可伸缩复合材料悬臂梁的非线性动力学建模及分析*

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摘要　首先,根据 Reddy 给出的考虑高阶剪切效应的层合理论,气动弹性活塞理论,利用 Hamilton 原理,对考虑采用基于活塞理论的一阶非线性气动力和面内参数激励的联合作用下的轴向可伸缩复合材料悬臂梁进行非线性动力学进行建模,得到其偏微分动力学控制方程. 然后对控制方程无量纲化,利用 Galerkin 方法对控制方程进行了截断,得到三个可反映可伸缩悬臂梁横向振动的无量纲形式的常微分非线性动力学方程,只要选取适合的复合材料及其相关参数,使用数值方法就对模型在外伸和回收过程中的相关振动特性进行了分析.

关键词　可伸缩梁，　复合材料悬臂梁，　高阶剪切理论，　一阶活塞理论，　非线性动力学

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引言

轴向可伸缩机翼是可变体机翼中的一种,它通过变展弦比来实现机翼变形以适应不同的飞行环境. 和传统机翼相比,可以进行光滑无缝变形的轴向可伸缩机翼机具有以下优点：

1)、更大限度的提升飞行包络线,更好的适应具体的飞行环境.

2)、变形的灵活性及变形的可控制性,可使飞行器的飞行性能获得最大限度的提升.

3)、可以降低飞行阻力,提高飞行距离.

但是,人们尚未对轴向可伸缩机翼伸缩过程中的动力学特性完全清楚. 一方面,机构的轴向伸缩运动极易诱发其产生横向振动及失稳. 另一方面,系统常处于高速的工作环境中,在外界扰动下将不可避免的带来非线性振动问题. 因此,在轴向可伸缩机翼设计过程中,考虑并分析其伸缩过程的结构非线性以及空气动力的非线性特性对机翼振动的影响在工程应用中具有非常重要的价值.

由于轴向可伸缩机翼所具有的优势,近年来,轴向可伸缩机翼的研究受到越来越多学者关注. 前苏联的 Bakashaev 设计了一架伸缩翼飞机[1],命名为 RK. 其伸缩部分外伸长度可达到未伸展时翼展长度的 2/3. Gevers 飞机公司申请了一个伸缩翼的专利[2]. 该机翼的翼展可以改变 100%. 雷声公司提出“压缩机翼”方案[3]. 弗吉尼亚理工大学(Virginia Tech)设计了可变后掠角的伸缩翼飞行器[4]. 孙麟、张伟对可变体机翼的非线性动力学进行了建模、理论分析与数值模拟研究[5]. 张谦、张伟设计、制作了轴向可伸缩机翼结构,并对其振动情况进行实验研究[6]. 关于可伸缩翼研究主要集中在对它进行设计、制作和试飞. 理论研究相对较少.

在研究中,可伸缩机翼可简化为可伸缩系统进行研究,对于可伸缩系统,一般可简化为可伸缩梁模型进行研究,Tabarrok 等人[7]对轴向运动悬臂梁的动力学行为进行了研究,推导其动力学方程,得到四个非线性偏微分方程和一个代数方程. 1995 年,Yuji Matsuzaki 等人[8]分别从理论分析和实验的角度对悬臂梁的外伸和回收过程中的振动进行了研究. 李山虎等人[9]对轴向运动悬臂梁的独立模态振动控制进行了研究. 王亮等人[10]使用哈密顿原理建立了端部带主动振子和跨内含有主动控制力的轴向运动悬臂梁的振动方程.

现有的研究,大多只针对伸出部分进行研究.

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由于可伸缩机翼结构较一般伸缩系统复杂,外伸部分也会对固定部分的振动产生影响,同时,目前机翼的选材上,也大多使用复合材料. 因而,针对可伸缩机翼,本文将以其视为实际工程背景,以在一定速度下运动的复合材料悬臂梁作为其力学模型,利用 Hamilton 原理,运用 Galerkin 方法即可得到该模型常微分形式的非线性动力学方程,然后只要选取适合的复合材料及其相关参数,使用数值方法就可对模型的在外伸和收缩时的相关特性进行分析.

