柔性悬臂梁非线性非平面运动的多脉冲轨道分析*

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摘要　首先建立了柔性悬臂梁非线性非平面运动的偏微分方程;然后运用Galerkin和多尺度方法得到平均方程,并利用规范形理论进一步将方程化简;最后用能量相位法求出多脉冲跳跃的能量函数序列. Dynamics软件数值计算表明:在系统中确实存在着由多脉冲跳跃而导致的Smale马蹄型混沌.

关键词　柔性悬臂梁,多脉冲轨道,非线性非平面运动,混沌动力学

前　言

柔性悬臂梁广泛应用于航空航天、机械等工程领域,因此对它的研究有着重要的意义.目前国内外对非线性非平面柔性悬臂梁的研究成果还非常有限. Crespo[1,2]和Nayfeh[3,4]等学者建立了运动偏微分方程,运用Galerkin方法和多尺度法研究了悬臂梁的响应,此外他们还从试验的角度观察到了一些模态的响应.由于研究工具的有限,对高维非线性系统的全局分析主要有两种方法,一种是由Kovacic和Wiggins发展的广义Melnikov方法,Wiggins[5]曾经用Melnikov方法把两个自由度的Hamilton系统分为3类;另外一种方法就是由Haller[6~8]和Wiggins综合了几何奇异摄动理论、高维Melnikov理论和横截理论而提出的能量相位法.

1　运动方程的建立和简化

我们用Hamilton原理推导梁的非平面运动方程,梁的长度为L,梁的单位质量为m,梁的一端固定,另一端受两个横向载荷$2F_2(s)\cos\Omega_2 t$, $2F_3(s)\cos\Omega_2 t$和一个轴向载荷$2F_1\cos\Omega_1 t$.对此弹性梁我们做如下假设,悬臂梁为均匀弹性梁,梁在变形前是一直梁,变形过程中长度不可伸长.于是就得到非平面运动柔性悬臂梁的控制方程为

$$\ddot{v}+\bar{c}\dot{v}+\beta_y v^{iv}+F_1\cos(\Omega_1 t)v''=$$
$$(1-\beta_y)\left[w''\int_1^s v''w''\mathrm{d}s-w'''\int_0^s v''w'\,\mathrm{d}s\right]'-$$
$$\frac{1}{\beta_\gamma}(1-\beta_y)^2\left[w''\int_0^s\int_1^s v''w''\mathrm{d}s\mathrm{d}s\right]''-$$
$$\beta_y[v'(v'v''+w'w'')']'-$$
$$\frac{1}{2}\left[v'\int_1^s\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left\{\int_0^s(v'^2+w'^2)\mathrm{d}s\right\}\mathrm{d}s\right]'-$$
$$F_1\cos(\Omega_1 t)[v'(v'^2+w'^2)]'+$$
$$F_2(s)\cos(\Omega_2 t),\tag{1a}$$
$$\ddot{w}+\bar{c}\dot{w}+w^{iv}+F_1\cos(\Omega_1 t)w''=$$
$$-(1-\beta_y)\left[v''\int_1^s v''w''\mathrm{d}s-v'''\int_0^s w''v'\,\mathrm{d}s\right]'-$$
$$\frac{1}{\beta_\gamma}(1-\beta_y)^2\left[v''\int_0^s\int_1^s v''w''\mathrm{d}s\mathrm{d}s\right]''-$$
$$[w'(v'v''+w'w'')']'-$$
$$\frac{1}{2}\left[w'\int_1^s\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left\{\int_0^s(v'^2+w'^2)\mathrm{d}s\right\}\mathrm{d}s\right]'-$$
$$F_1\cos(\Omega_1 t)[w'(v'^2+w'^2)]'+$$
$$F_3(s)\cos(\Omega_2 t).\tag{1b}$$

对上述方程无量纲化,运用多尺度方法得到直角坐标形式的主共振-主参数共振和1∶2内共振的平均方程

$$\dot{x}_1=-\frac{1}{2}cx_1-(\sigma_1+\alpha_1F_1)x_2+$$
$$\frac{1}{16}(2\alpha_2-3\alpha_3)x_2(x_1^2+x_2^2)-$$
$$\beta_1x_2(x_3^2+x_4^2),\tag{2a}$$
$$\dot{x}_2=(\sigma_1-\alpha_1F_1)x_1-\frac{1}{2}cx_2-$$
$$\frac{1}{16}(2\alpha_2-3\alpha_3)x_1(x_1^2+x_2^2)+$$
$$\beta_1x_1(x_3^2+x_4^2),\tag{2b}$$

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$$\dot{x}_3=-\frac{1}{2}cx_3-\frac{1}{2}\sigma_2x_4-\beta_2x_4(x_1^2+x_2^2)+\frac{1}{8}(2\alpha_2-3\alpha_3)x_4(x_3^2+x_4^2), \quad (2c)$$

$$\dot{x}_4=-\frac{1}{2}f_2+\frac{1}{2}\sigma_2x_3-\frac{1}{2}cx_4+\beta_2x_3(x_1^2+x_2^2)-\frac{1}{8}(2\alpha_2-3\alpha_3)x_3(x_3^2+x_4^2). \quad (2d)$$

利用规范形理论对方程(2)进一步化简得如下形式

$$\dot{u}_1=u_2,$$
$$\dot{u}_2=-\mu_1u_1-\mu_2u_2+\eta_1u_1^3+\beta_1u_1I^2, \quad (3a)$$
$$\dot{I}=-\bar{\mu}I-\bar{f}_2\sin\gamma,$$
$$I\dot{\gamma}=\bar{\sigma}_2I-\eta_2I^3+\beta_1Iu_1^2-\bar{f}_2\cos\gamma. \quad (3b)$$

