地基波动影响下非线性粘弹性桩的混沌运动分析*

任九生 程昌钧
（上海大学力学系，上海市应用数学和力学研究所，上海 200072）

摘要 研究了在地基波动影响下非线性粘弹性桩中的混沌运动. 假定桩体材料满足 Leaderman 非线性粘弹性本构关系，得到在轴向载荷作用下满足 Winkler 条件的地基土波动方程、桩与地基土耦合振动方程；利用 Galerkin 方法将非线性积分-微分方程简化，并进行了数值计算，揭示了非线性粘弹性桩包括混沌运动在内的动力学行为.

关键词 非线性粘弹性桩，地基波动，Galerkin 方法，混沌运动

引言

桩基础具有承载力大，稳定性好，沉降量小及能够适应各种地质条件和各种载荷情况等诸多优点，在建筑工程中得到了广泛的应用. 和桩基的静力设计理论相比，桩基动力学及桩基抗震的工程经验相对不足. 1987 年 Mizuno[1]综述了日本地震过程中桩基的破坏形式，如液化、振动效应等，指出需要对桩基进行详细的动力学分析，以提供桩基设计更适当的理论分析，使桩基的抗震性能得到不断的完善. 包括桩基的动力响应分析、稳定性分析、振动分析及动力学性质分析的一些理论、计算方法和结论可见 Novak[2]的综述报告. 对粘弹性桩及粘弹性梁、柱等结构的分析往往采用微分型，Boltzmann 叠加原理或 Leaderman 本构关系，对其动力学行为的研究常常采用 Galerkin 方法.[3] Cederbaum 和 Moon[4]应用多尺度法分析了粘弹性柱的稳定性，Surie 和 Cederbaum[5]利用 Liapunov 的概念研究了几何非线性粘弹性柱的稳定性，陈立群和程昌钧[6,7]分析了非线性粘弹性柱和非线性粘弹性梁的稳定性和混沌运动. 刘宗贤等[8,9]得到了地基波动影响下端承桩的横向地震反应解析解.

本文分析了在轴向周期载荷作用与地基波动影响下，满足 Leaderman 非线性粘弹性本构关系的粘弹性桩在横向运动中的混沌运动. 文中首先导出了满足 Winkler 条件的地基土波动方程、桩与地基土耦合振动方程——耦合的非线性积分－偏微分方程；然后采用 Galerkin 方法进行简化，得到了一个简化后的动力学系统；应用动力学系统中的数值方法，得到了不同载荷参数、几何参数、材料参数时粘弹性桩横向运动的时程曲线、功率谱、相图、Poincare 截面图及位移分叉图. 数值结果表明横向运动中的粘弹性桩有可能发生混沌运动，且其载荷参数、几何参数、材料参数等对其运动方式有较大的影响.

1 粘弹性桩的数学模型

考虑一长为 L，半径为 d，横截面积为 A，单位长度质量为 ρ 的桩体，设桩的轴向向上为 x 的正方向，横向为 z，桩底中心处为原点，承受一轴向周期载荷 $P(t)$ 的作用，记其横向位移为 $w(x,t)$，截面弯矩为 $M(x,t)$，并设地基土满足 Winkler 条件，并记其横向位移为 $u(x,t)$，则地基土波动方程、桩与地基土耦合振动方程分别为

$$v_h \frac{\partial u^2(x,t)}{\partial x^2} - \frac{c_h}{\rho_h}\frac{\partial u(x,t)}{\partial x} - \frac{\partial u^2(x,t)}{\partial t^2} = 0 \tag{1}$$

$$\rho A \frac{\partial w^2(x,t)}{\partial t^2} + \frac{\partial M^2(x,t)}{\partial x^2} + P(t)\frac{\partial w^2(x,t)}{\partial x^2} + k_t(w+u) + c_f\left(\frac{\partial w(x,t)}{\partial t} + \frac{\partial u(x,t)}{\partial t}\right) = 0 \tag{2}$$

式中 $v_h, c_h, \rho_h, k_t, c_f$ 分别为地基土的剪切波速、材

2004-10-29 收到第1稿，2004-12-20 收到修改稿.
* 国家自然科学基金资助项目(50278051)；上海市重点学科建设资助项目

料阻尼系数、质量密度、抗力系数、非线性辐射阻尼系数,且有

$$M(x,t) = -\int_{-d/2}^{d/2} z\sigma(x,z,t)\mathrm{d}z \qquad (3)$$

其中

$$\sigma(x,z,t) = E_0 g(x,z,t) + \int_0^t \dot{E}(t-\tau)g(x,z,t)\mathrm{d}\tau \qquad (4)$$