1　可伸缩复合材料悬臂梁动力学方程的建立

图 1 所示为一矩形截面复合材料层合梁,该梁左侧固定,右端自由. 梁的初始长度为 l_0,总厚度为 h,$u_0(x(t),t)$ 为梁中面沿 x 轴正方向的位移,$w_0(x(t),t)$ 为梁的中面横向位移,即沿 z 轴正方向位移,梁的瞬时长度 $l(t)$ 为

$$l(t) = l_0 + \int_0^l \frac{dl(t)}{dt}\mathrm{d}t,$$

其中,$\frac{dl(t)}{dt}=\frac{dx(t)}{dt}$,梁的外伸回收速度为$\frac{dl(t)}{dt}$,梁的中面横向速度为

$$\frac{dw_0(x(t),t)}{dt}=\frac{\partial w_0}{\partial t}+\frac{\partial w_0}{\partial x}\frac{\partial x(t)}{\partial t},$$

梁在外伸和收缩过程中受到结构阻尼 $F=-c\frac{dw_0}{dt}$ 和一阶活塞气动弹性力 $\Delta p=-\frac{4q_d\gamma}{M_\infty}\frac{1}{V_{air}}\frac{dw_0}{dt}$ 的作用. 其中,q_d 为动压,具体形式为 $q_d=\frac{1}{2}\rho_{air}V_{air}^2$,$\rho_{air}$ 单位体积气流密度,V_{air} 为超音速气流流速,$\gamma=\frac{M_\infty}{\sqrt{M_\infty^2-1}}$为修正系数,$M_\infty$ 为马赫数.

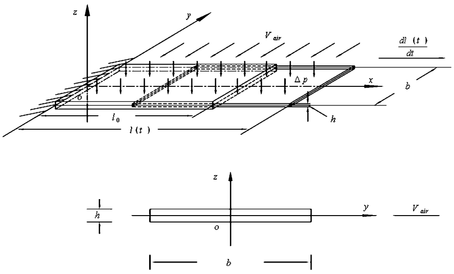

图 1　悬臂梁模型

Fig. 1　The model of cantilever beam

考虑模型做大变形振动,利用 Reddy 的高阶剪切变形理论,几何方程为

$$u_1(x,z,t)=u_0(x,t)+z\phi(x,t)-z^3c_1\left(\phi_x+\frac{\partial w_0(x,t)}{\partial x}\right) \tag{1a}$$

$$w(x,t)=w_0(x,t) \tag{1b}$$

其中,$x=x(t)$,$c_1=\frac{4}{3h^2}$;$\phi_x(x,t)$ 为梁弯曲引起的转角.

在图一所建坐标系下,该层合梁上任意一点的坐标可以表示如下:

$$\vec{R}=\left(x+u_0+z\phi_x-z^3c_1\left(\phi_x+\frac{\partial w_0}{\partial x}\right)\right)\vec{i}+(z+w_0)\vec{k} \tag{2}$$

x,z 是梁上任意一点在笛卡尔坐标系下的表示.

应变与位移关系考虑为几何非线性

$$\varepsilon_{xx}=\frac{\partial w_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{3a}$$

$$\gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x} \tag{3b}$$

根据 Hamilton 原理,得到系统广义力形式动力学方程:

$$\frac{dN_x}{dx}=I_0\frac{d^2x}{dt^2}+I_0\frac{d^2u_0}{dt^2}+(I_1-c_1I_3)\frac{d^2\phi_x}{dt^2}-c_1I_3\frac{\partial^3w_0}{\partial x\partial t^2} \tag{4a}$$

