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功能梯度材料圆板的非线性热振动及屈曲*

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摘要　采用弹性理论建立了功能梯度材料板的静力平衡方程，利用静力平衡方程确定了功能梯度材料板的中性面位置，在此基础上推导出了功能梯度材料板在均匀温度场中的非线性振动及屈曲微分方程组，求得了功能梯度材料圆板的非线性振动及屈曲的近似解，讨论分析了中性面位置、梯度指数、温度等因素对功能梯度材料圆板非线性振动及屈曲的影响. 把该方法计算结果与有限元计算结果进行了比较，验证了该方法的计算结果是可靠的. 算例分析表明，中性面位置对均匀温度场中功能梯度材料圆板的非线性振动及屈曲有一定影响.

关键词　功能梯度，　材料，　非线性，　振动，　屈曲，　温度

引 言

功能梯度材料是基于一种全新的材料设计概念合成的新型复合材料[1-6]，日本科学家于 1984 年提出了功能梯度材料的概念，即根据具体的要求，选择使用两种不同性能材料，通过连续平滑地改变两种材料的组织和结构，使其结合部位的界面消失，从而得到功能相应于组织变化而变化的均质材料，最终减小或消除结合部位的性能不匹配因素. 对于陶瓷和金属混合而成的功能梯度材料，由于陶瓷具有低传热系数而用于抵抗高温，金属则由于其良好的延展性而防止了短时间内温度剧变产生的应力而导致断裂破坏，因此被广泛地应用在航空航天等实际工程中. 所以，功能梯度材料板壳的力学性能引起了工程设计人员的极大关注. 但是，有关研究功能梯度材料板壳的文献都没有确定功能梯度材料板壳中性面的真实位置，而是假设了功能梯度材料板壳相对于中性面具有几何和弹性对称[7-12]，然后建立功能梯度材料板壳的振动及屈曲的微分方程，然而一般功能梯度材料板壳中性面与板壳中面是不重合的，这种研究方法显然是具有局限性的. 基于上述原因，本文首先确定了功能梯度材料板的中性面位置，建立了功能梯度材料板在均匀温度场中的非线性振动及屈曲的微分方程组，讨论分析了有关因素对圆板非线性振动及屈曲的影响.

1　振动及屈曲微分方程

对于图 1 所示均匀温度场中的功能梯度材料板，板的下侧为金属材料，上侧为陶瓷材料，中间为两种材料组成的混合物，由于金属材料与陶瓷材料的泊松比相近，可令它们的泊松比均为 μ. 设金属材料的弹性模量、热膨胀系数、密度分别为 E_m、α_m、ρ_m，陶瓷材料的弹性模量、热膨胀系数、密度分别为 E_c、α_c、ρ_c，则板内任一点的弹性模量、热膨胀系数、密度分别为

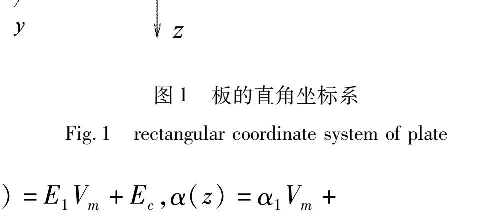

图 1　板的直角坐标系

Fig. 1　rectangular coordinate system of plate

$$E(z)=E_1V_m+E_c,\alpha(z)=\alpha_1V_m+\alpha_c,\rho(z)=\rho_1V_m+\rho_c \qquad (1)$$

式中，$E_1=E_m-E_c$，$\alpha_1=\alpha_m-\alpha_c$，$\rho_1=\rho_m-\rho_c$，$V_m$ 为金属材料组分的体积比例系数.

可设功能梯度材料板中金属材料组分的体积比例系数为板厚方向坐标 z 的幂函数为

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$$V_m = \left(\frac{z - z_0}{h} + \frac{1}{2}\right)^k \tag{2}$$

式中,k 为梯度指数,z_0 为板中面与中性面之间的距离.

