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Chetaev 型非完整系统 Nielsen 方程 Lie 对称性导致的一种守恒量*

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摘要 研究 Chetaev 型非完整系统 Nielsen 方程 Lie 对称性导致的一种守恒量,给出无限小群变换下 Chetaev 型非完整系统 Nielsen 方程 Lie 对称性的确定方程,得到 Chetaev 型非完整系统 Nielsen 方程 Lie 对称性直接导致的一种守恒量及其存在条件,并举例说明结果应用.

关键词 非完整系统, Nielsen 方程, Lie 对称性, 守恒量

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引言

对称性原理是物理学中更高层次的法则[1]. 寻求守恒量是对称性理论的重要用途之一,动力学系统的对称性与守恒量研究在数学、力学和物理学中具有极其重要的理论意义和实际意义. 近代分析力学中寻求约束力学系统守恒量的方法主要有 Noether 对称性[2]、Lie 对称性[3]和 Mei 对称性[4],这三种对称性导致的守恒量主要有 Noether 守恒量、Hojman 守恒量和 Mei 守恒量[5]. 近年来,人们对各类力学系统 Noether 对称性、Lie 对称性和 Mei 对称性导致的 Noether 守恒量、Hojman 守恒量和 Mei 守恒量的研究取得了一系列重要成果[6-14]. 随着其研究的不断发展,寻找新守恒量的研究受到了人们的关注. 文献[15]给出了 Lagrange 系统 Lie 对称性直接导致的一种新守恒量,文献[16]进一步研究了 Lagrange 系统 Noether 对称性和 Mei 对称性导出这种新守恒量,Nucci[17]在文献[16]的基础上对其中算例具有的这种新守恒量做了更细致的研究.

约束动力学系统的各种运动方程可分为三大重要体系:Lagrange 体系、Nielsen 体系和 Appell 体系. 关于 Nielsen 体系中 Nielsen 方程的对称性与守恒量已有许多研究. 王树勇和梅凤翔[18]给出了 Nielsen 方程 Noether 对称性与 Mei 对称性的关系,方建会[19]等研究了非保守力学系统 Nielsen 方程的 Mei 对称性,乔永芬[20]等研究了非完整系统相对论变质量 Nielsen 方程的 Mei 对称性和守恒量,许学军[21]等给出了非保守 Nielsen 方程的 Mei 对称性导致的非 Noether 守恒量,文献[22]研究了相对运动动力学系统 Nielsen 方程 Lie 对称性与 Hojman 守恒量. 然而,Nielsen 方程对称性导致文献[15]给出的新守恒量的研究成果还未见报道. 本文在文献[15]的基础上研究 Chetaev 型非完整系统 Nielsen 方程 Lie 对称性导致的这种新守恒量,给出无限小群变换下 Chetaev 型非完整系统 Nielsen 方程 Lie 对称性的确定方程,得到 Chetaev 型非完整系统 Nielsen 方程 Lie 对称性直接导致的这种新守恒量的形式及其存在条件.

1 Nielsen 方程的 Lie 对称性

设力学系统的位形由 n 个广义坐标 $q_s(s=1,\cdots,n)$ 确定,$L=L(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 为系统的 Lagrange 函数,$Q_s=Q_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 为非势广义力,其运动受到 g 个双面理想 Chetaev 型非完整约束

$$f_\beta(t,\boldsymbol{q},\dot{\boldsymbol{q}})=0,\ (\beta=1,\cdots,g), \tag{1}$$

约束(1)加在虚位移 δq_s 上的 Chetaev 条件为

$$\frac{\partial f_\beta}{\partial \dot{q}_s}\delta q_s=0. \tag{2}$$

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由 D'Alembert-Lagrange 原理和虚位移上的 Chetaev 条件(2),用 Lagrange 乘子法可得系统的 Nielsen 方程为

$$\frac{\partial}{\partial \dot{q}_s}\frac{\mathrm{d}L}{\mathrm{d}t}-2\frac{\partial L}{\partial q_s}=Q_s+\lambda_\beta\frac{\partial f_\beta}{\partial \dot{q}_s}, \tag{3}$$

其中 λ_β 为约束乘子,方程(3)可写为

$$N_s(L)=Q_s+\Lambda_s,\ (s=1,\cdots,n), \tag{4}$$

其中 N_s 为 Nielsen 算子

$$N_s=\frac{\partial}{\partial \dot{q}_s}\frac{\mathrm{d}}{\mathrm{d}t}-2\frac{\partial}{\partial q_s}, \tag{5}$$