$$\frac{\partial w_0}{\partial x}N_{xx,x}+N_{xx}\frac{\partial^2w_0}{\partial x^2}+Q_{x,x}+c_1P_{xx,xx}-c_2R_{x,x}+(\Delta P-F)=I_0\ddot{w}_0+c_1I_3\frac{\partial\ddot{u}_0}{\partial x}+c_1(I_4-c_1I_6)\frac{\partial\ddot{\phi}_x}{\partial x}-c_1^2I_6\frac{\partial^2\ddot{w}_0}{\partial x^2} \tag{4b}$$

$$M_{xx,x}-c_1P_{xx,x}-Q_x+c_2R_x=(I_1-c_1I_3)\ddot{u}_0+(I_2-2c_1I_4+c_1^2I_6)\ddot{\phi}_x-c_1(I_4-c_1I_6)\frac{\partial\ddot{w}_0}{\partial x} \tag{4c}$$

根据系统中广义力和广义位移的关系,对应系统广义位移形式的偏微分动力学方程可表示为

$$-I_0\frac{d^2x}{dt^2}-I_0\frac{d^2u_0}{dt^2}+A_{11}\frac{\partial u_0}{\partial x^2}+A_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x^2}=0 \tag{5a}$$

$$(D_{11}-2c_1F_{11}+c_1^2H_{11})\frac{\partial^2\phi_x}{\partial x^2}+(c_1^2H_{11}-c_1F_{11})\frac{\partial^3w_0}{\partial x^3}-(A_{55}-2c_2D_{55}+c_2^2F_{55})\left(\phi_x+\frac{\partial w_0}{\partial x}\right)=J_1\frac{d^2u}{dt^2}+K_2\frac{d^2\phi_x}{dt^2}-c_1J_4\frac{\partial^3w_0}{\partial x\partial t^2} \tag{5b}$$

$$A_{11}\frac{\partial^2 u_0}{\partial x^2}\frac{\partial w_0}{\partial x}+A_{11}\frac{\partial u_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}+\frac{3}{2}A_{11}(\frac{\partial w_0}{\partial x})^2\frac{\partial^2 w_0}{\partial x^2}+(c_1F_{11}-c_1^2H_{11})\frac{\partial^3\phi_x}{\partial x^3}-c_1^2H_{11}\frac{\partial^4 w_0}{\partial x^4}+(A_{55}-6c_1D_{55}+9c_1^2F_{55})(\frac{\partial\phi_x}{\partial x}+\frac{\partial^2 w_0}{\partial x^2})+(\Delta P-F)=I_0\frac{d^2w_0}{dt^2}-c_1^2I_6\frac{\partial^4 w_0}{\partial x^2\partial t^2}+c_1I_3\frac{\partial^3 u_0}{\partial x\partial t^2}+c_1J_4\frac{\partial^3\phi_x}{\partial x\partial t^2} \tag{5c}$$

其中,$c_1=\frac{4}{3h^2}$,c 为横向振动阻尼系数,

$$(A_{11},B_{11},D_{11},E_{11},F_{11},H_{11})=\sum_{k=1}^{N}\int_{z_k}^{zk+1}\overline{Q}_{11}^k(1,z,z^2,z^3,z^4,z^6)\mathrm{d}z$$

$$(A_{55},D_{55},F_{55})=\sum_{k=1}^{N}\int_{z_k}^{zk+1}\overline{Q}_{13}^k(1,z^2,z^4)\mathrm{d}z$$

$$I_i=\sum_{k=1}^{N}\int_{z_k}^{zk+1}\rho_k(z)^i\mathrm{d}z$$

$$J_i=I_i-c_1I_{i+2},K_2=I_2-2c_1I_4+c_1^2I_6$$

将(5a)式求解可得

$$\frac{\partial^2 u_0}{\partial x}=-\frac{1}{2}\left(\frac{\partial\phi_x}{\partial x^2}\right)+\frac{1}{2l}\int_0^1\left(\frac{\partial w_0}{\partial x}\right)^2\mathrm{d}x+\frac{I_0}{A_{11}}\frac{d^2x}{dt^2}(x-\frac{l}{2}) \tag{6}$$