为桩身材料满足 Leaderman 非线性粘弹性本构关系的轴向应力分量,$E(t)$ 为材料的松弛函数,$E_0 = E(0)$ 为材料的初始弹性模量;非线性函数

$$g(\varepsilon(x,z,t)) = \varepsilon(x,z,t) + \beta\varepsilon^2(x,z,t) + \gamma\varepsilon^3(x,z,t) \qquad (5)$$

其中 $\varepsilon(x,z,t)$ 为桩的轴向应变,材料常数 β,γ 应使得在小应变时 $g(\varepsilon)\to\varepsilon$. 小变形条件下,轴向应变满足的几何关系为

$$\varepsilon(x,z,t) = -z\frac{\partial^2 w(x,t)}{\partial x^2} \qquad (6)$$

式(3),(4),(5) 和式(6) 代入式(2) 可得

$$\rho A\ddot{w} + E_0\left[\frac{d^3}{12}w_{,xxxx} + \frac{3\gamma d^5}{80}(w_{,xxx}w_{,xxx} + w_{,xx}w_{,xxxx})\right] + \int_0^t \dot{E}(t-\tau)\times \left[\frac{d^3}{12}w_{,xxxx} + \frac{3\gamma d^5}{80}(w_{,xxx}w_{,xxx} + w_{,xx}w_{,xxxx})\right]\mathrm{d}\tau + P(t)w_{,xx} + k_t(w+u) + c_f\left(\frac{\partial w}{\partial t} + \frac{\partial u}{\partial t}\right) = 0 \qquad (7)$$

考虑嵌岩端承桩,桩底处应满足固定端边界条件[8,9]

$$w(x,t)|_{x=0} = w_{,x}(x,t)|_{x=0} = 0 \qquad (8)$$

对于嵌固于承台或地基梁中的桩头,可假定其转角为零,则桩头应满足边界条件[10]

$$w_{,x}(x,t)|_{x=L} = 0,$$
$$-Pw_{,x}(x,t)|_{x=L} + M_{,x}(x,t)|_{x=L} = 0 \qquad (9)$$

地基土应满足边界条件[8,9]

$$u(x,t)|_{x=0} = u_{,x}(x,t)|_{x=L} = 0 \qquad (10)$$

假设初始状态时桩体处于静止状态,则相应的初始条件为

$$w(x,t)|_{t=0} = w^0,$$
$$\dot{w}(x,t)|_{t=0} = \dot{w}^0 \qquad (11)$$

$$u(x,t)|_{t=0} = u^0,$$
$$\dot{u}(x,t)|_{t=0} = \dot{u}^0 \qquad (12)$$

2 问题的求解

方程(1),(7) 及其边界条件(8),(9),(10) 和初始条件(11),(12) 构成了考虑桩与地基土耦合振动的基桩动力学行为的初边值问题,这是一组耦合的非线性积分 - 偏微分方程组,得到问题的解析解是困难的,我们采用 Galerkin 方法对问题进行简化求解. 根据边界条件,问题的解可取如下形式

$$w(x,t) = \sum_{k=1}^{\infty} w_k(t)\left(1-\cos\frac{k\pi x}{L}\right) \qquad (13)$$

$$u(x,t) = \sum_{k=1}^{\infty} u_k(t)\left(1-\cos\frac{k\pi x}{L}\right) \qquad (14)$$

仅考虑 $k=1$ 的项,即仅对问题进行一阶 Galerkin 截断.[3] 若材料是标准线性固体,则有松弛函数满足的条件

$$e(t) = \frac{E(t)}{E_0} = a + b\exp(-\alpha t), a+b=1 \qquad (15)$$

引入无量纲参量

$$\beta_1 = \frac{E_0 d^3\pi^4}{12\rho L^4 A}, \beta_2 = \frac{9E_0\gamma d^5\pi^8}{320\rho AL^8},$$
$$\beta_3 = \frac{\pi^2}{\rho AL^2}, \beta_4 = \frac{c_f}{\rho A},$$
$$\beta_5 = \frac{k_t}{\rho A}, \beta_6 = \frac{c_h}{\rho_h}, \beta_7 = \frac{v_h\pi^2}{L^2}$$