Λ_s 为广义非完整约束反力

$$\Lambda_s=\Lambda_s(t,\boldsymbol{q},\dot{\boldsymbol{q}})=\lambda_\beta(t,\boldsymbol{q},\dot{\boldsymbol{q}})\frac{\partial f_\beta}{\partial \dot{q}_s} \tag{6}$$

假设系统(3)非奇异,即

$$\det\left(\frac{\partial^2 L}{\partial \dot{q}_s\partial \dot{q}_k}\right)\neq 0, \tag{7}$$

可求出所有广义加速度,记作

$$\ddot{q}_s=\alpha_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}),\ (s=1,\cdots,n). \tag{8}$$

引入时间和广义坐标的无限小变换

$$\begin{aligned}&t^*=t+\varepsilon\xi_0(t,\boldsymbol{q},\dot{\boldsymbol{q}})\\&q_s^*(t^*)=q_s(t)+\varepsilon\xi_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}),\ (s=1,\cdots,n).\end{aligned} \tag{9}$$

其中 ε 为无限小参数,ξ_0,ξ_s 为无限小变换生成元.

根据力学系统的 Lie 对称性理论[1],非完整系统 Nielsen 方程 Lie 对称性的确定方程为

$$X^{(2)}[N_s(L)]=X^{(1)}(Q_s+\Lambda_s),\ (s=1,\cdots,n). \tag{10}$$

其中

$$X^{(2)}=X^{(1)}+[(\dot{\xi}_s-\dot{q}_s\dot{\xi}_0)-\ddot{q}_s\dot{\xi}_0]\frac{\partial}{\partial \ddot{q}_s}, \tag{11}$$

$$X^{(1)}=X^{(0)}+(\dot{\xi}_s-\dot{q}_s\dot{\xi}_0)\frac{\partial}{\partial \dot{q}_s}, \tag{12}$$

$$X^{(0)}=\xi_0\frac{\partial}{\partial t}+\xi_s\frac{\partial}{\partial q_s}. \tag{13}$$

方程(8)对应的 Lie 对称性的确定方程表示为

$$\ddot{\xi}_s-\dot{q}_s\ddot{\xi}_0-2\dot{\xi}_0\alpha_s=\frac{\partial\alpha_s}{\partial t}\xi_0+\frac{\partial\alpha_s}{\partial q_k}\xi_k+\frac{\partial\alpha_s}{\partial \dot{q}_k}(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0). \tag{14}$$

可以证明 Lie 对称性的确定方程的两种表示(10)和(14)是等价的.

于是,可得到如下判据:

判据1 如果存在无限小生成元 ξ_0,ξ_s 满足确定方程(10)或(14),那么相应不变性为与非完整系统(1),(3)相应的完整系统(4)或(8)的 Lie 对称性.

非完整约束力学方程(1)在无限小变换(9)下的不变性归为如下限制方程

$$X^{(1)}[f_\beta(t,\boldsymbol{q},\dot{\boldsymbol{q}})]-0,\ (\beta=1,\cdots,g). \tag{15}$$

判据2 如果存在无限小生成元 ξ_0,ξ_s 满足确定方程(10)或(14)以及限制方程(15),那么相应不变性为非完整系统(1),(3)的弱 Lie 对称性.

考虑到等时变分与非等时变分有如下关系

$$\delta q_s=\Delta q_s-\dot{q}_s\Delta t=\varepsilon(\xi_s-\dot{q}_s\xi_0), \tag{16}$$

将(16)式代入(2)式,有

$$\frac{\partial f_\beta}{\partial \dot{q}_s}(\xi_s-\dot{q}_s\xi_0)=0, \tag{17}$$

称方程(17)为附加限制方程.

判据3 如果存在无限小生成元 ξ_0,ξ_s 满足确定方程(10)或(14),限制方程(15)以及附加限制方程(17),那么相应不变性为非完整系统(1),(3)的强 Lie 对称性.

2 Nielsen 方程的 Lie 对称性的结构方程和守恒量

已有的研究表明[5],非完整系统的 Lie 对称性可以直接导致 Hojman 守恒量.下面研究非完整系统 Lie 对称性直接导致的一种新守恒量及其存在条件.

