_r^* + \bar{\Lambda}_r^* + \frac{\partial \bar{L}^*}{\partial q_r} - \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_r \partial q_k}\dot{q}_k - \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_r \partial t}\right) \tag{11}$$

其中

$$\bar{W}_{rk}\bar{U}^{kl} = \delta_r^l. \tag{12}$$

把(11)式代入(9)式得

$$W_{rk}\bar{U}^{kl}\left(\bar{Q}_l^* + \bar{\Lambda}_l^* + \frac{\partial \bar{L}^*}{\partial q_l} - \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_l \partial q_k}\dot{q}_k - \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_l \partial t}\right) = Q_r^* + \Lambda_r^* + \frac{\partial L^*}{\partial q_r} - \frac{\partial^2 L^*}{\partial\dot{q}_r \partial q_k}\dot{q}_k - \frac{\partial^2 L^*}{\partial\dot{q}_r \partial t} \tag{13}$$

由定义和(13)式得判据:对于受约束(1)的非完整系统(3),如果两组动力学函数 L^*,$\boldsymbol{Q}^*$,$\boldsymbol{\Lambda}^*$ 和 $\bar{L}^*$,$\bar{\boldsymbol{Q}}^*$,$\bar{\boldsymbol{\Lambda}}^*$ 满足方程(13),则系统具有 Lagrange 对称性.

3 Lagrange 对称性导致的守恒量

对于相对论性非完整系统的 Lagrange 对称性有如下命题:

命题 对受约束(1)的相对论性非完整系统(3),如果两组动力学函数 $\boldsymbol{Q}^*$,$\boldsymbol{\Lambda}^*$ 和 $\bar{\boldsymbol{Q}}^*$,$\bar{\boldsymbol{\Lambda}}^*$ 满足条件

$$\frac{\partial}{\partial\dot{q}_l}(\bar{Q}_s^* + \bar{\Lambda}_s^*) = A_s^r \frac{\partial}{\partial\dot{q}_l}(Q_r^* + \Lambda_r^*) \tag{14}$$

则系统的 Lagrange 对称性可导致守恒量

$$I_L = tr(A)^m = \mathrm{const} \tag{15}$$

其中 A 为以 A_s^r 为元素的矩阵,

$$A_s^r = \bar{W}_{sk}U^{kr},\quad W_{rk}U^{ks} = \delta_r^s \tag{16}$$

m 为任意常数.

证明 将(16)式代入(13)式得

$$\frac{\partial^2 \bar{L}^*}{\partial\dot{q}_l \partial q_k}\dot{q}_k + \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_l \partial t} - \frac{\partial \bar{L}^*}{\partial q_l} - \bar{Q}_l^* - \bar{\Lambda}_l^* = A_l^r\left(\frac{\partial^2 L^*}{\partial\dot{q}_r \partial q_k}\dot{q}_k + \frac{\partial^2 L^*}{\partial\dot{q}_r \partial t} - \frac{\partial L^*}{\partial q_r} - \Lambda_r^* - Q_r^*\right) \tag{17}$$

对(17)式求关于 $\dot{q}_s$ 的偏导数得

$$\frac{\partial^3 \bar{L}^*}{\partial\dot{q}_l \partial q_k \partial\dot{q}_s}\dot{q}_k + \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_l \partial q_s} + \frac{\partial^3 \bar{L}^*}{\partial\dot{q}_l \partial\dot{q}_s \partial t} - \frac{\partial^2 \bar{L}^*}{\partial q_l \partial\dot{q}_s} - \frac{\partial \bar{Q}_l^*}{\partial\dot{q}_s} - \frac{\partial \bar{\Lambda}_l^*}{\partial\dot{q}_s} = \frac{\partial A_l^r}{\partial\dot{q}_s}\left(\frac{\partial^2 L^*}{\partial\dot{q}_r \partial q_k}\dot{q}_k + \frac{\partial^2 L^*}{\partial\dot{q}_r \partial t} - \frac{\partial L^*}{\partial q_r} - \Lambda_r^* - Q_r^*\right) + A_l^r\left(\frac{\partial^3 L^*}{\partial\dot{q}_r \partial q_k \partial\dot{q}_s}\dot{q}_k + \frac{\partial^2 L^*}{\partial\dot{q}_r \partial q_s} + \frac{\partial^3 L^*}{\partial\dot{q}_r \partial t \partial\dot{q}_s} - \frac{\partial^2 L^*}{\partial q_r \partial\dot{q}_s} - \frac{\partial^2 \Lambda_r^*}{\partial\dot{q}_s} - \frac{\partial^2 Q_r^*}{\partial\dot{q}_s}\right) \tag{18}$$