并作如下的变量变换

$$y_0 = t, y_1 = w_1(t), y_2 = \dot{w}_1(t),$$
$$y_3 = \int_0^t \dot{e}(t-\tau)w_1(\tau)\mathrm{d}\tau,$$
$$y_4 = \int_0^t \dot{e}(t-\tau)w_1^3(\tau)\mathrm{d}\tau,$$
$$y_5 = u_1, y_6 = \dot{u}_1(t) \qquad (16)$$

则耦合的积分 - 微分方程组(1),(7) 可化为如下常微分方程组

$$\dot{y}_0 = 1$$
$$\dot{y}_1 = y_2$$
$$\dot{y}_2 = -\beta_1 y_1 - \beta_2 y_1^3 - \beta_1 y_3 - \beta_2 y_4 + \beta_3 y_1 P_0\sin 2\pi t - \beta_4 y_2 - \beta_4 y_6 - \beta_5 y_1 - \beta_5 y_5$$
$$\dot{y}_3 = -\alpha(by_1 + y_3)$$

$$\dot{y}_4 = -\alpha(by_1^3 + y_4)$$
$$\dot{y}_5 = y_6$$
$$\dot{y}_6 = -\beta_6 y_6 - \beta_7 y_5 \tag{17}$$

式中已设轴向周期载荷具有形式 $p(t) = p_0 \sin 2\pi t$.

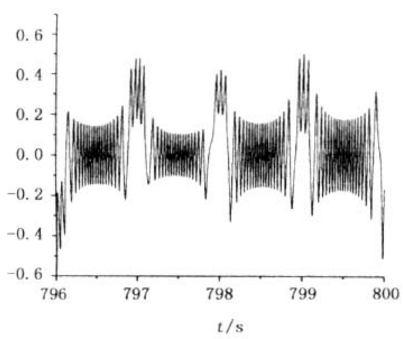
由初始条件(11),(12),可知方程组(17)满足初始条件

$$\{y_1(0), y_2(0), y_3(0), y_4(0), y_5(0), y_6(0)\} = (w_1^0, \dot{w}_1^0, 0, 0, u_1^0, \dot{u}_1^0) \tag{18}$$

3　数值结果和结论

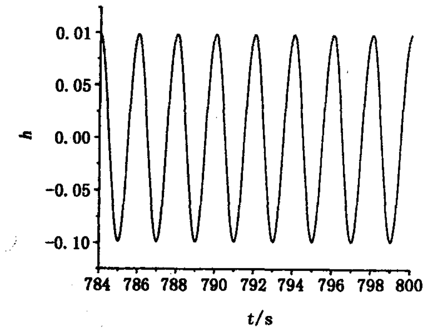
应用 Lunge-Kutta-Merson 方法对方程组(17),(18)进行数值求解,并应用非线性动力学的数值分析方法,可得到各种参数情况下系统的时程曲线、相图、Poincare 截面图及基桩横向位移对载荷参数 P_0,材料参数 α,几何参数——桩的长径比 $l = L/d$ 的分叉图.计算中我们取如下参数:$E_0 = 3.0 \times 10^4$ MPa, $\rho = 4/\pi \times 2.5 \times 10^9$ kg/m, $d = 1.0$ m, $\gamma = 2000$, $a = 0.9$, $v_h = 100$ m/s, $c_h = 0.05$, $\rho_h = 2.5 \times 10^3$ kg/m^3, $k_t = 2.75$, $G_h = 5.5$ MPa, $c_f = 0.05$.

图(1)～图(4)给出了基桩发生横向混沌运动的时程曲线、相图、Poincare 截面图及地基土运动的时程曲线;图(5)～图(7)给出了基桩横向位移随载荷参数 P_0,桩身长径比 $l = L/d$,材料参数 α 的分叉图.



