第 10 卷第 2 期 2012 年 6 月
1672-6553/2012/10(2)/117-4
动 力 学 与 控 制 学 报
JOURNAL OF DYNAMICS AND CONTROL
Vol. 10 No. 2
Jun. 2012

事件空间离散完整系统的 Noether 理论*

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摘要　研究差分离散变分原理和事件空间中离散完整系统的 Noether 理论. 运用差分离散变分方法,通过群的无限小变换,得到了事件空间中离散完整系统的差分离散变分原理,并建立了离散的运动方程. 得到了系统的 Noether 对称性的判据方程和 Noether 守恒量的形式以及其存在的条件. 举例说明结果的应用.

关键词　差分离散变分原理,　离散完整系统,　事件空间,　Noether 对称性

引 言

研究事件空间的动力学具有十分重要的意义. 1960 年,Synge 在他的著作中研究了各种空间(包括事件空间)中完整保守系统的分析动力学[1]. 1983 年,Rumjantsev[2] 建立了事件空间中非完整保守系统带乘子的参数方程. 梅凤翔[3] 得到了事件空间中非完整有势的变分原理和参数方程,并在其著作[4 -5]中研究了事件空间中完整系统的各种对称性和守恒量. 随后,事件空间的对称性理论被推广到非完整系统、Birkhoff 系统、变质量系统等连续动力学系统中,并取得了大量成果[6 -13]. 然而目前,事件空间中的离散动力学还很少被研究过.

2002 年,郭汉英等[14 -18] 提出了一种离散方法 -差分离散变分方法,提出了 Euler - Lagrange 上同调的概念. 这种方法将差分视作一个几何对象,它与连续情况中的导数有着相似的性质. 本文将运用这种方法得到了离散完整系统的运动方程,并将 Noether 理论扩展到事件空间中的离散完整系统,进而给出 Noether 对称性的判据方程和对应的 Noether 守恒量.

1　差分离散变分原理和系统的离散运动方程

构建一个 $(n+1)$ 维的位形空间(即事件空间),此空间中点的坐标是广义坐标 $q_s\ (s=1,\cdots,n)$ 和时间 t. 引入记号

$$x_1 = t,\quad x_{s+1} = q_s,\quad (s=1,\cdots,n) \tag{1}$$

则所有的变量 $x_\alpha\ (\alpha=1,\cdots,n+1)$ 都可以表示为某个参数 τ 的已知函数. 令 $x_\alpha = x_\alpha(\tau)$ 是 C^2 类曲线,使得 $dx_\alpha/d\tau = x'_\alpha$,这里 x'_α 不能同时为零,则有

$$\dot{x}_\alpha = dx_\alpha/dt = x'_\alpha/x'_1 \tag{2}$$

其中 $\dot{x}_\alpha$ 表示对 t 的导数,x'_α 表示对 τ 的导数. 事件空间中 Lagrange 函数 $\Lambda(x_\alpha, x'_\alpha)$ 和非势广义力 P_α 可以用下面的方程定义

$$\Lambda(x_\alpha, x'_\alpha) = x'_1 L(x_\alpha, x'_2/x'_1, \cdots, x'_{n+1}/x'_1) \tag{3}$$

$$P_1 = -Q_s x'_{s+1},$$
$$P_{s+1} = x'_1 Q_s(x_\alpha, x'_2/x'_1, \cdots, x'_{n+1}/x'_1) \tag{4}$$

系统的运动方程为

$$\frac{d}{d\tau}\frac{\partial\Lambda}{\partial x'_\alpha} - \frac{\partial\Lambda}{\partial x_\alpha} = P_\alpha \quad (\alpha=1,2,\cdots,n+1) \tag{5}$$

系统的 Hamilton 作用量可以表示为

$$S = \int_{\tau_1}^{\tau_2} \Lambda(x_\alpha, x'_\alpha)\mathrm{d}\tau \tag{6}$$

引入参数 τ 和变量 x_α 的无限小变换

$$\tau^* = \tau + \delta_\tau\tau = \tau + \varepsilon\xi_0(\tau, x_\beta, x'_\beta),$$
$$x_\alpha^*(\tau^*) = x_\alpha(\tau) + \delta_\tau x_\alpha = x_\alpha(\tau) + \varepsilon\xi_\alpha(\tau, x_\beta, x'_\beta)$$
$$(\alpha, \beta = 1, 2, \cdots, n+1) \tag{7}$$

其中 ε 为无限小参数,δ_τ 表示全变分,ξ_0, ξ_α 是无限小参数.