联立(6)式和(9)式得

$$-W_{rk}\ddot{q}_k = \frac{\partial^2 L^*}{\partial\dot{q}_r \partial q_k}\dot{q}_k + \frac{\partial^2 L^*}{\partial\dot{q}_r \partial t} - \frac{\partial L^*}{\partial q_r} - Q_r^* - \Lambda_r^* \tag{19}$$

将(19)式代入(18)式得

$$\frac{\partial \bar{W}_{ls}}{\partial q_k}\dot{q}_k + \frac{\partial \bar{W}_{ls}}{\partial t} + \frac{\partial^2 \bar{L}^*}{\partial\dot{q}_l \partial q_s} - \frac{\partial^2 \bar{L}^*}{\partial q_l \partial\dot{q}_s} - \frac{\partial \bar{Q}_l^*}{\partial\dot{q}_s} - \frac{\partial \bar{\Lambda}_l^*}{\partial\dot{q}_s} = -\frac{\partial A_l^r}{\partial\dot{q}_s}W_{rk}\ddot{q}_k + A_l^r\frac{\partial W_{rs}}{\partial q_k}\dot{q}_k + A_l^r\left(\frac{\partial W_{rs}}{\partial t} + \frac{\partial^2 L^*}{\partial\dot{q}_r \partial q_s} - \frac{\partial^2 L^*}{\partial q_r \partial\dot{q}_s} - \frac{\partial \Lambda_r^*}{\partial\dot{q}_s} - \frac{\partial Q_r^*}{\partial\dot{q}_s}\right) \tag{20}$$

对 $A_l^r W_{rs}$ 求关于 $\dot{q}_k$ 的偏导数

$$\frac{\partial(A_l^r W_{rs})}{\partial\dot{q}_k} = \frac{\partial(\bar{W}_{ls}U^{sr}W_{rs})}{\partial\dot{q}_k} = \frac{\partial^3 \bar{L}^*}{\partial\dot{q}_l \partial\dot{q}_s \partial\dot{q}_k} = \frac{\partial A_l^r}{\partial\dot{q}_k}W_{rs} + A_l^r\frac{\partial W_{rs}}{\partial\dot{q}_k} \tag{21}$$

即

$$\frac{\partial A_l^r}{\partial\dot{q}_k}W_{rs} = \frac{\partial \bar{W}_{lk}}{\partial\dot{q}_s} - A_l^r\frac{\partial W_{rk}}{\partial\dot{q}_s} \tag{22}$$

由(16)式得

$$\bar{W}_{lk} = A_l^r W_{rk} \tag{23}$$

求(23)式关于 $\dot{q}_s$ 的偏导数得

$$\frac{\partial \bar{W}_{lk}}{\partial\dot{q}_s} = \frac{\partial A_l^r}{\partial\dot{q}_s}W_{rk} + A_l^r\frac{\partial W_{rk}}{\partial\dot{q}_s} \tag{24}$$

由(22)和(24)式得

$$\frac{\partial A_l^r}{\partial \dot{q}_k}W_{rs}=\frac{\partial \bar{W}_{lk}}{\partial \dot{q}_s}-A_l^r\frac{\partial W_{rk}}{\partial \dot{q}_s}=\frac{\partial A_l^r}{\partial \dot{q}_s}W_{rk} \tag{25}$$

根据(16)式有

$$\frac{\partial \bar{W}_{ls}}{\partial q_k}=\frac{\partial(\bar{W}_{ls}U^{sr}W_{rs})}{\partial q_k}=\frac{\partial(A_l^rW_{rs})}{\partial q_k}=\frac{\partial A_l^r}{\partial q_k}W_{rs}+A_l^r\frac{\partial W_{rs}}{\partial q_k} \tag{26}$$

即

$$-\frac{\partial A_l^r}{\partial q_k}W_{rs}\dot{q}_k=A_l^r\frac{\partial W_{rs}}{\partial q_k}\dot{q}_k-\frac{\partial \bar{W}_{ls}}{\partial q_k}\dot{q}_k \tag{27}$$