(a) 基桩横向位移

(a)Transverse motion for the pile



(b) 地基土横向位移

(b)Transverse motion for the soil

图 1　桩体混沌运动时程曲线

(p = 50 MPa, l = 10.0, α = 0.0001)

Fig. 1　Time history curve for chaotic motion of the pile

(p = 50 MPa, l = 10.0, α = 0.0001)

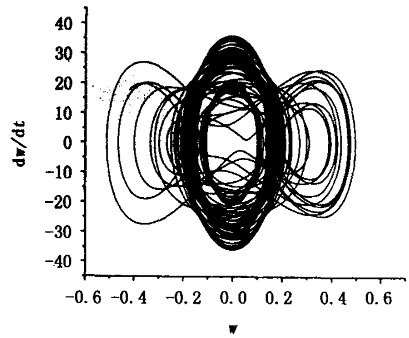


图 2　混沌运动相图

(p = 50 MPa, l = 10.0, α = 0.0001)

Fig. 2　Phase plane portrait for chaotic motion

(p = 50 MPa, l = 10.0, α = 0.0001)

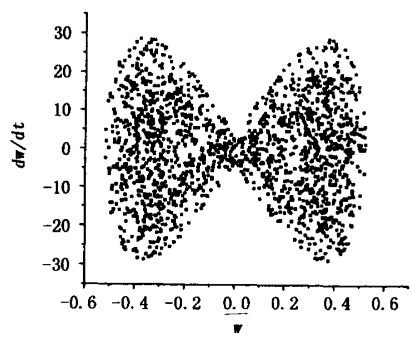


图 3　混沌运动 Poincare 截面图

(p = 50 MPa, l = 10.0, α = 0.0001)

Fig. 3　Poincare map for chaotic motion

(p = 50 MPa, l = 10.0, α = 0.0001)

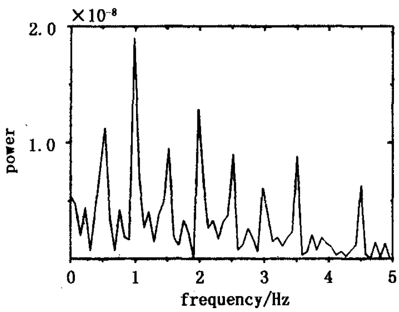


图4　FFT频谱图

(p = 50 MPa, l = 10.0, α = 0.0001)

Fig.4　Power spectrum for chaotic motion

(p = 50 MPa, l = 10.0, α = 0.0001)

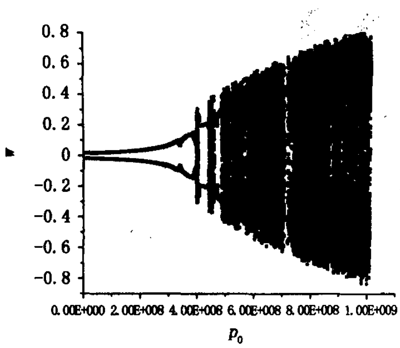


图5　位移-载荷分叉图

(l = 10.0, α = 0.00001)

Fig.5　Bifurcation plot for load

(l = 10.0, α = 0.00001)

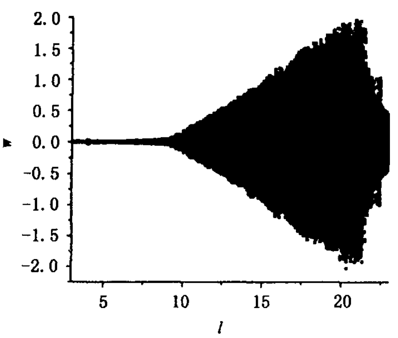


图6　位移-长径比分叉图

(p = 50 MPa, α = 0.000001)

Fig.6　Bifurcation plot for $l = L/d$

(p = 50 MPa, α = 0.00001)

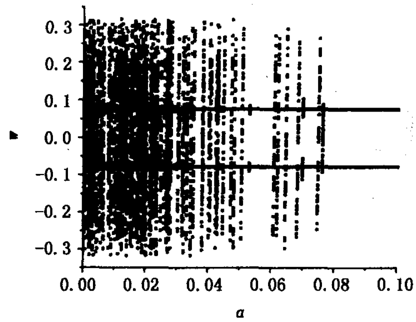


图7　位移-材料参数分叉图

(p = 50 MPa, l = 10.0)

Fig.7　Bifurcation plot for α

(p = 50 MPa, l = 10.0)

由图可见,基桩的横向运动形式可以是周期运动、准周期运动,并有可能发生混沌运动,如当(p = 50 MPa, l = 10.0, α = 0.00001)时,桩体的横向运动为混沌运动;载荷参数 P_0,几何参数 $l = L/d$,材料参数 α 等均对桩体的运动形式有较大的影响.随着载荷参数的增加,桩体的运动形式分别为周期运动、准周期运动,当载荷参数大于某一临界值时,运动形式为混沌运动,但混沌运动中有周期运动的窗口.随着桩体长径比的增加,桩体的运动形式分别为周期运动、准周期运动及混沌运动.材料参数 α 的增加有抑制桩体混沌运动的效应,即随着材料参数 α 的增加,桩体的运动形式分别是混沌运动、准周期运动及周期运动;反过来,随材料参数 α 的减小,桩体的运动形式分别为周期运动、准周期运动及混沌运动.