如果无限小变换方程(7)是系统(5)的广义准对称变换,可有

$$\delta_\tau S = -\int_{\tau_1}^{\tau_2} P_\alpha \delta x_\alpha \mathrm{d}\tau \tag{8}$$

在离散力学中,参数 τ 被离散为间隔为(τ_1,

2011-11-14 收到第 1 稿,2012-03-05 收到修改稿.
* 山东省自然科学基金资助项目(ZR2011AM012),中国石油大学(华东)研究生创新工程项目(CXYB11 -12)

τ_2)的点序列$\{\tau_k\}$, $k=0,1,\cdots,N$,离散的$x_\alpha(\tau)$, $P_\alpha(x_\beta,x'_\beta)$和$\Lambda(x_\alpha,x'_\alpha)$分别变为$x_\alpha^k=x_\alpha(\tau_k)$, $P_\alpha^k=P_\alpha(x_\beta^k,\frac{x_\beta^{k+1}-x_\beta^k}{\tau_{k+1}-\tau_k})$和$\Lambda_D^k=\Lambda_D(x_\alpha^k,\frac{x_\alpha^{k+1}-x_\alpha^k}{\tau_{k+1}-\tau_k})$,另外有定义$\Delta x_\alpha^k=\frac{x_\alpha^{k+1}-x_\alpha^k}{\tau_{k+1}-\tau_k}$.

引入参数τ_k和变量x_α^k的无限小变换

$$\tau_k^*=\tau_k+\delta_\tau\tau_k=\tau_k+\varepsilon\xi_0^k(\tau_k,x_\beta^k,\Delta x_\beta^k)$$
$$x_\alpha^{k*}=x_\alpha^k+\delta_\tau x_\alpha^k=x_\alpha^k+\varepsilon\xi_\alpha^k(\tau_k,x_\beta^k,\Delta x_\beta^k) \quad (9)$$

其中ε为无限小参数,而ξ_0^k,ξ_α^k为离散无限小生成元.

在无限小变换(9)的作用下,方程(6)和(8)分别变为

$$S_D=\sum_k(\tau_{k+1}-\tau_k)\Lambda_D^k \quad (10)$$
$$\delta_\tau S_D=-\sum_k(\tau_{k+1}-\tau_k)(P_\alpha^k\delta x_\alpha^k) \quad (11)$$

其中δx_α^k满足下面的关系

$$\delta x_\alpha^k=\delta_\tau x_\alpha^k-\delta_\tau\tau_k\Delta x_\alpha^k \quad (12)$$

由方程(10)和(11)可得

$$\delta_\tau\sum_k(\tau_{k+1}-\tau_k)\Lambda_D^k+\sum_k(\tau_{k+1}-\tau_k)(P_\alpha^k\delta x_\alpha^k)=\sum_k[\Lambda_D^k\delta_\tau(\tau_{k+1}-\tau_k)+(\tau_{k+1}-\tau_k)\delta_\tau\Lambda_D^k+(\tau_{k+1}-\tau_k)P_\alpha^k(\delta_\tau x_\alpha^k-\delta_\tau\tau_k\Delta x_\alpha^k)]=\sum_k(\tau_{k+1}-\tau_k)[\Lambda_D^k\Delta\delta_\tau\tau_k+\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}\delta_\tau x_\alpha^k+\frac{\partial\Lambda_D^k}{\partial\Delta x_\alpha^k}\delta_\tau\Delta x_\alpha^k+P_\alpha^k(\delta_\tau x_\alpha^k-\delta_\tau\tau_k\Delta x_\alpha^k)]=0 \quad (13)$$