根据弹性理论,功能梯度材料板在均匀温度场中的物理方程为

$$\begin{cases} \sigma_x = -\dfrac{E(z)z}{1-\mu^2}\left(\dfrac{\partial^2 w}{\partial x^2} + \mu\dfrac{\partial^2 w}{\partial y^2}\right) - \dfrac{E(z)\alpha(z)}{1-\mu}\Delta T \\ \sigma_y = -\dfrac{E(z)z}{1-\mu^2}\left(\dfrac{\partial^2 w}{\partial y^2} + \mu\dfrac{\partial^2 w}{\partial x^2}\right) - \dfrac{E(z)\alpha(z)}{1-\mu}\Delta T \\ \tau_{xy} = -\dfrac{E(z)z}{1+\mu^2}\dfrac{\partial^2 w}{\partial x\partial y} \end{cases} \tag{3}$$

式中,ΔT 为温度增量.

当 $\Delta T = 0$ 时,功能梯度材料板纯弯曲的横截面内力满足以下关系

$$\int_A \sigma_x \mathrm{d}A = 0, \quad \int_A \sigma_y \mathrm{d}A = 0 \tag{4}$$

把式(3)代入式(4)中可得

$$\int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}} E(z)\,\mathrm{d}z = 0 \tag{5}$$

把式(1)、式(2)代入式(5)中可求得

$$z_0 = \frac{(E_c - E_m)kh}{2(k+2)(E_m + kE_c)} \tag{6}$$

利用式(3)可以得到功能梯度材料板弯矩、扭矩表达式为

$$M_x = -\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right)\int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}}\frac{E(z)z^2}{1-\mu^2}\mathrm{d}z - \int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}}\frac{E(z)\alpha(z)z}{1-\mu^2}\Delta T\mathrm{d}z = -D\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) - M_T \tag{7a}$$

$$M_y = -\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right)\int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}}\frac{E(z)z^2}{1-\mu^2}\mathrm{d}z - \int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}}\frac{E(z)\alpha(z)z}{1-\mu^2}\Delta T\mathrm{d}z = -D\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right) - M_T \tag{7b}$$

$$M_{xy} = -\frac{\partial^2 w}{\partial x\partial y}\int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}}\frac{E(z)z^2}{1+\mu^2}\mathrm{d}z = -(1-\mu)D\frac{\partial^2 w}{\partial x\partial y} \tag{7c}$$

式中,

$$D = \frac{E_1}{1-\mu^2}\left[\frac{h}{k+1}\left(z_0 + \frac{h}{2}\right)^2 - \frac{2h^2}{(k+1)(k+2)}\left(z_0 + \frac{h}{2}\right) + \frac{2h^3}{(k+1)(k+2)(k+3)}\right] + \frac{E_c}{3(1-\mu^2)}\left[\left(z_0 + \frac{h}{2}\right)^3 - \left(z_0 - \frac{h}{2}\right)^3\right]$$

$$M_T = \frac{E_c\alpha_c\Delta T}{2(1-\mu)}\left[\left(z_0 + \frac{h}{2}\right)^2 - \left(z_0 - \frac{h}{2}\right)^2\right] + \frac{E_1\alpha_1 h\Delta T}{(2k+1)(1-\mu)}\left(z_0 + \frac{kh}{2k+2}\right) + \frac{(E_c\alpha_1 + E_1\alpha_c)h\Delta T}{(k+1)(1-\mu)}\left(z_0 + \frac{kh}{2k+4}\right)$$

由弹性理论可知,功能梯度材料板在外扰力作用下的内力应满足以下关系式

$$\begin{cases} \dfrac{\partial M_x}{\partial x} + \dfrac{\partial^2 M_{xy}}{\partial y} = Q_x \\ \dfrac{\partial M_{xy}}{\partial x} + \dfrac{\partial^2 M_y}{\partial y} = Q_y \end{cases} \tag{8}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y} - \rho h\frac{\partial^2 w}{\partial t^2} + q(x,y,t) = 0 \tag{9}$$

式中,$\rho h = \dfrac{(\rho_m + k\rho_c)}{k+1}$,$N_x$、$N_y$、$N_{xy}$ 为中面拉力及剪力,$q(x,y,t)$ 为外扰力.