把(25)和(27)式代入(20)式得

$$\frac{\partial^2\bar{L}^*}{\partial \dot{q}_l\partial q_s}-\frac{\partial^2\bar{L}^*}{\partial q_l\partial \dot{q}_s}-\frac{\partial \bar{Q}_l^*}{\partial \dot{q}_s}-\frac{\partial \bar{\Lambda}_l^*}{\partial \dot{q}_s}=-\frac{\partial A_l^r}{\partial \dot{q}_k}W_{rs}\ddot{q}_k-\frac{\partial A_l^r}{\partial q_k}W_{rs}\dot{q}_k-\frac{\partial \bar{W}_{ls}}{\partial t}+A_l^r\frac{\partial W_{rs}}{\partial t}+A_l^r\left(\frac{\partial^2L^*}{\partial \dot{q}_r\partial q_s}-\frac{\partial^2L^*}{\partial q_r\partial \dot{q}_s}-\frac{\partial \Lambda_r^*}{\partial \dot{q}_s}-\frac{\partial Q_r^*}{\partial \dot{q}_s}\right) \tag{28}$$

即

$$\frac{\partial^2\bar{L}^*}{\partial \dot{q}_l\partial q_s}-\frac{\partial^2\bar{L}^*}{\partial q_l\partial \dot{q}_s}-\frac{\partial \bar{Q}_l^*}{\partial \dot{q}_s}-\frac{\partial \bar{\Lambda}_l^*}{\partial \dot{q}_s}=-\frac{dA_l^r}{dt}W_{rs}+\frac{\partial A_l^r}{\partial t}W_{rs}-\frac{\partial \bar{W}_{ls}}{\partial t}+A_l^r\frac{\partial W_{rs}}{\partial t}+A_l^r\left(\frac{\partial^2L^*}{\partial \dot{q}_r\partial q_s}-\frac{\partial^2L^*}{\partial q_r\partial \dot{q}_s}-\frac{\partial \Lambda_r^*}{\partial \dot{q}_s}-\frac{\partial Q_r^*}{\partial \dot{q}_s}\right) \tag{29}$$

由式(27)得

$$A_l^r\frac{\partial W_{rs}}{\partial t}-\frac{\partial \bar{W}_{ls}}{\partial t}+\frac{\partial A_l^r}{\partial t}W_{rs}=0 \tag{30}$$

将(30)式代入(29)式得

$$\frac{\partial^2\bar{L}^*}{\partial \dot{q}_l\partial q_s}-\frac{\partial^2\bar{L}^*}{\partial q_l\partial \dot{q}_s}-\frac{\partial \bar{Q}_l^*}{\partial \dot{q}_s}-\frac{\partial \bar{\Lambda}_l^*}{\partial \dot{q}_s}=-\frac{dA_l^r}{dt}W_{rs}+A_l^r\left(\frac{\partial^2L^*}{\partial \dot{q}_r\partial q_s}-\frac{\partial^2L^*}{\partial q_r\partial \dot{q}_s}-\frac{\partial \Lambda_l^*}{\partial \dot{q}_s}-\frac{\partial Q_l^*}{\partial \dot{q}_s}\right) \tag{31}$$

定义矩阵 $T,\bar{T},A,U,W,\bar{W}$ 其元素分别为

$$T=\frac{\partial^2L^*}{\partial \dot{q}_r\partial q_s}-\frac{\partial^2L^*}{\partial q_r\partial \dot{q}_s},\bar{T}=\frac{\partial^2\bar{L}^*}{\partial \dot{q}_r\partial q_s}-\frac{\partial^2\bar{L}^*}{\partial q_r\partial \dot{q}_s}, \tag{32}$$

$$A=(A_s^r),U=(U^{sk}),W=(W_{rs}),\bar{W}=(\bar{W}_{rs}) \tag{33}$$

将条件(14),(32)和(33)式代入(31)式,得

$$\dot{A}=-\bar{T}U+ATU \tag{34}$$

因为 T 和 $\bar{T}$ 为反对称矩阵,U 和 $\bar{W}$ 为对称矩阵,因此对于任意正整数 m 有

$$\dot{A}(A)^{m-1}=(-\bar{T}U+\bar{W}UTU)(A)^{m-1}=-\bar{T}U(\bar{W}U)^{m-1}+\bar{W}UTU(\bar{W}U)^{m-1} \tag{35}$$

根据矩阵 $T,\bar{T},U$ 和 $\bar{W}$ 的特性及矩阵迹的性质得

$$tr[\bar{T}U(\bar{W}U)^{m-1}]=0,\quad tr[\bar{W}UTU(\bar{W}U)^{m-1}]=0 \tag{36}$$

即

$$tr[A(A)^{m-1}]=0 \tag{37}$$

即

$$\frac{d}{dt}[tr(A)^m]=0 \tag{38}$$

可得(15)式,命题得证.