命题1 如果非完整系统(1),(3)的 Lie 对称性的生成元 ξ_0,ξ_s 以及规范函数 $G=G(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 满足如下结构方程

$$\begin{aligned}&-\dot{\xi}_0\frac{\partial L}{\partial t}-X^{(1)}\left(\frac{\partial L}{\partial t}\right)+\dot{q}_sX^{(1)}(Q_s+\Lambda_s)+\\&\dot{\xi}_s(Q_s+\Lambda_s)+\dot{G}=0,\end{aligned} \tag{18}$$

则相应完整系统的 Lie 对称性直接导致一种守恒量

$$I=\dot{q}_sX^{(1)}\left(\frac{\partial L}{\partial t}\right)-X^{(0)}(L)+G=\mathrm{const}. \tag{19}$$

证明 求守恒量 I 对时间 t 的导数,得

$$\begin{aligned}\frac{\mathrm{d}I}{\mathrm{d}t}=&\dot{q}_s\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial X^{(1)}(L)}{\partial \dot{q}_s}\right]-\dot{q}_s\left\{\left(\frac{\partial\xi_0}{\partial \dot{q}_s}\right)'\frac{\partial L}{\partial t}+\right.\\&\frac{\partial\xi_0}{\partial \dot{q}_s}\left(\frac{\partial L}{\partial t}\right)'+\left(\frac{\partial\xi_k}{\partial \dot{q}_s}\right)'\frac{\partial L}{\partial q_k}+\frac{\partial\xi_k}{\partial \dot{q}_s}\left(\frac{\partial L}{\partial q_k}\right)'+\\&\left.\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)}{\partial \dot{q}_s}\right]\frac{\partial L}{\partial \dot{q}_k}+\frac{\partial(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)}{\partial \dot{q}_s}\left(\frac{\partial L}{\partial \dot{q}_k}\right)'\right\}+\\&\ddot{q}_sX^{(1)}\left(\frac{\partial L}{\partial \dot{q}_s}\right)-\frac{\mathrm{d}}{\mathrm{d}t}[X^{(0)}(L)]+\dot{G}.\end{aligned} \tag{20}$$

由(12)式易得

$$\begin{aligned}\frac{\partial X^{(1)}(L)}{\partial q_s}&=\frac{\partial}{\partial q_s}\left[\xi_0\frac{\partial L}{\partial t}+\xi_k\frac{\partial L}{\partial q_k}+(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)\frac{\partial L}{\partial \dot{q}_k}\right]\\&=X^{(1)}\left(\frac{\partial L}{\partial q_s}\right)+\left[\frac{\partial\xi_0}{\partial q_s}\frac{\partial L}{\partial t}+\frac{\partial\xi_k}{\partial q_s}\frac{\partial L}{\partial q_k}+\frac{\partial(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)}{\partial q_s}\frac{\partial L}{\partial \dot{q}_k}\right],\end{aligned} \tag{21}$$

结合(20)和(21)式,有

$$\begin{aligned}\frac{\mathrm{d}I}{\mathrm{d}t}=&\dot{q}_s\frac{\mathrm{d}}{\mathrm{d}t}\Big[\frac{\partial X^{(1)}(L)}{\partial\dot{q}_s}\Big]-\dot{q}_s\frac{\partial X^{(1)}(L)}{\partial q_s}-\\&\Big\{\Big(\frac{\partial\xi_0}{\partial\dot{q}_s}\Big)'\frac{\partial L}{\partial t}+\frac{\partial\xi_0}{\partial\dot{q}_s}\Big(\frac{\partial L}{\partial t}\Big)'+\Big(\frac{\partial\xi_k}{\partial\dot{q}_s}\Big)'\frac{\partial L}{\partial q_k}+\\&\frac{\partial\xi_k}{\partial\dot{q}_s}\Big(\frac{\partial L}{\partial q_k}\Big)'+\frac{\mathrm{d}}{\mathrm{d}t}\Big[\frac{\partial(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)}{\partial\dot{q}_s}\Big]\frac{\partial L}{\partial\dot{q}_k}+\\&\frac{\partial(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)}{\partial\dot{q}_s}\Big(\frac{\partial L}{\partial\dot{q}_k}\Big)'\Big\}+\dot{q}_s\frac{\partial X^{(1)}(L)}{\partial q_s}+\\&\ddot{q}_sX^{(1)}\Big(\frac{\partial L}{\partial\dot{q}_s}\Big)-\frac{\mathrm{d}}{\mathrm{d}t}[X^{(0)}(L)]+\dot{G}\\=&\dot{q}_s\Big\{N_s[X^{(1)}(L)]-N_s(\xi_0)\frac{\partial L}{\partial t}-N_s(\xi_k)\frac{\partial L}{\partial q_k}-\\&N_s(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)\frac{\partial L}{\partial\dot{q}_k}-\frac{\partial\xi_0}{\partial\dot{q}_s}\Big(\frac{\partial L}{\partial t}\Big)'-\frac{\partial\xi_k}{\partial\dot{q}_s}\Big(\frac{\partial L}{\partial q_k}\Big)'-\\&\Big(\frac{\partial\dot{\xi}_k}{\partial\dot{q}_s}-\dot{q}_k\frac{\partial\dot{\xi}_0}{\partial\dot{q}_s}\Big)\Big(\frac{\partial L}{\partial\dot{q}_k}\Big)'\Big\}+\dot{\xi}_0\dot{q}_s\Big(\frac{\partial L}{\partial\dot{q}_s}\Big)+\\&\dot{q}_sX^{(1)}\Big(\frac{\partial L}{\partial q_s}\Big)+\ddot{q}_sX^{(1)}\Big(\frac{\partial L}{\partial\dot{q}_s}\Big)-\frac{\mathrm{d}}{\mathrm{d}t}[X^{(0)}(L)]+\dot{G}.\end{aligned}\tag{22}$$

