几何非线性损伤粘弹性中厚板的动力学行为分析*

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摘要 根据Timoshenko几何变形假设和Boltzmann叠加原理,推导出控制损伤粘弹性Timoshenko中厚板的非线性动力方程以及简化的Galerkin截断方程组;然后利用非线性动力系统中的数值方法求解了简化方程组.通过分析可知,板在谐载荷的作用下,具有非常丰富的动力学特性.同时研究了板的几何参数、材料参数及载荷参数对损伤粘弹性中厚板动力学行为的影响.

关键词 损伤粘弹性固体,中厚板,几何非线性,非线性动力系统,分叉,混沌

引言

随着粘弹性材料在国防和民用工业中的广泛使用,粘弹性力学已成为国内外研究的热点之一.如今,粘弹性力学已成为固体力学的基础内容,成为现代连续介质力学的一个重要组成部分.由于粘弹性结构的大变形和/或非线性本构关系将导致其数学模型中出现非线性项而成为非线性系统,并可能出现分叉和混沌运动.随着人们在数学上对分叉理论和混沌理论认识的提高,对粘弹性板的混沌运动方面已有大量的成果,1994年Touati等[1]采用时间历程、功率谱和最大李雅普诺夫指数说明了在小变形条件下非线性粘弹性板的混沌运动,随后他们用相图、Poincare截面、功率谱和最大李雅普诺夫指数方法研究了大变形条件下非线性材料粘弹性板的混沌运动,并考察了各种参数对混沌性态的影响[2].1995年Suire和Cederbaum[3]研究了大变形线性粘弹性梁的周期和混沌动力学行为,其粘弹性本构关系为Boltzmann叠加原理.1996年,Argyris[4]采用微分型本构关系研究了粘弹性梁的混沌运动.丁睿等[5]研究了非线性粘弹性板的混沌性态,朱媛媛等[6]研究了大变形粘弹性薄板的混沌行为.程昌钧等[7]采用时程曲线、相图、频闪图和李雅普诺夫指数谱等数值方法研究大变形粘弹性板运动,并发现板存在超混沌运动,同时说明混沌与超混沌是交替出现的.张能辉等[8]还用李雅普诺夫指数谱等数值分析方法研究了超音速流动中粘弹性薄板的混沌现象.李晶晶[8]等应用考虑高阶横向剪切变形的Reddy理论和Boltzmann叠加原理,分析了具有高阶横向剪切效应有限变形的粘弹性板的动力学行为.虽然人们对粘弹性结构的动力学行为的研究已经取得了十分可喜的成果,但对损伤粘弹性结构的动力学行为的研究目前还没有文献报道,因此,具有损伤效应的粘弹性结构非线性动力学行为的研究目前仍是一个函待完善的课题,这里面还有许多的工作有待人们去探索.

本文应用Timoshenko几何变形假设和Boltzmann叠加原理,在损伤增量沿板的厚度方向成三次函数分布的假设条件下,导出了控制损伤粘弹性中厚板的非线性动力学方程,由于考虑了横向剪切效应和几何非线性,所得的方程是一组非线性的积分偏微分方程组.得到这类初边值问题的精确解是十分困难的.这里我们应用Galerkin方法把非线性的积分偏微分方程组简化为一组积分常微分方程组;然后,应用非线性动力学中的数值分析方法,对Galerkin截断系统的动力学行为在数值上进行了分析,研究了载荷参数、结构几何参数和材料参数对损伤粘弹性中厚板动力学行为的影响.计算和分析表明,和位移的动力学行为一样,损伤增量也表现出丰富的动力学性质.

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1 损伤粘弹性矩形中厚板的数学模型及简化

(1) 损伤粘弹性力学的动力学方程

设 u_i,ε_{ij},$_{ij}$ 和D 分别是损伤粘弹性材料的位移、应变、应力分量及损伤变量,它们均是坐标 xi 和时间t 的函数.根据连续介质损伤力学的基本规律,在有限变形条件下,它们满足如下方程:

运动微分方程

$$\sigma_{ij,j} + f_i - \rho\ddot{u}_i = 0 \tag{1}$$

$$\rho k\dot{D} - \alpha D_{,ii} + \omega D + \xi(D - D^0) - \beta\varepsilon_{kk} + l = 0 \tag{2}$$

几何方程

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right) \tag{3}$$

本构方程[10]

$$\sigma_{ij} = C_1 \otimes \varepsilon_{ij} + C_2 \otimes \varepsilon_{kk}\delta_{ij} - \beta(D - D^0)\delta_{ij} \tag{4}$$

上式中 C_1 和 C_2 为粘弹性材料的性质函数,它们和蠕变函数 J_1 和 J_2 的关系为 $C_1 = L^{-1}[1/s^2\bar{J}_1)]$, $C_2 = L^{-1}[(\bar{J}_1 - \bar{J}_2)/s^2\bar{J}_1(\bar{J}_1 + 2\bar{J}_2)]$.式中,$\overline{(\cdot)}$和 L^{-1} 分别表示 Laplace 变换和逆变换,s 是变换参数.符号 $\otimes$ 是 Boltzmann 算子,定义为

$$\varphi_1(t) \otimes \varphi_2(t) = \varphi_1(0^+)\varphi_2(t) + \dot{\varphi}_1(t) * \varphi_2(t) = \varphi_1(0^+)\varphi_2(t) + \int_{0^+}^{t} \dot{\varphi}_1(t - \tau)\varphi_2(\tau)\mathrm{d}\tau$$

在式(1)～式(4)中,f_i 为已知体积力,ρ 为参考构形的已知密度,k 为已知平衡惯量,l 是已知外在平衡体积力.α,ω,ξ,β为材料的特征常数,D^0是初始损伤.