代入(5c),同时综合(5b)可得

$$(D_{11}-2c_1F_{11}+c_1^2H_{11})\frac{\partial\phi_x}{\partial x^2}+(c_1^2H_{11}-c_1F_{11})\frac{\partial^3 w_0}{\partial x^3}-(A_{55}-2c_2D_{55}+c_2^2F_{55})(\phi_x+\frac{\partial w_0}{\partial x})=J_1\frac{d^2u}{dt^2}+K_2\frac{d^2\phi_x}{dt^2}-c_1J_4\frac{\partial^3 w_0}{\partial x\partial t^2} \tag{7a}$$

$$I_0\frac{d^2l}{dt^2}\frac{\partial w_0}{\partial x}+A_{11}\frac{1}{2l}\int_0^l\left(\frac{\partial w_0}{\partial x}\right)^2\mathrm{d}x\frac{\partial^2 w_0}{\partial x^2}+I_0\frac{d^2l}{dt^2}(x-\frac{l}{2})\frac{\partial^2 w_0}{\partial x^2}+(c_1F_{11}-c_1^2H_{11})\frac{\partial^3\phi_x}{\partial x^3}-c_1^2H_{11}\frac{\partial^4 w_0}{\partial x^4}+(A_{55}-6c_1D_{55}+9c_1^2F_{55})(\frac{\partial\phi_x}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}) \tag{7b}$$

引入以下无量刚变量

$$u_0\to\frac{u_0}{l},w_0\to\frac{w_0}{h},x\to\frac{x}{l},l\to\frac{l}{h},\phi_x\to\phi_x,$$

$$I_0\to\frac{I_0}{\rho h},J_1\to\frac{J_1}{\rho h^2},c\to\frac{ch\tau}{Q_{11}},c_1I_3\to\frac{c_1I_3}{\rho h^2},$$

$$c_1J_4\to\frac{c_2J_4}{\rho h^3},c_1^2I_6\to\frac{c_1^2I_6}{\rho h^3},K_2\to\frac{K_2}{\rho h^3},A_{11}\to\frac{A_{11}}{Q_{11}h},$$

$$D_{11}\to\frac{D_{11}}{Q_{11}h^3},F_{11}\to\frac{F_{11}}{Q_{11}h^5},H_{11}\to\frac{H_{11}}{Q_{11}h^7},A_{55}\to\frac{A_{55}}{Q_{13}h},$$

$$D_{55}\to\frac{D_{55}}{Q_{13}h^3},F_{55}\to\frac{F_{55}}{Q_{13}h^5},t\to\sqrt{\frac{Q_{11}I_{yz}}{I_0bl_0^4}}\to\tau t$$

将无量纲变量代入(6)式,可得无量纲形式的动力学方程如下:

$$\frac{\partial^2\phi_x}{\partial x^2}+a_2\frac{\partial^3 w_0}{\partial x^3}-a_2\phi_x-a_3\frac{\partial w_0}{\partial x}=a_4\frac{d^2\phi_x}{dt^2}-a_5\frac{\partial^3 w_0}{\partial x\partial t^2} \tag{8a}$$

$$b_1\frac{\partial w_0}{\partial x}\frac{d^2l}{dt^2}+b_2\int_0^1(\frac{\partial w_0}{\partial x})^2dx\frac{\partial^2 w_0}{\partial x^2}+b_3\frac{d^2l}{dt^2}(x-\frac{1}{2})\frac{\partial^2 w_0}{\partial x^2}+b_4\frac{\partial^3\phi_x}{\partial x^3}-b_5\frac{\partial^4 w_0}{\partial x^2}+b_6\frac{\partial\phi_x}{\partial x}+b_7\frac{\partial^2 w_0}{\partial x^2}+b_8\frac{\partial w_0}{\partial t}+b_9\frac{\partial w_0}{\partial x}+b_{10}(\frac{\partial w}{\partial t})^3+b_{11}(\frac{\partial w}{\partial t})^2(\frac{\partial w}{\partial x})+b_{12}(\frac{\partial w}{\partial t})(\frac{\partial w}{\partial x})^2+b_{13}(\frac{\partial w}{\partial x})^3=b_{14}\frac{d^2w_0}{dt^2}-b\,15\frac{\partial^4 w_0}{\partial x^2\partial t^2}+b_{16}\frac{\partial^3\phi_x}{\partial x\partial t^2} \tag{8b}$$