由弹性理论可知板中面内点的应变表达式为

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \varepsilon_y = \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \gamma_{xy} = \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial w}{\partial y}\dfrac{\partial w}{\partial x} \end{cases} \tag{10}$$

式中,u(或 v)为中面内点沿 x(或 y)方向的位移.由式(10)可以得到相容方程为

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x\partial y} = \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{11}$$

把板中面上应力用板中面内点的应变表示为

$$\begin{cases} \sigma_x^0 = \dfrac{E(z)}{1-\mu^2}[\varepsilon_x + \mu\varepsilon_y - (1+\mu)\alpha(z)\Delta T] \\ \sigma_y^0 = \dfrac{E(z)}{1-\mu^2}[\varepsilon_y + \mu\varepsilon_x - (1+\mu)\alpha(z)\Delta T] \\ \tau_{xy}^0 = \dfrac{E(z)}{2(1+\mu)}\gamma_{xy} \end{cases} \tag{12}$$

由式(12)可以得到功能梯度材料板的中面拉力为

$$\begin{cases} N_x = \dfrac{Eh}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y) - N_T \\ N_y = \dfrac{Eh}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x) - N_T \\ N_{xy} = \dfrac{Eh}{2(1+\mu)}\gamma_{xy} \end{cases} \tag{13}$$

式中，

$$E=E_c+\frac{E_1}{K+1},$$

$$N_T=\frac{E_c\alpha_c h\Delta T}{1-\mu}+\frac{E_1\alpha_1 h\Delta T}{(2k+1)(1-\mu)}+\frac{(E_1\alpha_c+E_c\alpha_1)h\Delta T}{(k+1)(1-\mu)}.$$

由式(13)还可以得到板中面点应变的另一种表达式为

$$\begin{cases}\varepsilon_x=\frac{1}{Eh}(N_x-\mu N_y)+\frac{(1-\mu)N_T}{Eh}\\ \varepsilon_y=\frac{1}{Eh}(N_y-\mu N_x)+\frac{(1-\mu)N_T}{Eh}\\ \gamma_{xy}=\frac{2(1+\mu)}{Eh}N_{xy}\end{cases}\tag{14}$$

再令

$$N_x=\frac{\partial^2\varphi}{\partial y^2},N_y=\frac{\partial^2\varphi}{\partial x^2},N_{xy}=-\frac{\partial^2\varphi}{\partial x\partial y}\tag{15}$$

把式(7)、式(8)、式(15)代入式(9)中，把式(14)、式(15)代入式(11)中，即可得到功能梯度材料板的非线性热振动及屈曲微分方程组为

$$\begin{cases}D\nabla^4 w+\nabla^2 M_T+\rho h\frac{\partial^2 w}{\partial t^2}=(\frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2\varphi}{\partial y^2}\frac{\partial^2 w}{\partial x^2}-2\frac{\partial^2\varphi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y})+q(x,y,t)\\ \nabla^4\varphi+(1-\mu)\nabla^2 N_T=Eh[(\frac{\partial^2 w}{\partial x\partial y})^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}]\end{cases}\tag{16}$$

若功能梯度材料圆板在均匀温度场中发生轴对称非线性振动及屈曲时，引入极坐标可把式(16)化为

$$\begin{cases}D\frac{d}{dr}(\nabla^2 w)+\frac{dM_T}{dr}=\frac{1}{r}\frac{d\varphi}{dr}\frac{dw}{dr}+\frac{1}{r}\int_0^r[q(r,t)-\rho h\frac{d^2 w}{dt^2}]dr\\ \frac{d}{dr}(\nabla^2\varphi)+(1-\mu)\frac{dN_T}{dr}=-\frac{Eh}{2r}(\frac{dw}{dr})^2\end{cases}\tag{17}$$

式中，$\nabla^2=\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}$

2 非线性热振动及屈曲近似解

以图2所示均匀温度场中的功能梯度材料圆板为例，研究其非线性固有振动及屈曲. 在圆板中心建立坐标原点，设其周边固支沿径向不可移动，其边界条件为

$$r=a,\quad w(a)=0\tag{18a}$$

$$r=0,\frac{d\varphi}{dr}=0;$$

$$r=a,\frac{d^2\varphi}{dr^2}-\frac{u}{r}\frac{d\varphi}{dr}+(1-\mu)N_T=0\tag{18b}$$