(2) 损伤粘弹性中厚板的数学模型

假定损伤粘弹性中厚板的厚度为 h,板承受横向谐载荷 $q(x,y,t)$.设板沿3个坐标轴 x,y,z 方向的位移分别为 u,v 和 w,中平面的法线绕 x,y 轴的转角分别为 φ 和 ψ.

根据 Timoshenko 假设,设板内任一点在任一时刻的位移分量 u,v,w 为

$$\left.\begin{aligned} u(x,y,z,t) &= u^0(x,y,t) - z\varphi(x,y,t) \\ v(x,y,z,t) &= v^0(x,y,t) - z\psi(x,y,t) \\ w(x,y,z,t) &= w^0(x,y,t) \end{aligned}\right\} \tag{5}$$

式中:u^0,v^0,w^0 分别是中面上沿 x,y,z 方向的位移分量.

如果板的转角较大,则由 von Kármán 理论,中厚板的应变可以分解为两部分,即平均应变和弯曲应变

$$\begin{aligned} &\varepsilon_x = \varepsilon_x^0 - z\varphi_{,x}, \varepsilon_y = \varepsilon_y^0 - z\varphi_{,y}, \\ &\gamma_{xy} = \gamma_{xy}^0 - z(\varphi_{,y} - \psi_{,x}), \\ &\gamma_{xz} = w_{,x} - \varphi, \gamma_{yz} = w_{,y} - \psi \end{aligned} \tag{6}$$

其中 ε_0,ε_{0_y} 和 γ_{xy}^0 是平均应变,且有

$$\begin{aligned} &\varepsilon_x^0 = u_{,x}^0 + w_{,x}^2/2, \\ &\varepsilon_y^0 = v_{,y}^0 + w_{,y}^2/2; \\ &\gamma_{xy}^0 = u_{,y}^0 + v_{,x}^0 + w_{,x}w_{,y} \end{aligned} \tag{7}$$

为了便于分析,可以假设损伤是坐标的3次函数,即有

$$D(x_i,t) - D^0(x_i) = D(x_\alpha,t)\left(\frac{z^3}{3} - \frac{h^2}{4}z\right) \tag{8}$$

这里和今后,凡下标为希腊字母者,取值为 x 和y.根据上面的方程,容易得到线性各向同性损伤粘弹性板的应力应变关系

$$\begin{aligned} \sigma_x &= (C_1 + C_2) \otimes (\varepsilon_x^0 - z\varphi_{,x}) + C_2 \otimes (\varepsilon_y^0 - z\psi_{,y}) - \beta D\left(\frac{z^3}{3} - \frac{h^2}{4}z\right) \\ \sigma_y &= C_2 \otimes (\varepsilon_x^0 - z\varphi_{,x}) + (C_1 + C_2) \otimes (\varepsilon_y^0 - z\psi_{,y}) - \beta D\left(\frac{z^3}{3} - \frac{h^2}{4}z\right) \\ \tau_{xy} &= \frac{1}{2}C_1 \otimes [\gamma_{xy}^0 - z(\varphi_{,y} + \psi_{,x})] \\ \tau_{xz} &= \frac{1}{2}C_1 \otimes [w_{,x} - \varphi), \tau_{yz} = \frac{1}{2}C_1 \otimes (w_{,y} - \psi) \end{aligned} \tag{9}$$

并且内力分量有表示式

$$N_x = \int_{-h/2}^{h/2} \sigma_x \mathrm{d}z = h[(C_1 + C_2) \otimes (u_{,x}^0 + w_{,x}^2/2) + C_2 \otimes (v_{,y}^0 + w_{,y}^2/2)]$$

$$N_y = \int_{-h/2}^{h/2} \sigma_y \mathrm{d}z = h[C_2 \otimes (u_{,x}^0 + w_{,x}^2/2) + (C_1 + C_2) \otimes (v_{,y}^0 + w_{,y}^2/2)]$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} \mathrm{d}z = \frac{h}{2}C_1 \otimes (u_{,y}^0 + v_{,x}^0 + w_{,x}w_{,y})$$

$$Q_x \doteq \int_{-h/2}^{h/2} \tau_{xz} \mathrm{d}z = \frac{h}{2}C_1 \otimes (w_{,x} - \varphi)$$

$$Q_y = \int_{-h/2}^{h/2} \tau_{yz} \mathrm{d}z = \frac{h}{2} C_1 \otimes (w_{,y} - \psi)$$

$$M_x = \int_{-h/2}^{h/2} \sigma_x z \mathrm{d}z = -\frac{h^3}{12}[(C_1 + C_2) \otimes \varphi_{,x} + C_2 \otimes \psi_{,y} - \frac{h^2}{5}\beta D)]$$

$$M_y = \int_{-h/2}^{h/2} \sigma_y z \mathrm{d}z = -\frac{h^3}{12}[C_2 \otimes \varphi_{,x} + (C_1 + C_2) \otimes \psi_{,y} - \frac{h^2}{5}\beta D)]$$

$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} z \mathrm{d}z = -\frac{h^3}{24} C_1 \otimes (\varphi_{,y} + \psi_{,x}) \tag{10}$$

式中，N_x, N_y, N_{xy} 为板的薄膜内力，$Q_x, Q_y, M_x, M_y, M_{xy}$ 分别为板的横向剪力、弯矩和扭矩.