运用Leibniz法则[17]

$$\Delta(f_kg_k)=\Delta f_k\cdot g_k+f_{k+1}\cdot\Delta g_k \quad (14)$$

以及下面的关系

$$\delta_\tau\Delta x_\alpha^k=\Delta\delta_\tau x_\alpha^k-\Delta x_\alpha^k\Delta\delta_\tau\tau_k \quad (15)$$

方程(13)可被写为

$$\sum_k(\tau_{k+1}-\tau_k)[\Delta(\Lambda_D^{k-1}\delta_\tau\tau_k)-\Delta\Lambda_D^{k-1}\delta_\tau\tau_k+\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}\delta_\tau x_\alpha^k+\frac{\partial\Lambda_D^k}{\partial\Delta x_\alpha^k}(\Delta\delta_\tau x_\alpha^k-\Delta x_\alpha^k\Delta\delta_\tau\tau_k)+P_\alpha^k(\delta_\tau x_\alpha^k-\delta_\tau\tau_k\Delta x_\alpha^k)]=\sum_k(\tau_{k+1}-\tau_k)[\Delta(\Lambda_D^{k-1}\delta_\tau\tau_k)-\Delta\Lambda_D^{k-1}\delta_\tau\tau_k+\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}\delta_\tau x_\alpha^k+\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\delta_\tau x_\alpha^k)-\Delta\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\delta_\tau x_\alpha^k-\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\Delta x_\alpha^{k-1}\delta_\tau\tau_k)+\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\Delta x_\alpha^{k-1})\delta_\tau\tau_k+P_\alpha^k(\delta_\tau x_\alpha^k-\delta_\tau\tau_k\Delta x_\alpha^k)]=(\tau_{k+1}-\tau_k)\{\Delta(\Lambda_D^{k-1}\delta_\tau\tau_k+\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\delta_\tau x_\alpha^k-\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\Delta x_\alpha^{k-1}\delta_\tau\tau_k)+(\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}-\Delta\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}+P_\alpha^k)\delta_\tau x_\alpha^k+[\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\Delta x_\alpha^{k-1})-\Delta\Lambda_D^{k-1}-P_\alpha^k\Delta x_\alpha^k]\delta_\tau\tau_k\}=0 \quad (16)$$

考虑到固定边界条件$\delta_\tau\tau_0=\delta_\tau\tau_N=0$和$\delta_\tau x_\alpha^0=\delta_\tau x_\alpha^N$,再由方程(16),可得到系统的离散运动方程及能量方程为

$$\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}})-\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}=P_\alpha^k \quad (17)$$

$$\Delta\Lambda^{k-1}-\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\Delta x_\alpha^{k-1})+P_\alpha^k\Delta x_\alpha^k=0 \quad (18)$$

2 事件空间中离散完整系统的 Noether 理论

为了得到离散 Noether 守恒量及其存在的条件,首先给出 Noether 等式。

根据方程(13)和(15),可得

$$\Lambda_D^k\Delta\delta_\tau\tau_k+\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}\delta_\tau x_\alpha^k+\frac{\partial\Lambda_D^k}{\partial\Delta x_\alpha^k}\delta_\tau\Delta x_\alpha^k+P_\alpha^k(\delta_\tau x_\alpha^k-\delta_\tau\tau_k\Delta x_\alpha^k)=\Lambda_D^k\Delta\delta_\tau\tau_k+\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}\delta_\tau x_\alpha^k+\frac{\partial\Lambda_D^k}{\partial\Delta x_\alpha^k}(\Delta\delta_\tau x_\alpha^k-\Delta x_\alpha^k\Delta\delta_\tau\tau_k)+P_\alpha^k(\delta_\tau x_\alpha^k-\delta_\tau\tau_k\Delta x_\alpha^k)=0 \quad (19)$$