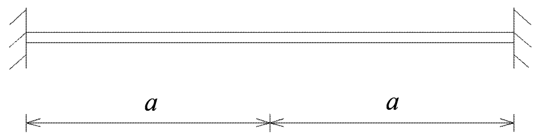

图2 周边固支圆板

Fig. 2 circular plate with peripheral clamped support

参阅有关弹性力学专著可设满足式(18a)的图2所示功能梯度材料圆板的非线性固有热振动振型函数为

$$w(r,t)=T(t)(1-\frac{r^2}{a^2})^2\tag{19}$$

把式(19)代入式(17)第二分式中可以得到

$$\frac{d\varphi}{dr}=\frac{EhT^2}{6a}[\frac{(5-3\mu)r}{(1-\mu)a}-\frac{6r^3}{a^3}+\frac{4r^5}{a^5}-\frac{r^7}{a^7}]-N_T r\tag{20}$$

在式(17)第一分式中令 $q(r,t)=0$ 且利用伽辽金原理可得

$$\int_0^r[D\frac{d}{dr}(\nabla^2 w)+\frac{dM_T}{dr}-\frac{1}{r}\int_0^r\rho h\frac{d^2 w}{dr^2}rdr-\frac{1}{r}\frac{d\varphi}{dr}\frac{dw}{dr}](\frac{r}{a}-\frac{r^3}{a^3})rdr=0\tag{21}$$

把式(19)、式(20)代入式(21)中可得圆板非线性固有热振动微分方程为

$$\frac{d^2T}{dt^2}+\omega_0^2T+\beta T^3=0\tag{22}$$

式中，$\omega_0^2=(\frac{320D}{3a^4}-\frac{20N_T}{3a^2})/\rho h,\beta=\frac{10(23-9\mu)E}{63(1-\mu)\rho a^4}$.

在式(22)中引入“人工摄动参数”且令 $\tau=\omega t$ 可以得到

$$\omega^2\frac{d^2T}{d\tau^2}+\omega_0^2T+\varepsilon\beta T^3=0\tag{23}$$

令式(23)的初始条件为

$$\tau=0,T(0)=b,\frac{dT(0)}{d\tau}=0\tag{24}$$

令

$$\begin{cases}T(\tau)=T_0(\tau)+\varepsilon T_1(\tau)+\varepsilon^2T_2(\tau)+\cdots\\ \omega=\omega_0+\varepsilon\omega_1+\varepsilon^2\omega_2+\cdots\end{cases}\tag{25}$$

把式(25)代入式(23)中可以得到下式

$$\begin{cases}\dfrac{\mathrm{d}^2T_0}{\mathrm{d}\tau^2}+T_0=0\\ \dfrac{\mathrm{d}^2T_1}{\mathrm{d}\tau^2}+T_1=-\dfrac{2\omega_1}{\omega_0}\dfrac{\mathrm{d}^2T_0}{\mathrm{d}\tau^2}-\dfrac{\beta}{\omega_0^2}T_0^3\\ \dfrac{\mathrm{d}^2T_2}{\mathrm{d}\tau^2}+T_2=-\dfrac{2\omega_1}{\omega_0}\dfrac{\mathrm{d}^2T_1}{\mathrm{d}\tau^2}-\dfrac{3\beta}{\omega_0^2}T_0^2T_1-\\ \quad\dfrac{(\omega_1^2+2\omega_0\omega_2)}{\omega_0^2}\dfrac{\mathrm{d}^2T_0}{\mathrm{d}\tau^2}\end{cases} \tag{26}$$

把式(23)的解表示为系数待定的傅立叶级数

$$T(\tau)=T_0(\tau)+\varepsilon T_1(\tau)+\varepsilon^2T_2(\tau)+\cdots=b\cos\tau+\sum_{j=1}^{\infty}\varepsilon^j(c_j+b_j\cos\tau+\sum_{i=2}^{\infty}a_{ij}\cos i\tau)+\cdots \tag{27}$$