假设板的质量密度为 ρ，并忽略板平面内的惯性力，则由薄板微元体的平衡，可得到5个位移分量，即中面位移 $u^0, v^0, w^0 = w$ 和关于 x, y 方向的转角和表示的运动微分方程组

$$(C_1 + C_2) \otimes (u^0_{,xx} + w_{,x} w_{,xx}) + C_2 \otimes (v^0_{,yx} + w_{,y} w_{,yx}) + \frac{1}{2} C_1 \otimes (u^0_{,yy} + v^0_{,xy} + w_{,xy} w_{,y} + w_{,x} w_{,yy}) = 0$$

$$C_2 \otimes (u^0_{,xy} + w_{,x} w_{,xy}) + (C_1 + C_2) \otimes (v^0_{,yy} + w_{,y} w_{,yy}) + \frac{1}{2} C_1 \otimes (u^0_{,yx} + v^0_{,xx} + w_{,xx} w_{,y} + w_{,x} w_{,yx}) = 0$$

$$h\{\frac{1}{2} C_1 \otimes (\nabla^2 w - \varphi_{,x} - \psi_{,y}) = [(C_1 + C_2) \otimes (u^0_{,xx} + w_{,x} w_{,xx}) + C_2 \otimes (v^0_{,yx} + w_{,y} w_{,yx}) + \frac{1}{2} C_1 \otimes (u^0_{,yy} + v^0_{,xy} + w_{,xy} w_{,y} + w_{,x} w_{,yy})] w_{,x} + [C_2 \otimes (u^0_{,xy} + w_{,x} w_{,xy}) + (C_1 + C_2) \otimes (v^0_{,yy} + w_{,y}, w_{,yy}) + \frac{1}{2} C_1 \otimes (u^0_{,yx} + v^0_{,xx} + w_{,xx} w_{,y} + w_{,x} w_{,yx})] w_{,y} + [(C_1 + C_2) \otimes (u^0_{,x} + w^2_{,x}/2) + (C_1 + C_2) \otimes (v^0_{,y} + w^2_{,y}/2]) w_{,yy} + C_1 \otimes (u^0_{,y} + v^0_{,x} + w_{,x} w_{,y}) \times w_{,xy}\} = \rho h w_{,tt} - q$$

$$\frac{h^2}{6}(C_1 + C_2) \otimes \varphi_{,xx} + C_2 \otimes \psi_{,yx} - \frac{h^2}{5}\beta D_{,x}) + \frac{h^2}{12} C_1 \otimes (\varphi_{,yy} + \psi_{,xy}) + C_1 \otimes (w_{,x} - \varphi) = \frac{\rho h^2}{6} \varphi_{,tt}$$

$$\frac{h^2}{6}(C_2 \otimes \varphi_{,xy} + (C_1 + C_2) \otimes \psi_{,yy} - \frac{h^2}{5}\beta D_{,y}) + \frac{h^2}{12} C_1 \otimes (\varphi_{,yx} + \psi_{,xx}) + C_1 \otimes (w_{,y} - \psi) = \frac{\rho h^2}{6} \psi_{,tt} \tag{11}$$

另外，我们还有损伤的运动微分方程为

$$-\rho k \ddot{D} + \alpha D_{,\alpha\alpha} - \omega \dot{D} - \xi D - \frac{84\beta}{17h^2}(\varphi_{,x} + \psi_{,y}) = 0 \tag{12}$$

为了方便，假定板为四边简支矩形板，则有下面的边界条件

$$u^0 = v^0 = w = M_x = 0 (x = 0, a)$$

$$u^0 = v^0 = w = M_y = 0 (x = 0, b)$$

$$D = D^* = 0 (x = 0, a) \text{和} (y = 0, b)) \tag{13}$$

假设初始条件为

$$w|_{t=0} = w^0, \dot{w}|_{t=0} = \dot{w}^0,$$

$$\varphi|_{t=0} = \varphi^0, \dot{\varphi}|_{t=0} = \dot{\varphi}^0$$

$$\psi|_{t=0} = \psi^0, \dot{\psi}|_{t=0} = \dot{\psi}^0$$

$$D|_{t=0} = D^0, \dot{D}|_{t=0} = \dot{D}^0 \tag{14}$$

这里，各等式右端的函数分别是各物理量在初始时刻的值，它们都仅是坐标 x_α 的已知函数. 式(11) ~ 式(14)是控制损伤粘弹性板动力学行为的初边值问题，可见它们是1组非线性积分偏微分方程组为了得到这个初 - 边值问题的解，我们采用 Galerkin 平均化的方法将问题进行简化. 根据边界条件式(13)，式(11) ~ 式(12)的解可取为如下的形式

$$u^0(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \bar{u}^0(t)_{nm} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b}$$

$$v^0(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \bar{v}^0(t)_{nm} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b}$$

$$w(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \bar{w}(t)_{nm} \sin\frac{n\pi x}{a} \times \sin\frac{m\pi y}{b}$$

$$\varphi(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \bar{\varphi}(t)_{nm} \cos\frac{n\pi x}{a} \sin\frac{m\pi y}{b}$$

$$\psi(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \bar{\psi}(t)_{nm} \sin\frac{n\pi x}{a} \cos\frac{m\pi y}{b}$$

$$D(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\bar{D}(t)_{nm}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}\tag{15}$$

