载流导线在磁场中混沌运动区域分析

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摘要　在考虑电磁场对结构变形影响基础上,假设导线变形为小变形,采用弦的模型建立了载流导线在磁场中的周期激励作用下的横向振动控制方程. 利用伽辽金原理及 Melnikov 方法推导出了载流导线发生混沌运动的临界条件,并讨论了导线张力、导线距离、电流等因素对载流导线混沌运动区域的影响. 得到了如下的结论:载流导线的混沌运动区域随导线张力、导线距离的增大而变大;电流小于某一值时,载流导线混沌运动区域随电流增大而减小.

关键词　载流导线,磁场,混沌运动

引言

具有强磁场、高电流的设备已广泛应用于各种工程领域中,当载流导线受到周期激励作用时会产生剧烈振动,使载流导体产生严重变形甚至损坏,所以对载流导体在磁场中混沌运动进行研究是非常必要的,近二十年来有关磁场中载流直导线及薄板的弹性变形问题的研究较为活跃[1~4]. Wolfe P. 及 Healey TT 采用弦的模型对均布磁场中沿场方向的载流直导线的稳定性问题进行了深入研究[5]. 本文则采用弦的模型对磁场中载流导线的混沌运动进行了研究,利用伽辽金原理及 Melnikov 方法推导出了载流导线可能发生混沌运动的临界条件,并讨论分析了导线张力、电流等因素对载流导线可能发生混沌运动区域的影响.

1　基本方程

如图 1 所示,设磁场由相距的两根无限长的刚性直导线中的电流产生,其间一两端固定的张紧导线平衡位置沿轴,长为,载流,采用弦模型可得在周期激励作用下的横向振动控制方程为[6,7]:

$$\frac{T_0}{\sqrt{1+(\frac{\partial W}{\partial x})^2}}\frac{\partial^2 W}{\partial x^2}+\frac{\mu I_0 iW}{\pi^2(a^2-w^2)}[1+(\frac{\partial W}{\partial x})^2]=\rho A\frac{\partial^2 W}{\partial x^2}+\gamma\frac{\partial W}{\partial x}-q(x,t) \quad (1)$$

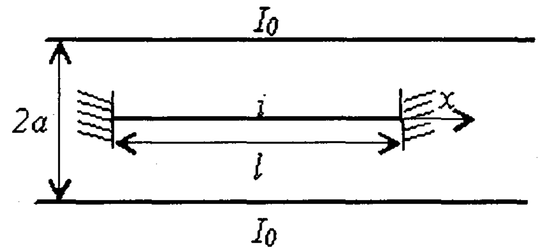

图 1　载流梁模型

Fig. 1　The model of beam carrying electric current

式中,T_0 为载流导线张力,μ 为空气中的磁导率,γ 为阻尼系数,ρ 为密度,A 为载流导线面积,$W(x,t)$ 为导线横振位移,$q(x,t)$ 为外扰力.

考虑当 $\frac{\partial W}{\partial x}<1$,$\frac{W}{a}<1$,可把式(1)化为:

$$T_0[1-\frac{1}{2}(\frac{\partial W}{\partial x})^2]\frac{\partial^2 W}{\partial x^2}+\frac{\mu I_0 iW}{\pi a^2}[1+(\frac{W}{a})^2+(\frac{\partial W}{\partial x})^2]=\rho A\frac{\partial^2 W}{\partial x^2}+\gamma\frac{\partial W}{\partial x}-q(x,t) \quad (2)$$

设

$$W(x,t)=Z(t)\sin\frac{\pi x}{l} \quad (3)$$

$$q(x,t)=(P_0+P_1\cos\omega_0 t)\sin\frac{\pi x}{l} \quad (4)$$

把式(3)、式(4)代入式(1)并利用伽辽金原理得:

$$\frac{d^2 z}{dt^2}+\alpha z-\beta z_3=\frac{4}{\pi\rho A}(P_0+P_1\cos\omega_0 t)-\frac{\gamma}{\rho A}\frac{dz}{dt} \quad (5)$$

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上式中

$$\alpha=\frac{T_0a^2\pi^3-\mu I_0il^2}{\pi\rho a^2l^2A}$$

$$\beta=\frac{T_0a^4\pi^5+2a^2l^2\pi^2\mu I_0i+6\mu I_0il^4}{8\pi\rho a^4l^4A}$$

2 发生混沌运动的区域带

对式(5)进行无量纲化可以得到下式：

$$\frac{d^2\varphi}{d\tau^2}+\varphi-\varphi^3=\varepsilon\left(f_0+f_1\cos\Omega\tau-\eta\frac{d\varphi}{d\tau}\right)\qquad(6)$$