为了使式(27)满足初始条件式(24),可补充条件

$$c_j+b_j+\sum_{i=1}^{\infty}a_{ij}=0 \tag{28}$$

把式(27)代入式(26)中利用系数待定法及式(28)可以求得

$$\omega=\lim_{\varepsilon\to1}(\omega_0+\varepsilon\omega_1+\varepsilon^2\omega_2)=\omega_0(1+\frac{3\beta b^2}{8\omega_0^2}-\frac{15\beta^2b^4}{256\omega_0^4}) \tag{29}$$

$$T(t)=\lim_{\varepsilon\to1}(T_0+\varepsilon T_1+\varepsilon^2T_2)=b\cos\omega t+(\frac{\beta b^3}{32\omega_0^2}\cos3\omega t-\frac{\beta b^3}{32\omega_0^2}\cos\omega t)+(\frac{20\beta^2b^5}{1024\omega_0^4}\cos\omega t-\frac{21\beta^2b^3}{1024\omega_0^4}\cos3\omega t+\frac{\beta^2b^5}{1024\omega_0^2}\cos5\omega t) \tag{30}$$

在式(19)中把时间用 $T(t)$ 板中心挠度 f 代替,且在式(22)中略去惯性项可得功能梯度材料圆板的热屈曲关系式

$$N_T=\frac{16D}{a^2}+\frac{(23-9\mu)Eh}{42(1-\mu)a^2}f^2 \tag{31}$$

3　算例分析

为了验证本文计算方法正确性,分别用 ANSYS 和本文方法(即式(31)、式(29))计算了图 2 所示温度载荷作用下周边固支圆板中点挠度 f 和板非线性振动频率 w,并比较了直接考虑 $z_0=0$,即认为中面与中性面重合的情况. 圆板半径 $a=1000$mm,板厚 $h=100$mm. 陶瓷材料的弹性模量和热膨胀系数、密度分别为,$\alpha_c=7.4\times10^{-6}/°C$,$E_c=380GPa$,$\rho_c=2.5\times10^3\mathrm{kg/m^3}$ 金属材料的弹性模量和热膨胀系数、密度分别为 $\alpha_m=23\times10^{-6}/°C$,$E_m=70GPa$,$\rho_m=2.7\times10^3\mathrm{kg/m^3}$ 泊松比均为 $\mu=0.3$. 分别取 0.25,0.5. 有限元建立模型求解,单元为 8 节点 SOLID46 实体层状单元,定义 50 层材料层来模拟功能梯度材料的材料性能的变化,顶层 $E_c=380GPa$,$\alpha_c=7.4\times10^{-6}/°C$,$\rho_c=2.5\times10^3\mathrm{kg/m^3}$,$\mu=0.3$ 底层 $E_m=70GPa$,$\alpha_m=23\times10^{-6}/°C$,$\rho_m=2.7\times10^3\mathrm{kg/m^3}$,$\mu=0.3$,中间层按照式(1)、式(2)来确定、和. 采用 Large Displacement static analysis 进行求解. $k=0.25$,$\Delta T=800°$和 $k=0.5$,$\Delta T=800°$时圆板节点平面外位移如图 3 和图 4 所示. 本文计算结果与有限元结果比较如表 1、表 2 所示.

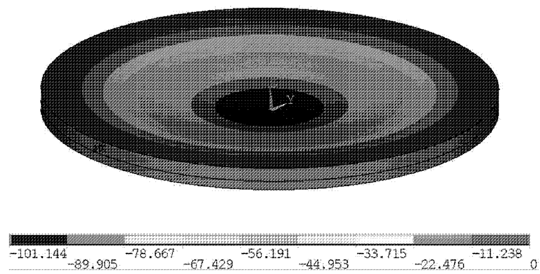

图 3　节点平面外位移($k=0.25$)