为了方便，假定横向载荷 q 表示为

$$q(x,y,t)=\bar{q}(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}\tag{16}$$

将式(15)和式(16)代入到式(11)中，同时分别令 $n=1,1$、$m=1,3$，可得二阶 Galerkin 截断模型如下

$$\begin{aligned}
&A_1\otimes\bar{u}_{11}^0=0,A_2\otimes\bar{u}_{13}^0=0,\\
&B_1\otimes\bar{v}_{11}^0=0,B_2\otimes\bar{v}_{13}^0=0\\
&A_3\otimes\bar{w}_{11}+\bar{w}_{11}(A_4\otimes\bar{w}_{11}^2)+\bar{w}_{11}[A_{41}\otimes\\
&\quad\bar{w}_{11}\bar{w}_{13})]+\bar{w}_{11}(A_{42}\otimes\bar{w}_{13}^2)+\bar{w}_{13}\times\\
&\quad A_5\otimes\bar{w}2_{11})+\bar{w}_{13}[A_{51}\otimes(\bar{w}_{11}\bar{w}_{13})]+\\
&\quad A_6\otimes\bar{\varphi}_{11}+A_7\otimes\bar{\psi}_{11}=A_8\ddot{\bar{w}}_{11}-\frac{ab}{4}\bar{q}\\
&B_3\otimes\bar{w}_{13}+\bar{w}_{11}(B_4\otimes\bar{w}_{11}^2)+\bar{w}_{11}[B_{41}\otimes\\
&\quad(\bar{w}_{11}\bar{w}_{13})]+\bar{w}_{13}(B_5\otimes\bar{w}_{13}^2)+B_6\otimes\bar{\varphi}_{13}+\\
&\quad B_7\otimes\bar{\psi}_{13}=B_8\ddot{\bar{w}}_{11}\\
&A_9\otimes\bar{w}_{11}+A_{10}\otimes\bar{\varphi}_{11}+A_{11}\otimes\bar{\psi}_{11}+\\
&\quad A_{12}\bar{D}_{11}=A_{13}\ddot{\bar{\varphi}}_{11}\\
&B_9\otimes\bar{w}_{13}+B_{10}\otimes\bar{\varphi}_{13}+B_{11}\otimes\bar{\psi}_{13}+\\
&\quad B_{12}\otimes\bar{D}_{13}=A_{13}\otimes\ddot{\bar{\varphi}}_{11}\\
&A_{14}\otimes\bar{w}_{11}+A_{15}\otimes\bar{w}_{11}+A_{16}\otimes\bar{\psi}_{11}+\\
&\quad A_{17}\otimes\bar{D}_{11}=A_{18}\ddot{\bar{\varphi}}_{11}\\
&B_{14}\otimes\bar{w}_{13}+B_{15}\otimes\bar{\varphi}_{13}+B_{16}\otimes\bar{\psi}_{13}+\\
&\quad B_{17}\otimes\bar{D}_{13}=B_{18}\ddot{\bar{\varphi}}_{13}\\
&A_{19}\bar{D}_{11}+A_{20}\dot{D}_{11}+A_{21}\ddot{D}_{11}+A_{22}\bar{\varphi}_{11}+\\
&\quad A_{23}\bar{\psi}_{11}=0\\
&B_{19}\bar{D}_{13}+B_{20}\dot{D}_{13}+B_{21}\ddot{D}_{13}+B_{22}\bar{\varphi}_{13}+\\
&\quad B_{23}\bar{\psi}_{13}=0
\end{aligned}\tag{17}$$

式(17)中的各系数均是板的形状参数和材料参数的有关函数．限于篇幅，这里略去了它们的表达式．在式(17)中，令 $\bar{w}_{13}=\bar{\varphi}_{13}=\bar{\psi}_{13}=0$，即令 $n=1,m=1$，我们可得一阶 Galerkin 截断模型．从式(17)中可以看出，变量 u 和 v 与挠度、损伤及转角都是非耦合的．

2　问题求解

引入无量纲化参数，并作如下的变量变换

$$\begin{aligned}
&\alpha_1=a/b,\beta_1=b/h,w=\bar{w}/h,\\
&u=\bar{u}/h,v=\bar{v}/h,D_{11}=h^3\bar{D}_{11},\\
&D_{13}=h^3\bar{D}_{13},\beta_2=C_1(0)/(\rho V_c^2),\\
&\beta_3=\beta/(\rho V_c^2),\beta_4=\alpha/(\rho k V_c^2),\\
&\beta_5=\xi h^2/(\rho k V_c^2),\beta_6=wh/(\rho k V_c),\\
&\beta_7=\beta h^2/(\rho k V_c^2),\tau=tV_c/h,\\
&\tau_0=t_0V_c/h,c_1(\tau)=C_1(\tau)/C_1(0),\\
&q_0=\bar{q}/C_1(0),y_0=t,y_1=w_{11},\\
&y_2=\dot{w}_{11},y_3=\varphi_{11},y_4=\dot{\varphi}_{11},\\
&y_5=\psi_{11},y_6=\dot{\psi}_{11},\\
&y_7=\int_0^t\dot{c}_1(t-\tau)w_{11}(\tau)\mathrm{d}\tau,\\
&y_8=\int_0^t\dot{c}_1(t-\tau)\varphi_{11}(\tau)\mathrm{d}\tau,\\
&y_9=\int_0^t\dot{c}_1(t-\tau)\psi_{11}(\tau)\mathrm{d}\tau,\\
&y_{10}=\int_0^t\dot{c}_1(t-\tau)w_{11}^2(\tau)\mathrm{d}\tau,\\
&y_{11}=w_{13},y_{12}=\dot{w}_{13},y_{13}=\varphi_{13},\\
&y_{14}=\dot{\varphi}_{13},y_{15}=\psi_{13},y_{16}=\dot{\psi}_{13},\\
&y_{17}=\int_0^t\dot{c}_1(t-\tau)w_{13}(\tau)\mathrm{d}\tau,\\
&y_{18}=\int_0^t\dot{c}_1(t-\tau)w_{13}^2(\tau)\mathrm{d}\tau,\\
&y_{19}=\int_0^t\dot{c}_1(t-\tau)w_{11}(\tau)\dot{w}_{13}(\tau)\mathrm{d}\tau,\\
&y_{20}=\int_0^t\dot{c}_1(t-\tau)\varphi_{13}(\tau)\mathrm{d}\tau,\\
&y_{21}=\int_0^t\dot{c}_1(t-\tau)\psi_{13}(\tau)\mathrm{d}\tau,\\
&y_{22}=D_{11},y_{23}=D_{13},\\
&y_{24}=\dot{D}_{11},y_{25}=\dot{D}_{13}
\end{aligned}\tag{18}$$