考虑到

$$\delta_\tau\tau_k=\varepsilon\xi_0^k,\quad \delta_\tau x_\alpha^k=\varepsilon\xi_\alpha^k \quad (20)$$

再由ε的任意性,可知

$$\Lambda_D^k\Delta\xi_0^k+\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}\xi_\alpha^k+\frac{\partial\Lambda_D^k}{\partial\Delta x_\alpha^k}(\Delta\xi_\alpha^k-\Delta x_\alpha^k\Delta\xi_0^k)+P_\alpha^k(\xi_\alpha^k-\xi_0^k\Delta x_\alpha^k)=0 \quad (21)$$

命题 事件空间中对于满足方程(17)和(18)的离散完整系统,如果存在离散规范函数$G_N^k=G_N^k(x_\alpha^k,\Delta x_\alpha^k)$满足方程

$$\Lambda_D^k\Delta\xi_0^k+\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}\xi_\alpha^k+\frac{\partial\Lambda_D^k}{\partial\Delta x_\alpha^k}(\Delta\xi_\alpha^k-\Delta x_\alpha^k\Delta\xi_0^k)+P_\alpha^k(\xi_\alpha^k-\xi_0^k\Delta x_\alpha^k)+\Delta G_N^k=0 \quad (22)$$

那么系统的 Noether 对称性将导致离散 Noether 守恒量

$$I_D=\Lambda_D^{k-1}\xi_0^k+\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{K-1}}(\xi_\alpha^k-\Delta x_\alpha^{k-1}\xi_0^k)+G_N^k=\mathrm{const} \quad (23)$$

证明 运用 Leibniz 法则,方程(22)变为

$$\Lambda_D^k\Delta\xi_0^k+\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}\xi_\alpha^k+\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}(\Delta\xi_\alpha^k-\Delta x_\alpha^k\Delta\xi_0^k)+P_\alpha^k(\xi_\alpha^k-$$

$$\xi_0^k\Delta x_\alpha^k)+\Delta G_N^k=\Delta(\Lambda_D^{k-1}\xi_0^k)-\Delta\Lambda_D^{k-1}\xi_0^k+\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}\xi_\alpha^k+$$
$$\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\xi_\alpha^k)-\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}})\xi_\alpha^k-\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{K-1}}\Delta x_\alpha^{k-1}\xi_0^k)+$$
$$\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\Delta x_\alpha^{k-1})\xi_0^k+P_\alpha^k(\xi_\alpha^k-\xi_0^k\Delta x_\alpha^k)+\Delta G_N^k=$$
$$\Delta(\Lambda_D^{k-1}\xi_0^k+\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\xi_\alpha^k-\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\Delta x_\alpha^{k-1}\xi_0^k+G_N^k)-$$
$$[\Delta\Lambda_D^{k-1}-\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\Delta x_\alpha^{k-1})+P_\alpha^k\Delta x_\alpha^k]\xi_0^k+$$
$$[\frac{\partial\Lambda_D^k}{\partial x_\alpha^k}\Delta x_\alpha^{k-1}-\Delta(\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}})+P_\alpha^k]\xi_\alpha^k \qquad (24)$$

再将方程(17)和(18)代入方程(24)中可得

$$\Delta(\Lambda_D^{k-1}\xi_0^k+\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\xi_\alpha^k-\frac{\partial\Lambda_D^{k-1}}{\partial\Delta x_\alpha^{k-1}}\Delta x_\alpha^{k-1}\xi_0^k+G_N^k)=0 \qquad (25)$$

命题得证.