式中，$z=\varphi\sqrt{\frac{\alpha}{\beta}}$，$t=\frac{\tau}{\sqrt{\alpha}}$，$\Omega=\frac{\omega_0}{\sqrt{\alpha}}$，$f_0=\frac{4\sqrt{\beta}P_0}{\varepsilon\pi\rho A\alpha^{\frac{3}{2}}}$，$f_1=\frac{4\sqrt{\beta}P_1}{\varepsilon\pi\rho A\alpha^{\frac{3}{2}}}$，$\eta=\frac{\gamma}{\varepsilon\rho A\alpha^{\frac{1}{2}}}$.

由式(6)可知无扰动系统为：

$$\frac{d^2\varphi}{d\tau^2}+\varphi-\varphi^3=0\qquad(7)$$

再由式(7)可以求得其同宿轨道为：

$$\varphi(\tau)=\pm th\frac{\tau}{\sqrt{2}}\qquad(8)$$

用 Melnikov 函数来测量两轨道间的距离为[8]：

$$M(\tau_0)=\int_{-\infty}^{\infty}[f_0+f_1\cos\Omega(\tau+\tau_0)-\eta\varphi]\varphi' d\tau_0=$$

$$-\frac{2\sqrt{2}}{2}\eta\pm2f_0\pm\sqrt{2}\pi\Omega f_1\mathrm{csch}\frac{\pi\Omega}{\sqrt{2}}\cos\Omega\tau_0\qquad(9)$$

由式(9)可以得到：

(1)若 $f_0>f_1$ 时，函数 $g(x)=\frac{\mathrm{sh}x}{x}-\frac{f_1}{f_0}\geqslant0$，所以系统(6)产生马蹄的阈值条件为：

$$\frac{\sqrt{2}}{3}/\left(\frac{f_0}{f_1}+\frac{\pi\Omega\mathrm{csch}\frac{\pi\omega}{\sqrt{2}}}{\sqrt{2}}\right)\leqslant\frac{f_1}{\eta}\leqslant$$

$$\frac{\sqrt{2}}{3}/\left(\frac{f_0}{f_1}-\frac{\pi\omega\mathrm{csch}\frac{\pi\Omega}{\sqrt{2}}}{\sqrt{2}}\right)\qquad(10)$$

(2)若 $0<f_0<f_1$ 时，可知存在唯一的 Ω_0 满足

$$\frac{\pi\Omega_0}{\sqrt{2}}\mathrm{csch}\frac{\pi\Omega_0}{\sqrt{2}}=0\qquad(11)$$

所以，当 $0<\Omega<\Omega_0$ 时，系统(6)发生混沌运动时条件为：

$$\frac{f_1}{\eta}\geqslant\frac{\sqrt{2}}{3}/\left(\frac{f_1}{f_0}+\frac{\pi\Omega\mathrm{csch}\frac{\pi\omega}{\sqrt{2}}}{\sqrt{2}}\right)\qquad(12)$$

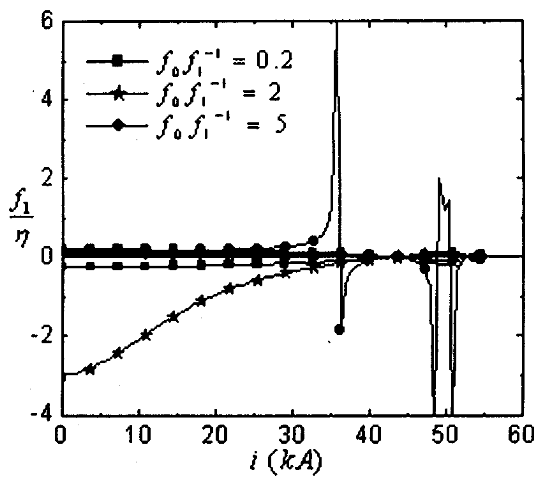
当 $\Omega>\Omega_0$ 时，系统(6)发生混沌运动的条件与式(10)一致.