考虑如下关系式

$$\begin{aligned}N_s[X^{(1)}(L)]=&N_s(\xi_0)\frac{\partial L}{\partial t}+\xi_0N_s\Big(\frac{\partial L}{\partial t}\Big)+\dot{\xi}_0\frac{\partial^2L}{\partial t\partial\dot{q}_s}+\\&\frac{\partial\xi_0}{\partial\dot{q}_s}\Big(\frac{\partial L}{\partial t}\Big)'+N_s(\xi_k)\frac{\partial L}{\partial q_k}+\xi_kN_s\Big(\frac{\partial L}{\partial q_k}\Big)+\\&\dot{\xi}_k\frac{\partial^2L}{\partial q_k\partial\dot{q}_s}+\frac{\partial\xi_k}{\partial\dot{q}_s}\Big(\frac{\partial L}{\partial q_k}\Big)'+N_s(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)\frac{\partial L}{\partial\dot{q}_k}+\\&(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)N_s\Big(\frac{\partial L}{\partial\dot{q}_k}\Big)+(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)\frac{\partial^2L}{\partial\dot{q}_k\partial\dot{q}_s}+\\&\Big(\frac{\partial\dot{\xi}_k}{\partial\dot{q}_s}-\dot{q}_k\frac{\partial\dot{\xi}_0}{\partial\dot{q}_s}\Big)\Big(\frac{\partial L}{\partial\dot{q}_k}\Big)'-\dot{\xi}_0\Big(\frac{\partial L}{\partial\dot{q}_s}\Big)',\end{aligned}\tag{23}$$

$$\begin{aligned}X^{(2)}[N_s(L)]=&\xi_0N_s\Big(\frac{\partial L}{\partial t}\Big)+\xi_kN_s\Big(\frac{\partial L}{\partial q_k}\Big)+\\&(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)N_s\Big(\frac{\partial L}{\partial\dot{q}_k}\Big)+(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)\frac{\partial^2L}{\partial\dot{q}_s\partial q_k}+\\&[(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)'-\ddot{q}_k\dot{\xi}_0]\frac{\partial^2L}{\partial\dot{q}_k\partial\dot{q}_s},\end{aligned}\tag{24}$$

由(23)和(24)式得

$$\begin{aligned}X^{(2)}[N_s(L)]=&N_s[X^{(1)}(L)]-N_s(\xi_0)\frac{\partial L}{\partial t}-\\&N_s(\xi_k)\frac{\partial L}{\partial q_k}-N_s(\dot{\xi}_k-\dot{q}_k\dot{\xi}_0)\frac{\partial L}{\partial\dot{q}_k}-\frac{\partial\xi_0}{\partial\dot{q}_s}\Big(\frac{\partial L}{\partial t}\Big)'-\\&\frac{\partial\xi_k}{\partial\dot{q}_s}\Big(\frac{\partial L}{\partial q_k}\Big)'-\Big(\frac{\partial\dot{\xi}_k}{\partial\dot{q}_s}-\dot{q}_k\frac{\partial\dot{\xi}_0}{\partial\dot{q}_s}\Big)\Big(\frac{\partial L}{\partial\dot{q}_k}\Big)'.\end{aligned}\tag{25}$$