2 Galerkin 离散

为了后续研究和分析,需要将偏微分方程(7)离散为常微分方程,在这里直接使用 Galerkin 原理将(7)进行离散. 设横向位移的模态函数为

$$X_n(x)=[\cosh(\lambda_nx)-\cos(\lambda_nx)]-\sigma_n[\sinh(\lambda_nx)-\sin(\lambda_nx)] \tag{9}$$

转角 ϕ_x 的模态函数为

$$Y_n(x)=\sin(\frac{1}{2}(2n-1)\pi x) \tag{10}$$

其中

$$\sigma_n=\frac{(\cosh(\lambda_n)+\cos(\lambda_n))}{(\sinh(\lambda_n)+\sin(\lambda_n))},$$

λ_n 则是由超越方程 $1+\cosh\lambda_n\cos\lambda_n=0$ 的根给出的长期项

$$\lambda_n=\begin{cases}1.875 & n=1\\4.694 & n=2\\7.855 & n=3\\(n-0.5)\pi & n\geqslant 4\end{cases}.$$

设轴向外伸梁横向位移的形式

$$w_0\sum_{n=1}^{n}X_n(x)w_n(t) \tag{11}$$

转角 ϕ_x 的表达形式为

$$\phi_x = \sum_{n=1}^{n} Y_n(x)\gamma_n(t) \tag{12}$$

对(8a)做一阶 Galerkin 截断,对(8b)做二阶 Galerkin 截断,利用三角函数的正交性,得到常微分形式的非线性动力学方程为

$$c_1\ddot{\gamma}_1 + c_2\gamma_1 + c_3 w_1 + c_4 w_2 + c_5\dot{w}_1 + c_6\dot{w}_2 + c_7\ddot{w}_1 + c_8\ddot{w}_2 = 0 \tag{13a}$$

$$\begin{aligned} &d_1 w_1 + d_2 w_2 + d_3 w_1^2 w_2 + d_4 w_1 w_2^2 + d_5 w_1^3 + d_6 w_2^3 + \\ &d_7\dot{w}_1 + d_8\dot{w}_2 + d_9\ddot{w}_1 + d_{10}\ddot{w}_2 + d_{11}\dot{w}_1^3 + d_{12}\dot{w}_2^3 + \\ &d_{13}w_1\dot{w}_1^2 + d_{14}w_1\dot{w}_2^2 + d_{15}w_1^2\dot{w}_1 + d_{16}w_1^2\dot{w}_2 + \\ &d_{17}w_2\dot{w}_2^2 + d_{18}w_2\dot{w}_1^2 + d_{19}w_2^2\dot{w}_2 + d_{20}\dot{w}_1 w_2^2 + \\ &d_{21}\dot{w}_1^2\dot{w}_2 + d_{22}\dot{w}_1\dot{w}_2^2 + d_{23}w_1 w_2\dot{w}_2 + \\ &d_{24}w_1\dot{w}_2\dot{w}_2 + d_{28}\gamma_1 + d_{29}\ddot{\gamma}_1 = 0 \end{aligned} \tag{13b}$$