推论 1: 对于相对论完整系统,如果广义力满足

$$\frac{\partial}{\partial \dot{q}_l}\bar{Q}_s=A_s^r\frac{\partial}{\partial \dot{q}_l}Q_r \tag{39}$$

则系统的 Lagrange 对称性可以导致守恒量(15).

推论 2: 对于相对论 Lagrange 系统,如果系统的广义有势力 $\bar{Q}_{sv}$ 和 Q_{rv} 满足

$$\frac{\partial}{\partial \dot{q}_l}\bar{Q}_{sv}=A_s^r\frac{\partial}{\partial \dot{q}_l}Q_{rv} \tag{40}$$

则由系统的 Lagrange 对称性可以导致守恒量(15).

4　算例

为了验证以上推导,给出以下算例. 假设一系统的相对论性广义动能为

$$L^*=m_0c^2\left(1-\sqrt{1-\frac{\dot{q}_1^2+\dot{q}_2^2}{c^2}}\right), \tag{41}$$

约束方程为

$$f=\dot{q}_2-t\dot{q}_1=0 \tag{42}$$

广义力

$$Q_1^*=Q_2^*=0 \tag{43}$$

则由(4)式和(6)式得

$$L_1^*=\frac{d}{dt}\frac{\partial}{\partial \dot{q}_1}L^*-\frac{\partial L^*}{\partial q_1}-Q_1^*-\Lambda_1^*=m_0\frac{\ddot{q}_1}{\sqrt{1-\frac{\dot{q}_1^2+\dot{q}_2^2}{c^2}}}+m_0\dot{q}_1\frac{(\dot{q}_1\ddot{q}_1+\dot{q}_2\ddot{q}_2)/c^2}{[1-(\dot{q}_1^2+\dot{q}_2^2)/c^2]^{\frac{3}{2}}}-Q_1^*-\lambda\frac{\partial f}{\partial \dot{q}_1} \tag{44}$$

$$L_2^*=\frac{d}{dt}\frac{\partial}{\partial \dot{q}_2}L^*-\frac{\partial L^*}{\partial q_2}-Q_2^*-\Lambda_2^*=m_0\frac{\ddot{q}_2}{\sqrt{1-\frac{\dot{q}_1^2+\dot{q}_2^2}{c^2}}}+m_0\dot{q}_2\frac{(\dot{q}_1\ddot{q}_1+\dot{q}_2\ddot{q}_2)/c^2}{[1-(\dot{q}_1^2+\dot{q}_2^2)/c^2]^{\frac{3}{2}}}-$$

$$Q_2^* - \lambda \frac{\partial f}{\partial \dot{q}_2} \tag{45}$$

由(3),(42)-(45)式得

$$\lambda = \frac{m_0}{1+t^2} \frac{\dot{q}_1}{\sqrt{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2}} \tag{46}$$

即

$$\Lambda_1^* = -\frac{m_0 t}{1+t^2} \frac{\dot{q}_1}{\sqrt{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2}} \tag{47}$$

$$\Lambda_2^* = \frac{m_0}{1+t^2} \frac{\dot{q}_1}{\sqrt{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2}} \tag{48}$$

将(43)式,(47)和(48)式分别代入(44)式和(45)式得

$$L_1^* = m_0 \frac{\ddot{q}_1}{\sqrt{1-\frac{\dot{q}_1^2+\dot{q}_2^2}{c^2}}} + m_0\dot{q}_1 \frac{(\dot{q}_1\ddot{q}_1+\dot{q}_2\ddot{q}_2)/c^2}{[1-(\dot{q}_1^2+\dot{q}_2^2)/c^2]^{\frac{3}{2}}} + \frac{m_0 t}{1+t^2} \frac{\dot{q}_1}{\sqrt{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2}} \tag{49}$$

$$L_2^* = m_0 \frac{\ddot{q}_2}{\sqrt{1-\frac{\dot{q}_1^2+\dot{q}_2^2}{c^2}}} + m_0\dot{q}_2 \frac{(\dot{q}_1\ddot{q}_1+\dot{q}_2\ddot{q}_2)/c^2}{[1-(\dot{q}_1^2+\dot{q}_2^2)/c^2]^{\frac{3}{2}}} - \frac{m_0}{1+t^2} \frac{\dot{q}_1}{\sqrt{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2}} \tag{50}$$