对于标准线性固体，材料的松弛函数 $c_1(t)$ 满足下面的条件

$$c_1(t)=c_0+c_1\exp(-\alpha t),$$

$$c_1(0) = c_0 + c_1 = 1,$$
$$\dot{c}_1(t-\tau) = \Psi_1(t)\cdot\Psi_2(\tau) = -c_1\exp(-\alpha t)\cdot\alpha\exp(\alpha\tau) \quad (19)$$

显见方程(17)是一个非线性自治微分方程,其相应的自治系统为

$$\dot{Y} = F(Y) \quad (20)$$
$$Y = \{y_0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}, y_{11}, y_{12}, y_{13}, y_{14}, y_{15}, y_{16}, y_{17}, y_{18}, y_{19}, y_{20}, y_{21}, y_{22}, y_{23}, y_{24}, y_{25}\}$$

其中

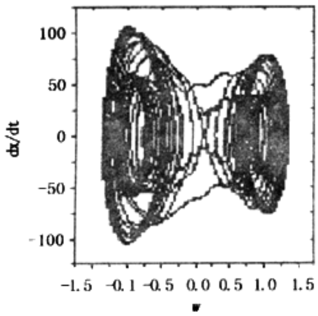
$$F_0 = 1, F_1 = y_2,$$
$$F_2 = k_1(y_1+y_7) + k_2(y_1^3 + y_1y_{10} + k_3(y_{11}^2y_1 + y_1y_{18}) + k_4(y_1^2y_{11} + y_1y_{19}) + k_5(y_1^2y_{11} + y_{10}y_{11}) + k_6(y_1y_{11}^2 - y_{11}y_{19}) + k_7(y_3 + y_8) + k_{71}(y_5+y_9) + \beta_2 q_0,$$
$$F_3 = y_4,$$
$$F_4 = -k_{13}y_{22} + k_{14}(y_1+y_7) - k_{15}(y_3 + y_8) - k_{151}(y_5+y_9),$$
$$F_5 = y_6,$$
$$F_6 = k_{191}(y_1+y_7) - k_{192}(y_3+y_8) - k_{193}(y_5+y_9) - k_{194}y_{22},$$
$$F_7 = -\alpha(c_1y_1 + y_7),$$
$$F_8 = -\alpha(c_1y_3 + y_8),$$
$$F_9 = -\alpha(c_1y_5 + y_9),$$
$$F_{10} = -\alpha(c_1y_1^2 + y_{10}),$$
$$F_{11} = y_{12},$$
$$F_{12} = -k_8(y_{11}+y_{17}) + k_9(y_1^2 + y_1y_{10}) + k_{10}(y_1^2y_{11} + y_1y_{19}) + k_{11}(y_{11}^3 + y_{11}y_{18}) + k_{12}(y_{13}+y_{20}),$$
$$F_{13} = y_{14},$$
$$F_{14} = -k_{16}y_{23} + k_{17}(y_{11}+y_{17}) + k_{18}(y_{13}+y_{20})$$
$$F_{15} = F_{16}$$
$$F_{16} = K_{201}(y_{11}+y_{17}) - k_{202}(y_{13}+y_{20}) - k_{203}(y_{15}+y_{21}) - k_{204}y_{23}$$
$$F_{17} = -\alpha(c_1y_{11} + y_{17}),$$
$$F_{18} = -\alpha(c_1y_{11}^2 + y_{18}),$$
$$F_{19} = -\alpha(c_1y_1y_{13} + y_{19}),$$
$$F_{20} = -\alpha(c_1y_{13} + y_{20}),$$
$$F_{21} = -\alpha(c_1y_{15} + y_{21}),$$
$$F_{22} = y_{24},$$
$$F_{23} = y_{25},$$
$$F_{24} = -k_{21}y_{22} - k_{22}y_{24} + k_{23}y_3 + k_{231}y_5,$$
$$F_{25} = -k_{24}y_{23} - k_{25}y_{25} + k_{26}y_{13} + k_{261}y_{15} \quad (21)$$