$$\begin{aligned} &e_1 w_1 + e_2 w_2 + e_3 w_1^2 w_2 + e_4 w_1 w_2^2 + e_5 w_1^3 + e_6 w_2^3 + \\ &e_7\dot{w}_1 + e_8\dot{w}_2 + e_9\ddot{w}_1 + e_{10}\ddot{w}_2 + e_{11}\dot{w}_1^3 + e_{12}\dot{w}_2^3 + \\ &e_{13}w_1\dot{w}_1^2 + e_{14}w_1\dot{w}_2^2 + e_{15}w_1^2\dot{w}_1 + e_{16}w_1^2\dot{w}_2 + \\ &e_{17}w_2\dot{w}_2^2 + e_{18}w_2\dot{w}_1^2 + e_{19}w_2^2\dot{w}_2 + e_{20}\dot{w}_1 w_2^2 + \\ &e_{21}\dot{w}_1^2\dot{w}_2 + e_{22}\dot{w}_1\dot{w}_2^2 + e_{23}w_1 w_2\dot{w}_2 + e_{24}w_1\dot{w}_1 w_2 + \\ &e_{25}\dot{w}_1 w_2\dot{w}_2 + e_{26}w_1\dot{w}_1\dot{w}_2 + e_{27}\dot{w}_1 w_2\dot{w}_2 + \\ &e_{28}\gamma_1 + e_{29}\ddot{\gamma}_1 = 0 \end{aligned} \tag{13c}$$

3　结果分析

取初始长度 $l_0 = 2\text{m}$,宽 $b = 0.2\text{m}$,厚 $h = 0.001\text{m}$,飞行速度 $V_{air} = 865\text{m/s}$,阻尼系数 $c = 300\text{N}\cdot\text{s/m}$ 的等直截面复合材料层合梁进行研究,对应无量纲初始条件选取如下

$$w_1 = 0.1, \dot{w}_1 = 0.01, w_2 = 0.5, \dot{w}_2 = 0.5$$

对应拉伸弹性模量,剪切弹性模量及密度给出如下

$$Q_{11}^{(1)} = Q_{11}^{(3)} = 3.712\text{GPa},$$
$$Q_{11}^{(2)} = 0.841\text{GPa},$$
$$Q_{13}^{(1)} = Q_{13}^{(2)} = Q_{13}^{(3)} = 0.414\text{GPa},$$
$$\rho^{(1)} = \rho^{(2)} = \rho^{(3)} = 1800\text{kg/m}^3,$$

其中 $\rho^{(1)}$,$\rho^{(2)}$ 和 $\rho^{(3)}$ 代表层合梁上,中,下三层材料的密度,$Q_{11}^{(1)}$,$Q_{11}^{(2)}$,$Q_{11}^{(3)}$ 代表机翼上,中,下三层材料的拉伸弹性摸量;$Q_{13}^{(1)}$,$Q_{13}^{(2)}$,$Q_{13}^{(3)}$ 代表机翼上、中、下三层材料的简切弹性摸量.

3.1　伸缩梁由 2 米外伸至 4 米时端点的振动情况

图 2 - 4 表示的是给定相同初速度 $v_0 = 0.001\text{m/s}$,不同加速度时,机翼外伸的振动图象,对应的加速度分别为 $a_1 = 0.001\text{m/s}^2$,$a_2 = 0.002\text{m/s}^2$,$a_3 = 0.004\text{m/s}^2$. 在图 2 - 4 中,梁端点振动的振幅均有增大的趋势,并且加速度越大,振动的振幅越大.

3.2　伸缩梁由 4 米回收至 2 米时端点的振动情况

在梁回收过程中,给定相同初速度 $v_0 = -0.2\text{m/s}$,不同减加速度 $a_1 = 0.001\text{m/s}^2$,$a_2 = 0.002\text{m/s}^2$,$a_3 = 0.003\text{m/s}^2$ 以后,通过数值模拟可得梁端点的振动情况,分别由图 5 - 7 表示. 由图 5 - 7 可以看出,梁端点的振幅在整体上呈下降趋势,将图 5 - 7 进一步对比得出梁在回收过程中,减加速度越大,回收速度减小得越快,机翼的振幅也衰减得越快.