若有另一相对论性非完整学系统

$$\bar{Q}_1^* + \bar{\Lambda}_1^* = \frac{m_0 t}{1+t^2} \frac{\dot{q}_1^2}{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2} \tag{51}$$

$$\bar{Q}_2^* + \bar{\Lambda}_2^* = -\frac{m_0}{1+t^2} \frac{\dot{q}_1^2}{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2} \tag{52}$$

$$\bar{L}_1^* = -2m_0 \frac{\dot{q}_1\ddot{q}_1}{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2} - 2m_0\dot{q}_1^2 \frac{(\dot{q}_1\ddot{q}_1+\dot{q}_2\ddot{q}_2)/c^2}{[1-(\dot{q}_1^2+\dot{q}_2^2)/c^2]^2} - \frac{2m_0}{1+t^2} \frac{\dot{q}_1^2}{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2} = -\frac{2\dot{q}_1}{\sqrt{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2}} L_1^* \tag{53}$$

$$\bar{L}_2^* = -2m_0 \frac{\dot{q}_1\ddot{q}_1}{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2} - 2m_0\dot{q}_1\dot{q}_2 \frac{(\dot{q}_1\ddot{q}_1+\dot{q}_2\ddot{q}_2)/c^2}{[1-(\dot{q}_1^2+\dot{q}_2^2)/c^2]^2} - \frac{2m_0 t}{1+t^2} \frac{\dot{q}_1^2}{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2} = -\frac{2\dot{q}_1}{\sqrt{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2}} L_2^* \tag{54}$$

即

$$A_1^1 = A_2^2 = -\frac{2\dot{q}_1}{\sqrt{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2}} \tag{55}$$

由(43)式,(47)式,(48)式,(51)式和(52)式可得 $\bar{\boldsymbol{Q}}^*$,$\boldsymbol{Q}^*$ 和 $\bar{\boldsymbol{\Lambda}}^*$,$\boldsymbol{\Lambda}^*$ 满足条件(14),即

$$\frac{\partial}{\partial \dot{q}_1}(\bar{Q}_1^* + \bar{\Lambda}_1) = A_1^1 \frac{\partial}{\partial \dot{q}_1}(Q_1^* + \Lambda_1) \tag{56}$$

$$\frac{\partial}{\partial \dot{q}_2}(\bar{Q}_2^* + \bar{\Lambda}_2) = A_2^2 \frac{\partial}{\partial \dot{q}_2}(Q_2^* + \Lambda_2) \tag{57}$$

故由命题得

$$I_L = -\frac{4\dot{q}_1}{\sqrt{1-(\dot{q}_1^2+\dot{q}_2^2)/c^2}} = \text{const.} \tag{58}$$

5 小结

本文研究了相对论性非完整约束力学系统的 Lagrange 对称性理论,得到了相对论性非完整系统的 Lagrange 对称性定义和判据,给出了系统 Lagrange 对称性导致守恒量的条件和守恒量的形式.本文将 Lagrange 对称性理论的研究范畴扩展到相对论力学系统领域,对 Lagrange 对称性的完善和系统具有理论意义.当质点运动速度远小于光速时,本文结论将回归到文献[17]的结果.

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SYMMETRY AND CONSERVED QUANTITY OF LAGRANGIANS FOR RELATIVISTIC NONHOLONOMIC SYSTEM *

Zhang Bin[1†] Fang Jianhui[1] Zhang Dongai[2] Xu Ruili[1] Liu Xuefeng[1]

(1. *College of Science*, *China University of Petroleum* (*East China*), *Qingdao* 266580, *China*)

(2. *Dagou Middle School of Huining County in Gansu Province*, *Baiyin* 730721, *China*)

Abstract In this paper, we are study the symmetry of Lagrangians and the conserved quantities for a nonholonomic relativistic system. The Criterion of the symmetry for a nonholonomic relativistic system is given. Then the conditions under which there exist a conserved quantity and the form of the conserved quantity are obtained. And finally there is an example to illustrate the application of the results.

Key words relativisitc, nonholonomic system, symmetry of Lagrangians, conserved quantity

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† Corresponding author E-mail:teamup@ yeah. net