式(21)的系数也与板的几何性质和材料性质有关,限于篇幅,这里也略去了它们的表达式.在推导过程中已假定材料的泊松比 $\mu = \text{const}$,即 $C_2(t)/C_1(t) = \mu/(1-\mu) = \mu_1$.式(21)的初值,从初始条件(14),可写为

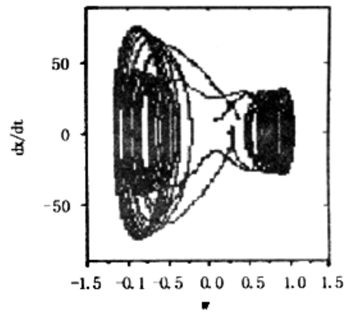
$$\{y_1(0), y_2(0), y_3(0), y_4(0), y_5(0), y_6(0), y_7(0), y_8(0), y_9(0), y_{10}(0), y_{11}(0), y_{12}(0), y_{13}(0), y_{14}(0), y_{15}(0), y_{16}(0), y_{17}(0), y_{18}(0), y_{19}(0), y_{20}(0), y_{21}(0), y_{22}(0), y_{23}(0), y_{24}(0), y_{25}(0)\} = \{w_1^0, \dot{w}_1^0, \varphi_1^0, \dot{\varphi}_1^0, \psi_1^0, \dot{\psi}_1^0, 0,0,0,0, w_3^0, \dot{w}_3^0, \varphi_3^0, \dot{\varphi}_3^0, \psi_3^0, \dot{\psi}_3^0, 0,0,0,0, D_1^0, D_3^0, \dot{D}_1, \dot{D}_3\} \quad (22)$$

3 数值结果和结论

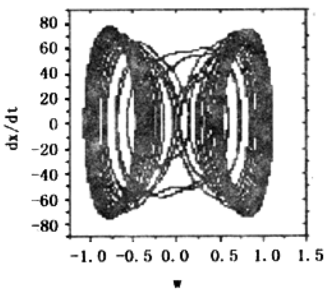
应用 Runge-Kutta-Merson 对给定的截断系统式(21)和式(22)进行数值求解,并应用非线性动力学的数值分析方法,可以得到系统的时程曲线,相平面图,Poincare截面和分叉图等.计算中我们首先取 $\alpha_1 = 1, \beta_2 = 10^5, \beta_3 = 6.67\times10^4, \beta_4 = 3.33\times10^5, \beta_5 = 5\times10^3, \beta_6 = 36.1, \beta_7 = 4.17\times10^3, \mu = 0.23, c_1 = 0.9, q_0 = q\sin(2\pi t)$,考察长厚比 β_1,材料参数 α 和载荷幅值 q 的影响.通过分析可知,1阶 Galerkin 截断系统的动力学性质和2阶 Galerkin 截断系统的性质定性一致(可参见图5),所以分析中我们只给出了2阶 Galerkin 截断的动力学图.图1和图2分别给出了不同参数时,2阶 Galerkin 截断系统的动力学图.