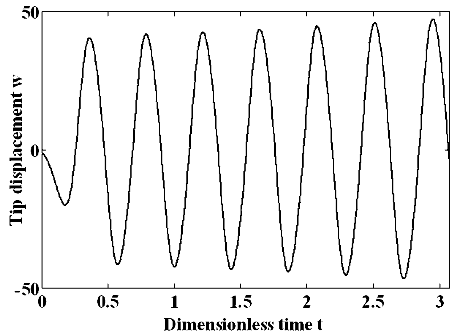

图 2　机翼端点的波形图

Fig. 2　The tip waveform of the wing

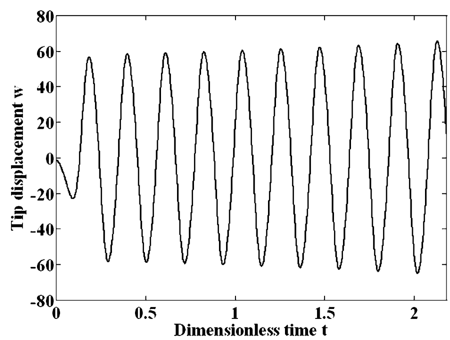

图 3　机翼端点的波形图

Fig. 3　The tip waveform of the wing

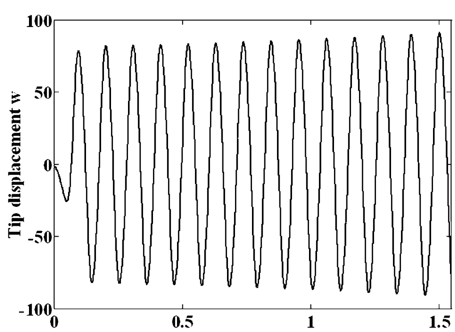

图 4　机翼端点的波形图

Fig. 4　The tip waveform of the wing

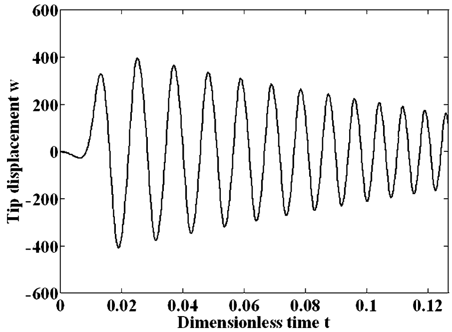

图5 机翼端点的波形图

Fig. 5 The tip waveform of the wing

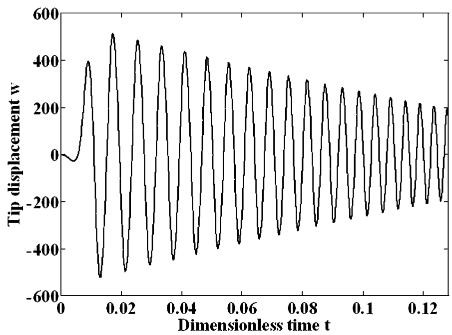

图6 机翼端点的波形图

Fig. 6 The tip waveform of the wing

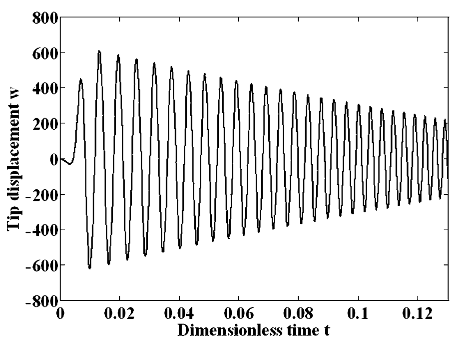

图7 机翼端点的波形图

Fig. 7 The tip waveform of the wing

4 结论

本文以可伸缩机翼为实际工程背景,以在一定速度下运动的复合材料悬臂梁作为其力学模型,建模中考虑高阶剪切效应的层合理论,气动弹性活塞力的影响,引入Hamilton原理对可伸缩机翼进行建模,得到其非线性运动控制方程,为了进一步对其进行数值分析并尽量保证结果的完整性,运用Galerkin方法分别对关于转角的控制方程做一阶Galerkin截断和关于横向位移的控制方程进行了二阶离散,得到常微分形式的非线性动力学方程.然后通过数值模拟得到以下结论:

1)、在梁加速外伸过程中,端点的振幅越来越大,梁的外伸速度对振动的影响较大,外伸速度越大,对机翼振动的影响越大;

2)、在梁减速回收过程中,端点的振幅越来越小,梁的回收速度对振动的影响较大,回收速度越大,对机翼振动的影响越大.