3 实例计算及讨论

为了讨论分析导线张力、电流等因素对载流导线可能发生混沌运动区域的影响，取计算参数如下 $l=1.789\mathrm{m}$，$A=3.14\times10^{-6}\mathrm{m}^2$，$\rho=7.8\times10^3\mathrm{kg/m}$，$\mu=4\pi\times10^{-7}\mathrm{H/m}$，$I_0=i$，$\omega=100\mathrm{rad/s}$. 当导线张力 T_0、距离 a、$f_0f_1^{-1}$ 变化时由式(10)~式(12)可以得到 $f_1\eta^{-1}-i$ 的变化曲线，如图 2 ~ 图 4 所示.

对图 2 ~ 图 4 进行分析，可以得到以下结论：

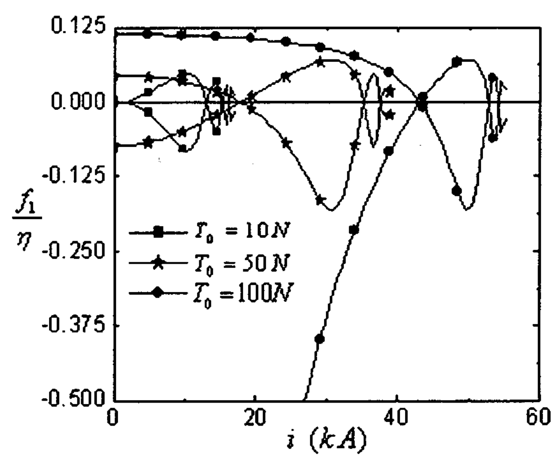
(1)当电流 i 小于某值时(为方便讨论，设该值为 i_0)，相同计算参数两分支曲线所夹面积是电流 i 的减函数，即磁场中载流导线混沌运动区域变



($T_0=100\mathrm{N}$，$a=2\mathrm{m}$)

图 2 $f_0f_1^{-1}$ 对混沌区域的影响

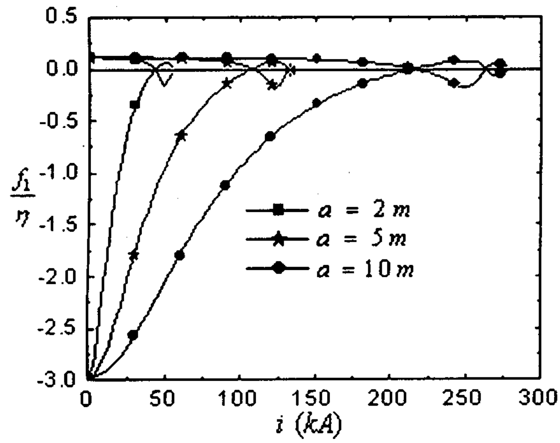
Fig. 2 The influence of $f_0f_1^{-1}$ on chaotic motion region



($f_0f_1^{-1}=2$，$a=2\mathrm{m}$)

图 3 导线张力对混沌区域的影响

Fig. 3 The influence of string tension on chaotic motion



$(f_0f_1^{-1} = 2, T_0 = 100N)$

图 4 导线距离对混沌区域的影响

Fig. 4 The influence of string distance on chaotic motion

小,但当电流 i 到达 i_0 时二者呈现出复杂的关系,并且 i_0 与 $f_0f_1^{-1}$、导线张力 T_0、距离 a 等因素有关,当上述三因素增大 i_0 时也增大 .

(2) 随着导线张力 T_0 的增大,相同导线张力 T_0 的两分支曲线所夹面积变大,即载流导线的混沌运动区域变大 .

(3) 随着距离 a 的增大,相同距离 a 的两分支曲线所夹面积变大,即载流导线的混沌运动区域变大 .

(4) 随着 $f_0f_1^{-1}$ 的变化,相同 $f_0f_1^{-1}$ 的两分支曲线所夹面积呈现出不规则变化,即载流导线的混沌运动区域与 $f_0f_1^{-1}$ 存在着不确定的关系 .

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ANALYSIS OF CHAOTIC MOTION REGION OF WIRE CARRYING ELECTRIC CURRENT IN MAGNETIC FIELD

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Abstract Considering the influence of electromagnetic field on the structural deformation, and supposing that the wire's deformation was small, a natural lateral vibrant control equation of wire carrying electric current under the periodical excitation in the magnetic field was established by chord modeling. The critical condition to chaotic motion of the wire carrying electric current in the magnetic field was researched with Melnikov Method and Galerkin Principle. And factors such as string tension, string distance and current that affected the chaotic motion region were discussed. The results showed that the chaotic motion region of wire carrying electric current increased with the increasing of string tension and string distance respectively, and decreased with the increasing of electric current when the electric current was below a certain value.

Key words wire carrying electric current, magnetic field, chaotic motion

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