参　考　文　献

1　Mizuno H. Pile damage during earthquake in Japan. In: Nogami T, ets. Dynamic response of pile foundation. New York: ASCE, 1987. 53～78

2　Novak M. Piles under dynamic loads. Proceeding: Second International Conference on Recent Advances in Geotechnical Earthquake Engineering and Sore Dynamics. University of Missori-Rolla, 1991. 2433～2456

3　陈立群,程昌钧. 基于Galerkin截断粘弹性结构动力学行为研究综述. 自然杂志, 1999, 21(1): 1～4 (Chen Liqun, Cheng Changjun. On investigations of dynamic behavior of viscoelastic structures based on the Galerkin method. *Nature Magazine*, 1999, 21(1): 1～4 (in Chinese))

4 Cederbaum G, Mond M. Stability properties of a viscoelastic column under a period force. *J Appl Mech*, 1992, 59:16～19

5 Suire G, Cederbaum G. Periodic and chaotic behavior of viscoelastic nonlinear bars under harmonic excitations. *Int J Mechanics Science*, 1995,37: 753～772

6 陈立群,程昌钧. 非线性粘弹性柱的稳定性和混沌运动.应用数学和力学,2000,21(9):890～896(Chen Liqun, Cheng Changjun. Stability and chaotic motion in columns of nonlinear viscoelastic material. *Applied Mathematics and Mechanics*,2000, 21(9): 890～896(in Chinese))

7 陈立群,程昌钧.非线性粘弹性梁的动力学行为.应用数学和力学,2000,21(9):897～902 (Chen Liqun, Cheng Changjun. Dynamic behavior of nonlinear viscoelastic beams. *Applied Mathematics and Mechanics*, 2000, 21(9): 897～902(in Chinese))

8 刘宗贤,李玉亭,傅文彬. 端承桩在层状地基波动与辐射阻尼影响下的横向地震反应分析.东北地震研究,1998,14(2):43～48 (Liu Zhongxian, Li Yuting, Fu Wenbin. Transverse earthquake response of pile under the motion of layered soils and pile viscosity. *Seismological Research of Northeast China*, 1998 14(2): 43～48 (in Chinese))

9 刘宗贤,李玉亭,傅文彬. 在常轴力与阻尼力作用及地基波动影响下桩的横向地震反应分析.地震学刊,1995(2):28～36 (Liu Zhongxian, Li Yuting, Fu Wenbin. Transverse earthquake response of pile under the axial force and pile viscosity. *Journal of Seismology*, 1995(2): 28～36 (in Chinese))

10 Nogami T, Novak M. Soil-pile interaction in horizontal vibration. *Earthquake Engineering and Strucatral Dynamics*,1977, 5:263～281

CHAOTIC MOTION ANALYSIS IN PILES OF NONLINEAR VISCOELASTIC MATERIALS CONSIDERING THE MOTION OF GROUND SOIL *

Ren Jiusheng Cheng Changjun

(*Department of Mechanics, Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai* 200072, *China*)

Abstract This paper investigated the chaotic motion for a nonlinear viscoelastic pile subjected to a periodical axial force considering the ground soil motion. The material of the pile was assumed to obey the nonlinear Leaderman viscoelastic relation. The equations for the ground soil motion satisfying the Winkler condition and the equations for the transverse motion of pile were derived as the coupled nonlinear integro-partial-differential equations. The equations were simplified into ordinary differential equations by the Galerkin method. Numerical results indicated that there were lots of dynamical behaviors including chaotic motion in the viscoelastic piles.

Key words nonlinear viscoelastic piles, motion of ground soil, the Galerkin method, chaotic motion

Received 29 October 2004, revised 20 December 2004.

* The project supported by the National Natural Science Foundation of China(50278051) and the Municipal Key Subject Programs of Shanghai.