Fig. 3　circular plate with peripheral clamped support $k=0.25$

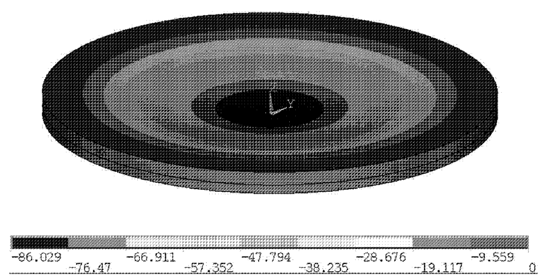

图 4　节点平面外位移($k=0.5$)

Fig. 4　the nodes displacement out of plane $k=0.5$

表 1　温度与挠度的非线性关系

Table 1　the nonlinear relation between temperature andn deflection

ΔT			600	700	800	900	1000
		$z=0.1h$	54.55	83.01	103.92	121.28	136.46
$k=0.25$	f	$z=0$	44.72	64.11	89.55	109.22	125.90
		ANSYS	53.31	81.86	101.15	119.65	133.02
		$z=0.12h$	33.62	67.01	88.60	105.88	120.71
$k=0.5$	f	$z=0$	29.15	47.01	65.03	87.12	104.64
		ANSYS	32.23	65.73	86.03	102.20	116.12

由表 1、表 2 可以看出,随着温度升高均匀温度场中功能梯度材料圆板的屈曲挠度将增大、非线

性固有振动频率将变小,这主要是由于温度升高将降低功能梯度材料圆板的弯曲刚度.随着梯度指数增大均匀温度场中功能梯度材料圆板的屈曲挠度将变小、非线性固有振动频率将变大,主要是由于梯度指数增大将增加功能梯度材料圆板的弯曲刚度.

由表1还可知道,采用有限元方法计算的功能梯度材料圆板屈曲挠度和本文方法计算的功能梯度材料圆板屈曲挠度非常相近,两种方法的计算结果吻合的非常好,充分验证了本文方法的可靠性.

表2 温度与频率的非线性关系

Table 2 the nonlinear relation between temperature andn deflection

ΔT			10°	20°	30°	40°	50°
$k=0.25$	$b=0.1h$	$z=0.1h$	2147	2126	2105	2083	2061
		$z=0$	2289	2270	2249	2230	2210
	$b=0.2h$	$z=0.1h$	2158	2138	2117	2096	2075
		$z=0$	2301	2281	2260	2241	2222
$k=0.5$	$b=0.1h$	$z=0.12h$	2386	2364	2343	2321	2298
		$z=0$	2605	2585	2566	2546	2526
	$b=0.2h$	$z=0.12h$	2399	2379	2357	2335	2313
		$z=0$	2618	2599	2579	2560	2539

如按有关文献不确定功能梯度材料板壳中性面的真实位置,而是假设功能梯度材料板壳相对于中性面具有几何和弹性对称,来研究功能梯度材料圆板的非线性振动及屈曲.算例分析表明,中性面位置对均匀温度场中功能梯度材料圆板的非线性振动及屈曲有较大的影响,这一点由表1、表2就可以看出.

4 结论

由以上分析可以得到以下结论:

(1)采用有限元方法和本文方法计算的结果非常相近,两种方法的计算结果吻合的非常好,充分验证了本文方法的可靠性.

(2)温度升高将降低功能梯度材料圆板的弯曲刚度,梯度指数增大将增加功能梯度材料圆板的弯曲刚度.

(3)中性面位置对均匀温度场中功能梯度材料圆板的非线性振动及屈曲有较大影响.

参 考 文 献

1 Zhong Z, Yu T. Vibration of a simply supported functionally graded piezoelectric rectangular plate. *Smart Mater Struct*, 2006,15:1404 ~ 1412

2 Zhong Z, Shang E T. Three-dimensional exact analysis of a simply supported functionally gradient piezoelectric plate. *Int J Solids Struct*, 2003,40(20):5335 ~ 5352

3 尚尔涛,仲政.功能梯度热释电材料平板柱形弯曲问题的精确解.应用力学学报,2003,20(4):122 ~ 125 (Shang E T, Zhong Z. Exact solutions for functionally graded piezothermoelectric plates in cylindrical bending. *Chinese Journal of Applied Mechanics*, 2003,20(4):122 ~ 125 (in Chinese))