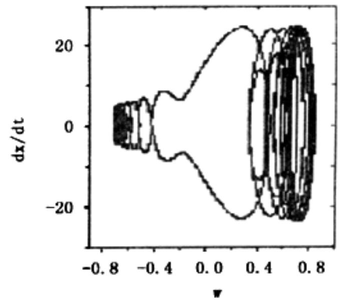
(a)　$\beta_1 = 4.2$



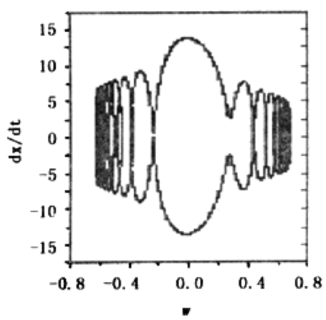
(b)　$\beta_1 = 4$



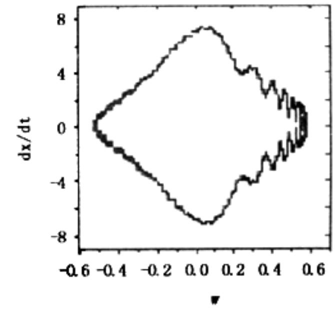
(c)　$\beta_1 = 3.8$



(d)　$\beta_1 = 3.6$



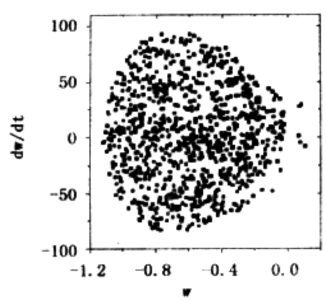
(e)　$\beta_1 = 3.4$



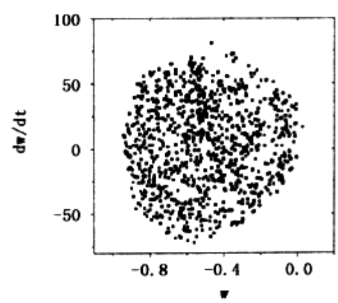
(f)　$\beta_1 = 3.2$

图 1　$\alpha = 0.2, q = 0.1$ 时,挠度的相平面图

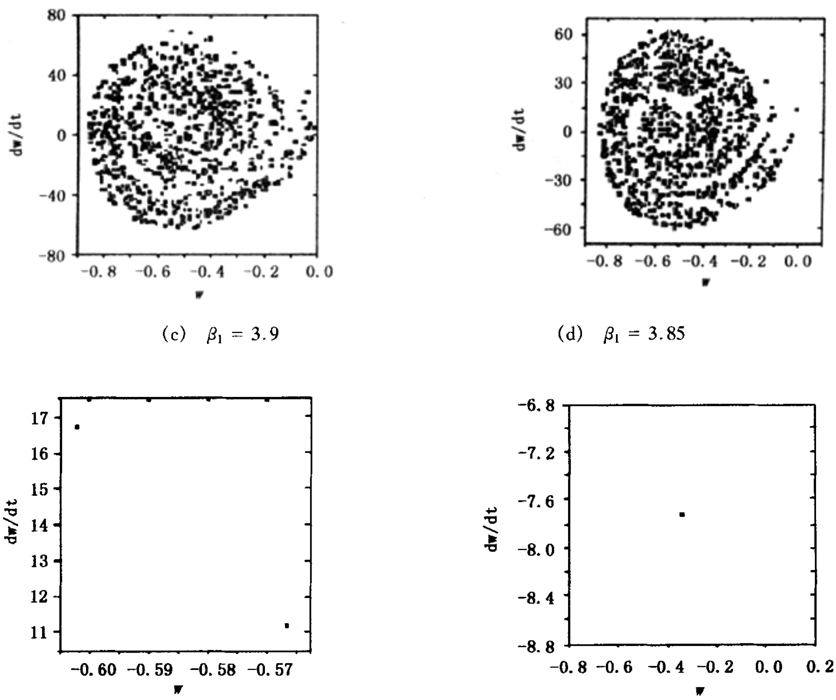
Fig. 1　Phase-trajectory of deflection for $\alpha = 0.2, q = 0.1$



(a)　$\beta_1 = 4.2$



(b)　$\beta_1 = 4$



(c) $\beta_1 = 3.9$　(d) $\beta_1 = 3.85$

(e) $\beta_1 = 3.8$　(f) $\beta_1 = 3.5$

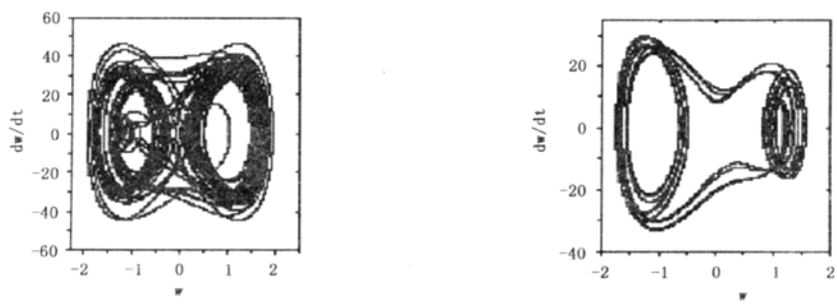
图2　$\alpha = 0.2, q = 0.1$时，挠度的Poincare截面

Fig.2　Poincare sections of deflection for $\alpha = 0.2, q = 0.1$

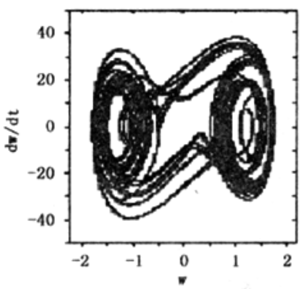
在$\beta_1 = 10, q = 0.01$时，图3给出了在不同α(松弛时间的倒数)时，系统(21)和式(22)的挠度相平面图.基于以上分析，可以看到增加α的值，将会抑制混沌运动的发生.

若取$\beta_1 = 10, q = 0.2$，考察不同载荷参数对动力系统式(21)～式(22)的影响.得到的相平面图示于图4中.由图可以看出，随着载荷参数q的增大，板将会由规则的周期运动向不规则的混沌运动转化.

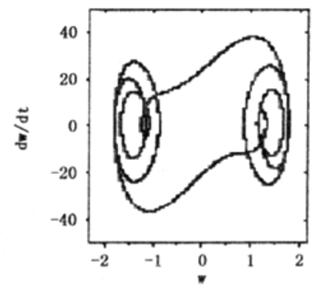
图5给出了当取$\beta_1 = 10, \alpha = 0.2$，时，系统随着载荷参数$q$变化的挠度分叉图.从图5可看出，1阶截断系统和2阶截断系统的动力性质定性基本一致.



(a) $\alpha = 0.2$　(b) $\alpha = 2$



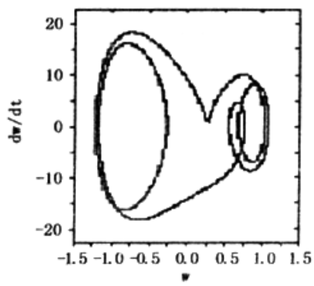
(c)　α =



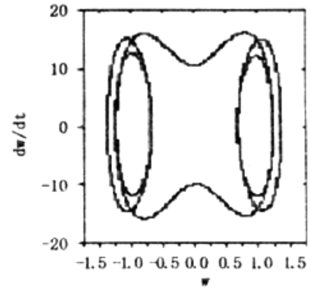
(d)　$\alpha = 10.0$

图 3　$\beta_1 = 10, q = 0.01$ 时,挠度相平面图

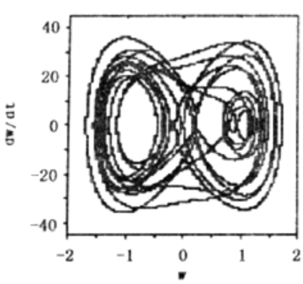
Fig. 3　Phase-trajectory diagrams of deflection for $\beta_1 = 10, q = 0.01$