将(25)式代入(22)式,得

$$\begin{aligned}\frac{\mathrm{d}I}{\mathrm{d}t}=&\dot{q}_sX^{(2)}[N_s(L)]+\dot{\xi}_0\dot{q}_s\Big(\frac{\partial L}{\partial\dot{q}_s}\Big)'+\dot{q}_sX^{(1)}\Big(\frac{\partial L}{\partial\dot{q}_s}\Big)+\\&\ddot{q}_sX^{(1)}\Big(\frac{\partial L}{\partial\dot{q}_s}\Big)-\frac{\mathrm{d}}{\mathrm{d}t}[X^{(0)}(L)]+\dot{G},\end{aligned}\tag{26}$$

即

$$\begin{aligned}\frac{\mathrm{d}I}{\mathrm{d}t}=&\dot{q}_sX^{(2)}[N_s(L)]+\dot{\xi}_sN_s(L)-X^{(1)}\Big(\frac{\partial L}{\partial t}\Big)-\\&\dot{\xi}_0\frac{\partial L}{\partial t}+\dot{G}.\end{aligned}\tag{27}$$

由(27)式,结合(4),(10)和(18)式可得

$$\frac{\mathrm{d}I}{\mathrm{d}t}=0.\tag{28}$$

命题 1 得证.

利用命题 1,判据 2 和判据 3 容易得到如下命题:

命题 2　对于 Chetaev 型非完整系统(1),(3),如果存在无限小生成元 ξ_0,ξ_s 满足确定方程(10)或(14)以及限制方程(15),且存在规范函数 $G=G(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 使得方程(18)成立,则非完整系统的弱 Lie 对称性直接导致守恒量(19).

命题 3　对于 Chetaev 型非完整系统(1),(3),如果存在无限小生成元 ξ_0,ξ_s 满足确定方程(10)或(14),限制方程(15)以及附加限制方程(17),且存在规范函数 $G=G(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 使得方程(18)成立,则非完整系统的强 Lie 对称性直接导致守恒量(19).

对于一般完整系统,$\Lambda_s=0$,由上述命题可得推论:

推论 1　对一般完整系统,如果存在无限小生成元 ξ_0,ξ_s 满足确定方程

$$X^{(2)}[N_s(L)]=X^{(1)}(Q_s),\ (s=1,\cdots,n),\tag{29}$$

且存在规范函数 $G=G(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 满足结构方程

$$-\dot{\xi}_0\frac{\partial L}{\partial t}-X^{(1)}\Big(\frac{\partial L}{\partial t}\Big)+\dot{q}_sX^{(1)}(Q_s)+\dot{\xi}_sQ_s+\dot{G}=0,\tag{30}$$

则一般完整系统的 Lie 对称性可直接导致守恒量(19).

对于完整保守系统,$Q_s=0,\Lambda_s=0$,由以上命题可得推论:

推论 2　对完整保守系统,如果存在无限小生成元 ξ_0,ξ_s 满足确定方程

$$X^{(2)}[N_s(L)]=0,\ (s=1,\cdots,n),\tag{31}$$

且存在规范函数 $G=G(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 满足结构方程

$$-\dot{\xi}_0\frac{\partial L}{\partial t}-X^{(1)}\Big(\frac{\partial L}{\partial t}\Big)+\dot{G}=0,\tag{32}$$

则完整保守系统的 Lie 对称性可直接导致守恒量(19).

3　算例

非完整系统的 Lagrange 函数为

$$L=\frac{1}{2}m(\dot{q}_1^2+\dot{q}_2^2+\dot{q}_3^2)-mgq_3^2, \tag{33}$$

系统非完整约束力学方程为

$$f=\dot{q}_1^2+\dot{q}_2^2-\dot{q}_3^2=0, \tag{34}$$

非势广义力 $Q_s=0$,试研究非完整系统 Nielsen 方程 Lie 对称性导致的一种守恒量.