4 Chen W Q, Ding H J. On free vibration of a functionally graded piezoelectric plates. *Acta Mechanica*, 2002:153 ~ 207

5 Wu X H, Chen C Q, Shen Y P, et al. A high order theory for functionally graded piezoelectric shells. *Int J Solids Struct*, 2002,39(20):5325 ~ 5344

6 王铁军,马连生,石朝锋.功能梯度中厚圆/环板轴对称弯曲问题的解析解.力学学报,2004,36(3):348 ~ 353 (Wang T J, Ma L S, Shi Z F. Analytical solutions for axisymmetric bending of functionally graded circular/annular plates. *Acta Mechanica Sinica*, 2004,36(3):348 ~ 353 (in Chinese))

7 马连生,赵永刚,杨静宁.功能梯度圆板的轴对称非线性分析—大挠度问题.兰州理工大学学报,2004,30(6):139 ~ 142 (Ma L S, Zhao Y G, Yang J N. Axisymmmtric nonlinear analysis of functionally graded circular plate: large deflection bending problem. *Journal of Lanzhou University of Technology*, 2004,30(6):139 ~ 142 (in Chinese))

8 马连生,赵永刚,杨静宁.径向压力作用下功能梯度圆板的过屈曲.兰州理工大学学报,2006,32(4):158 ~ 161 (Ma L S, Zhao Y G, Yang J N. Post-buckling of a functionally graded circular plate subjected to radial compression. *Journal of Lanzhou University of Technology*, 2006,32(4):158 ~ 161 (in Chinese))

9 沈惠申.功能梯度复合材料板壳结构的弯曲、屈曲和振动.力学进展,2004,34(1):53 ~ 60 (Shen H S. BENDING, Buckling and vibration of functionally graded plates and shells. *Advances In Mechanics*, 2004,34(1):53 ~ 60 (in Chinese))

10 刘进,武兰河,张晓炜.功能梯度材料板的弯曲问题.石家庄铁道学院学报,2003,16(2):1 ~ 5 (Liu J, Wu L H, Zhang X W. On bending of functionally graded rectangular plates. *Journal of Shijiazhuang Railway Institute*, 2003,16

(2):1 ~ 5 (in Chinese))

11 吴晓,杨立军,孙晋. 基础位移激励下碳纤维矩形索网的非线性振动. 动力学与控制学报,2009,7(4):339 ~ 343(Wu X, Yang L J, Sun J. The nonlinear vibration of rectangular cable net using carbon fiber cables under foundation displacement excitation. *Journal of Dynamics and control*, 2009,7(4):339 ~ 343 (in Chinese))

12 吴晓,杨立军,孙晋. 碳纤维缆索悬索桥竖向非线性自由振动研究. 动力学与控制学报,2010,8(1):67 ~ 73 (Wu X, Yang L J, Sun J. Vertical nonlinear free vibration of suspension bridges using carbon fiber cables. *Journal of Dynamics and control*, 2010,8(1):67 ~ 73 (in Chinese))

NONLINEAR THERMAL VIBRATION AND BUCKLING OF FUNCTIONALLY GRADED CIRCULAR PLATE*

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Abstract　The static equilibrium equation of a functionally graded circular plate was established by using elastic theory, and the neutral plane site of the functionally graded circular plate was determined. On this basis, the nonlinear vibration and buckling differential equations for the functionally graded circular plate in uniform temperature field were derived, the approximate solution to nonlinear thermal vibration and buckling of the functionally graded circular plate was obtained, the effects of neutral plane site, gradient index and temperature on nonlinear thermal vibration and buckling of the functionally graded circular plate were discussed and analyzed. The comparison of the calculation results by this method with these by finite element method verified the method was correct. Analysis on examples indicates that the neutral plane site has certain influence on nonlinear thermal vibration and buckling of the functionally graded circular plate in uniform temperature field.

Key words　functionally graded,　materials,　nonlinear,　vibration,　buckling,　temperature

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