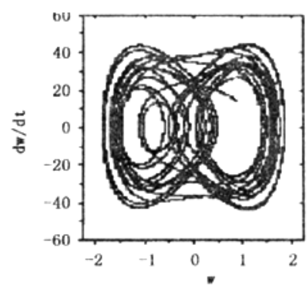
(a)　$q = 0.006$



(b)　$q = 0.007$



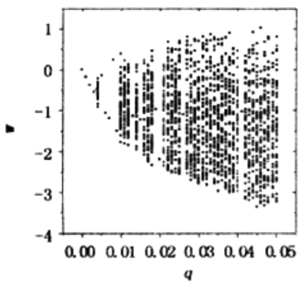
(c)　$q = 0.0075$



(d)　$q = 0.008$

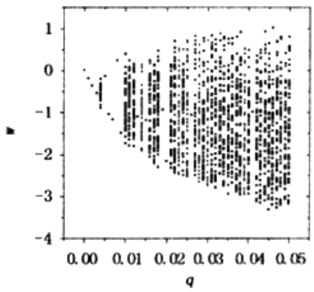
图 4　$\beta_1 = 10, q = 0.2$ 时,挠度相平面图

Fig. 4　Phase-trajectory diagrams of deflection for $\beta_1 = 10, q = 0.2$



(a)　1 阶截断系统

(a)　First-order truncated system



(b)　2 阶截断系统

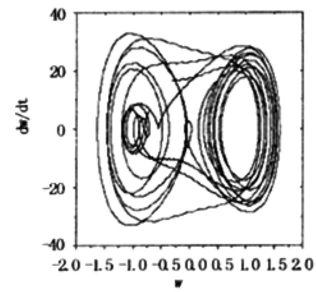
(b)　Second-order truncated system

图 5　$\beta_1 = 10, \alpha = 0.2$ 时,挠度的分叉图

Fig. 5　Bifurcation figures of deflection for $\beta_1 = 10, \alpha = 0.2$

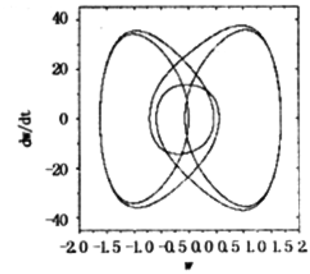
4 材料损伤对板动力学行为的影响

为了分析材料损伤对板动力学行为的影响，我们在相同的材料参数和板的几何参数条件下，比较了考虑损伤和无损伤的粘弹性板的动力学性质．取材料参数分别选定为 $\beta_1 = 10, \alpha = 0.2, q = 0.008$ and $\beta_3 = 1.334\times10^5, \beta_7 = 8.34\times10^3$（损伤参数 β_3 和 β_7 为上节给定的两倍），其它材料和上节给出的材料参数相同．图6给出了有损伤和无损伤粘弹性中厚板的动力学特性．可以看出，当 $q = 0.008$，有损伤的粘弹性板处于混沌运动，而无损伤的板在相同的载荷条件下仍处于稳定的周期运动，说明损伤增强了板的不稳定性，对板的结构稳定性是有害的．我们要减小损伤参数 β_3 和 β_7，从而保证结构的稳定性．由图可以看出，随着载荷参数 q 的增大，板将会由规则的周期运动向不规则的混沌运动转化．



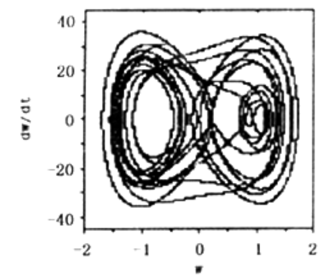
(a) 有损伤挠度的相平面图

(a) Phase-trajectory diagram of deflection with damage



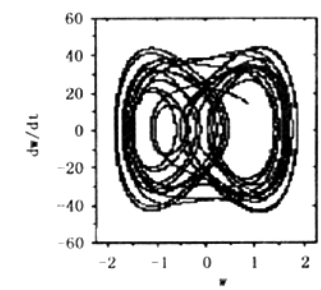
(b) 无损伤挠度的相平面图

(b) phase-trajectory diagram of deflection without damage



(c) 有损伤挠度的 Poincare 截面

(c) Poincare section of deflection with damage



(d) 无损伤挠度的 Poincare 截面

(b) Poincare section of deflection without damage

图6 有损伤和无损伤板的动力学行为的比较

Fig.6 Comparison of dynamical properties of plates with and without damage

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DYNAMICAL BEHAVIORS OF NONLINEAR VISCOELASTIC THICK PLATES WITH DAMAGE*

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Abstract Based on the deformation hypothesis of Timoshenko's plates and the Boltzmann's superposition principles for linear viscoelastic materials, the nonlinear equations governing the dynamical behavior of Timoshenko's viscoelastic thick plates with damage were derived, and the Galerkin method was applied to simplify the equations. The numerical methods in nonlinear dynamical systems were used to solve the simplified systems. It could be seen that there are plenty of dynamical properties for dynamical systems formed by this kind of viscoelastic thick plates with damage under a transverse harmonic load. The influences of load, geometry and material parameters on the dynamical behavior of the nonlinear system were investigated, and the effect of damage on the dynamical behavior of plate was discussed.

Key words viscoelastic solid with damage, thick plate, geometrical non-linearity, nonlinear dynamic system, bifurcation, chaos

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