参考文献

1 Tabarrok B, Leech C M, Kim Y I. On the dynamics of an axially moving beam. *Journal of the Franklin Institute*, 1974,297:201 ~ 220

2 Gevers D E. Multi-purpose aircraft. US Patent 1998,5:850 ~ 990

3 Andersen G R, Cowan D L, Piatak D J. Aeroelastic modeling, analysis and testing of a morphing wing structure. 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Honolulu, Hawaii, 2007:359 ~ 373

4 Arrison L, Birocco K, Gaylord C, et al. 2002-2003 AE/ME morphing wing design. Virginia Tech Aerospace Engineering Senior Design Project Spring Semester Final Report, 2003:1 ~ 89

5 Sun L, Zhang W, Cao D X, Yao M H. Dynamcial behaviour of telescoping wing in supersonic airflow. In: The Third International Conference on Dynamics, Vibration and Control , Hangzhou, 2010

6 Zhang W, Chen J E, Cao D X, Zhang Q. Experimental investigation on principal parametric resonance of prebuckling flexible cantilever beam with square section. ASME 2011 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Washington, 2011

7 Tabarrok B, Leech C M, Kim Y I. On the dynamics of an axially moving beam. *Journal of the Franklin Institute*, 1974,297:201 ~ 220

8 Matsuzaki Y, Taki Y, Toyama M. Vibration of a cantilevered beam during deployment and retrieval: analysis and experiment. *Smart Materials and Structures*, 1995,4:334 ~ 339

9 李山虎,杨靖波,黄清华,陈德成. 轴向运动悬臂梁的独立模态振动控制-I 近似理论分析. 应用力学学报, 2002, 19(1): 35 ~ 39 (Li S L, Yang J B, Huang Q H, Chen D C. Independent Model Space Vibration Control of An Axially Moving Cantilever Beam-Part I: Theoretical A-

nalysis of Approximation. *Chinese Journal of Applied Mechanics*, 2002, 19(1): 35 ~39 (in Chinese))

10 王亮, 陈怀海, 贺旭东, 游伟倩, 轴向运动变长度悬臂梁的振动控制, 振动工程学报, 2009,22(6):565 ~570 (Wang L, Chen H H, He X D, You W Q. Vibration control of an axially moving cantilever beam with varying length. *Journal of Vibration Engineering*, 2009,22(6):565 ~570 (in Chinese))

NONLINEAR DYNAMICS MODELING AND NUMERICAL ANALYSIS OF TELESCOPING-AND -TRANSLATING COMPOSITE LAMINATED CANTILEVER BEAM *

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Abstract Firstly, based on the Reddy higher-order shear deformation theory and the pneumatic elastic piston theory, the nonlinear governing equations of motion for an axially moving cantilever beam were established by using the generalized Hamilton's principle, and the first order nonlinear aerodynamic force and parametric excitation in-plane were obtained. After introducing dimensionless variables and parameters, the nonlinear governing equations became dimensionless equations. At last, according to Galerkin's approach, the governing equations of motion were simplified to three ordinary differential nonlinear dynamic equations. As long as the suitable composite material and relevant parameters are given, the relevant vibration characters of the modeling during deployment and retrieval can be analyzed by using numerical method.

Key words telescoping-and-translating beam, composite material laminated beam, higher-order shear deformation theory, pneumatic elastic piston theory, nonlinear dynamic

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