3　算例

事件空间中系统的离散 Lagrange 函数为

$$\Lambda_D^k=\Delta x_1^k\{\frac{1}{2}[(\frac{\Delta x_2^k}{\Delta x_1^k})^2+(\frac{\Delta x_3^k}{\Delta x_1^k})^2+(\frac{\Delta x_4^k}{\Delta x_1^k})^2]-x_4^k\} \qquad (26)$$

离散广义力为

$$P_1^k=(\frac{\Delta x_3^k}{\Delta x_1^k})^2\Delta x_2^k-(\frac{\Delta x_2^k}{\Delta x_1^k})^2\Delta x_3^k,$$
$$P_2^k=-(\frac{\Delta x_3^k}{\Delta x_1^k})^2\Delta x_1^k,P_3^k=(\frac{\Delta x_2^k}{\Delta x_1^k})^2\Delta x_1^k,P_4^k=0 \qquad (27)$$

试讨论系统的 Noether 对称性.

由方程(17)和(18),我们可以得到系统的离散运动方程

$$\Delta\{-\frac{1}{2}[(\frac{\Delta x_2^{k-1}}{\Delta x_1^{k-1}})^2+(\frac{\Delta x_3^{k-1}}{\Delta x_1^{k-1}})^2+(\frac{\Delta x_4^{k-1}}{\Delta x_1^{k-1}})^2]-x_4^{k-1}\}=(\frac{\Delta x_3^k}{\Delta x_1^k})^2\Delta x_2^k-(\frac{\Delta x_2^k}{\Delta x_1^k})^2\Delta x_3^k,$$
$$\Delta(\frac{\Delta x_2^{k-1}}{\Delta x_1^{k-1}})=-(\frac{\Delta x_3^k}{\Delta x_1^k})^2\Delta x_1^k,$$
$$\Delta(\frac{\Delta x_3^{k-1}}{\Delta x_1^{k-1}})=-(\frac{\Delta x_2^k}{\Delta x_1^k})^2\Delta x_1^k,\Delta(\frac{\Delta x_4^{k-1}}{\Delta x_1^{k-1}})+\Delta x_1^k=0,$$
$$\Delta\Lambda_D^{k-1}+\Delta\{-\frac{1}{2}[\frac{(\Delta x_2^{k-1})^2}{\Delta x_1^{k-1}}+\frac{(\Delta x_3^{k-1})^2}{\Delta x_1^{k-1}}+\frac{(\Delta x_4^{k-1})^2}{\Delta x_1^{k-1}}]+\Delta x_1^{k-1}x_4^{k-1}\}=0 \qquad (28)$$

取无限小生成元为

$$\xi_0^k=\xi_4^k=1,\xi_1^k=\xi_2^k=\xi_3^k=0 \qquad (29)$$

用式(29)和离散 Noether 等式(22),可得规范函数

$$G_N^k=x_1^k \qquad (30)$$

则可得事件空间中离散 Noether 守恒量为

$$I_D=\frac{\Delta x_4^{k-1}}{\Delta x_1^{k-1}}+x_1^k=\text{const} \qquad (31)$$

4　结论

运用差分离散变分方法,研究了事件空间中离散完整系统的 Noether 对称性及其守恒量. 用上述方法,将原来的连续系统离散化并尽可能保留原来系统的结构和性质. 连续力学系统的许多几何性质也同样存在于离散的系统中. 当 $\tau_{k+1}-\tau_k\to0$ 时,文中的结论可以自然回到连续情形下相应的结果.

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NOETHER THEOREM OF DISCRETE HOLONOMIC SYSTEMS IN EVENT SPACE *

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Abstract Difference discrete variation principle and Noether theorem of discrete holonomic systems in event space were studied. By the difference discrete variation approach, the difference discrete variation principle of discrete holonomic systems in event space was derived. The discrete equations of motion of the system were established. The criterion of Noether symmetry of the system was given. The discrete Noether conserved quantity and the condition for its existence were obtained. Finally, an example was discussed to show the applications of the results.

Key words difference discrete variation principle, discrete holonomic systems, event space, Noether symmetry

Received 14 November 2011, revised 5 March 2012.

* The project supported by the Natural Science Foundation of Shandong Province of China (ZR2011AM012) and the Graduate Innovation Project of China University of Petroleum (East China) (CXYB11-12)