将(33),(34)式代入方程(3),得

$$m\ddot{q}_1=2\lambda\dot{q}_1,$$
$$m\ddot{q}_2=2\lambda\dot{q}_2, \tag{35}$$
$$m\ddot{q}_3+2mgq_3=-2\lambda\dot{q}_3.$$

对(34)式求时间 t 的导数

$$\dot{q}_1\ddot{q}_1+\dot{q}_2\ddot{q}_2-\dot{q}_3\ddot{q}_3=0, \tag{36}$$

结合(35)式和(36)式,可得

$$\lambda=-\frac{mgq_3}{2\dot{q}_3}, \tag{37}$$

将(37)式代入方程(35),得

$$\ddot{q}_1=\alpha_1=-\frac{g\dot{q}_1q_3}{\dot{q}_3},\ddot{q}_2=\alpha_2=-\frac{g\dot{q}_2q_3}{\dot{q}_3},$$
$$\ddot{q}_3=\alpha_3=-gq_3. \tag{38}$$

由确定方程(14)和(38)式得

$$\ddot{\xi}_1-\dot{q}_1\ddot{\xi}_0-2\dot{\xi}_0\ddot{q}_1=-\frac{g\dot{q}_1\xi_3}{\dot{q}_3}-\frac{gq_3}{\dot{q}_3}(\dot{\xi}_1-\dot{q}_1\dot{\xi}_0)+\frac{g\dot{q}_1q_3}{\dot{q}_3^2}(\dot{\xi}_3-\dot{q}_3\dot{\xi}_0),$$

$$\ddot{\xi}_2-\dot{q}_2\ddot{\xi}_0-2\dot{\xi}_0\ddot{q}_2=-\frac{g\dot{q}_2\xi_3}{\dot{q}_3}-\frac{gq_3}{\dot{q}_3}(\dot{\xi}_2-\dot{q}_2\dot{\xi}_0)+\frac{g\dot{q}_2q_3}{\dot{q}_3^2}(\dot{\xi}_3-\dot{q}_3\dot{\xi}_0),$$

$$\ddot{\xi}_3-\dot{q}_3\ddot{\xi}_0-2\dot{\xi}_0\ddot{q}_3=-g\xi_3. \tag{39}$$

根据限制方程(15)得

$$(\dot{\xi}_1-\dot{q}_1\dot{\xi}_0)\dot{q}_1+(\dot{\xi}_2-\dot{q}_2\dot{\xi}_0)\dot{q}_2-(\dot{\xi}_3-\dot{q}_3\dot{\xi}_0)\dot{q}_3=0, \tag{40}$$

根据附加限制方程(17)得

$$(\xi_1-\dot{q}_1\xi_0)\dot{q}_1+(\xi_2-\dot{q}_2\xi_0)\dot{q}_2-(\xi_3-\dot{q}_3\xi_0)\dot{q}_3=0. \tag{41}$$

联立方程(39)、(40)和(41),求解方程组可以得到如下无限小生成元

$$\xi_0=0,\ \xi_1=\ddot{q}_1,\ \xi_2=\ddot{q}_2,\ \xi_3=\ddot{q}_3. \tag{42}$$

将生成元(42)代入结构方程(18),存在

$$G=0, \tag{43}$$

利用生成元(42),规范函数(43)和命题3可得

$$I=-2mg\dot{q}_3^2-2mg^2q_3^2=\text{const}. \tag{44}$$

4 结论

本文给出了无限小群变换下 Chetaev 型非完整系统 Nielsen 方程 Lie 对称性的确定方程,得到了 Chetaev 型非完整系统 Nielsen 方程 Lie 对称性直接导致的一种守恒量及其存在条件. 该守恒量是与 Chetaev 型非完整系统 Nielsen 方程 Lie 对称性直接导致的 Hojman 守恒量,间接导致的 Noether 守恒量和 Mei 守恒量不同的另一种守恒量. 本文结果更具一般意义,对非完整系统、一般完整系统和完整保守系统都适用. 若将 Nielsen 算子转换为与之等价的 Euler 算子,在 $Q_s=0$,$\Lambda_s=0$ 的条件下,本文结果将回到文献[15]的结果.

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A TYPE OF CONSERVED QUANTITY OF LIE SYMMETRY FOR NONHOLONOMIC SYSTEM OF NIELSEN EQUATION OF CHETAEV'S TYPE*

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Abstract This paper studied a type of conserved quantity deduced by the Lie symmetry for nonholonomic system with Chetaev-type of the Nielsen equation. Firstly, the determining equations of the Lie symmetry for nonholonomic system with Chetaev-type of the Nielsen equation were given under the infinitesimal transformation of groups. Secondly, the conditions of the existence of the type of conserved quantity of the system as well as its forms were obtained. And finally, an example was given to illustrate the application of the results.

Key words nonholonomic system, Nielsen equation, Lie symmetry, conserved quantity

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