引入如下的尺度变换

$$\mu_2\to\varepsilon\mu_2,\bar{\mu}\to\varepsilon\bar{\mu},\bar{f}_2\to\varepsilon\bar{f}_2, \quad (4)$$

得到带有扰动的规范形

$$\dot{u}_1=\frac{\partial H}{\partial u_2}+\varepsilon g^{u_1}=u_2,$$
$$\dot{u}=-\frac{\partial H}{\partial u_1}+\varepsilon g^{u_2}=-\mu_1u_1+\eta_1u_1^3+\beta_1u_1I^2-\varepsilon\mu_2u_2, \quad (5a)$$
$$\dot{I}=\frac{\partial H}{\partial\gamma}+\varepsilon g^I\text{-}\varepsilon\bar{f}_2\sin\gamma=-\varepsilon\bar{\mu}I-\varepsilon\bar{f}_2\sin\gamma,$$
$$I\dot{\gamma}=-\frac{\partial H}{\partial I}+\varepsilon g^\gamma-\varepsilon\bar{f}_2\cos\gamma=\bar{\sigma}_2I-\eta_2I^3+\beta_1Iu_1^2-\varepsilon\bar{f}_2\cos\gamma, \quad (5b)$$

其中 Hamilton 函数的形式如下

$$H(u_1,u_2,I,\gamma)=\frac{1}{2}u_2^2+\frac{1}{2}\mu_1u_1^2-\frac{1}{4}\eta_1u_1^4-\frac{1}{2}\beta_1I^2u_1^2-\frac{1}{2}\bar{\sigma}_2I^2+\frac{1}{4}\eta_2I^4, \quad (6)$$

而且

$$g^{u_1}=0,g^{u_2}=-\mu_2u_2,g^I=-\bar{\mu}I,g^\gamma=0. \quad (7)$$

2 解耦系统的动力学分析

当 $\varepsilon\to0$ 时，方程(5)是一个解耦的非线性系统，分析带有扰动项的前两个解耦方程

$$\dot{u}_1=u_2,$$
$$\dot{\mu}_2=-\mu_1u_1+\eta_1u_1^3+\beta_1I^2u_1-\varepsilon\mu_2u_2. \quad (8)$$

经分析知道当 $\eta_1>0$ 时，解耦系统有一个中心点和两个鞍点，即 $q_0=(0,0)$，$q_\pm(I)=(B,0)$，其中 $B=\pm\{[\bar{\mu}^2-\bar{\sigma}_1(1-\bar{\sigma}_1)-\beta_1I^2]/\eta_1\}^{1/2}$. 因此系统(8)会出现异宿分叉，求解方程(8)得出两条异宿曲线为

$$u_1(T_1)=\pm\sqrt{\frac{\varepsilon_1}{\eta_1}}\tanh\left[\frac{\sqrt{2\varepsilon_1}}{2}T_1\right],$$
$$u_2(T_1)=\pm\frac{\varepsilon_1}{\sqrt{2\eta_1}}\mathrm{sech}^2\left[\frac{\sqrt{2\varepsilon_1}}{2}T_1\right] \quad (9)$$

3 扰动系统的全局分析

在这一部分，我们分析方程(3)的后两个方程，考虑小扰动 ε 对流形 M 的影响，其中 $M=M_\varepsilon=\{(u,I,\gamma)\mid u=q_\pm(I),I_1\le I\le I_2,0\le\gamma<2\pi\}$，则方程(3)的后两个方程

$$\dot{I}=-\bar{\mu}I-\bar{f}_2\sin\gamma,$$
$$\dot{\gamma}=\bar{\sigma}_2-\eta_2I^2+\beta_1u_1^2-\frac{\bar{f}_2}{I}\cos\gamma. \quad (10)$$

引入如下尺度变换

$$\bar{\mu}\to\varepsilon\bar{\mu},\quad I=I_r+\sqrt{\varepsilon}h,$$
$$\bar{f}_2\to\varepsilon\bar{f}_2,\quad T_1\to\frac{T_1}{\sqrt{\varepsilon}} \quad (11)$$

将变换(11)代入方程(10)，可得

$$\dot{h}=-\bar{\mu}I_r-\bar{f}_2\sin\gamma-\sqrt{\varepsilon}h\bar{\mu},$$
$$\dot{\gamma}=-\frac{2\delta}{\eta_1}I_rh-\sqrt{\varepsilon}\left(\frac{\delta}{\eta_1}h^2+\frac{\bar{f}_2}{I_r}\cos\gamma\right) \quad (12)$$

方程(12)的未扰动部分是一个 Hamilton 系统，其 Hamilton 函数为

$$\hat{H}_D(h,\gamma)=-\bar{\mu}I_r\gamma+\bar{f}_2\cos\gamma+\frac{\delta}{\eta_1}I_rh^2. \quad (13)$$

经过分析发现未扰动部分有一个中心点和一个鞍点，即 $p_0=(0,\gamma_c)=(0,-\arcsin(\bar{\mu}I_r/\bar{f}_2))$ 和 $q_0=(0,\gamma_s)=(0,\pi+\arcsin(\bar{\mu}I_r/\bar{f}_2))$. 当考虑充分小的扰动时，$p_0$ 由中心点变为稳定焦点 p_ε，q_0 仍保持双曲奇点的性质即受扰动后仍为鞍点 q_ε.

4 用能量相位法分析多脉冲

根据 Haller 和 Wiggins 提出的能量相位法，我们写出能量差分函数

$$\Delta^n\hat{H}_D(\gamma)=\hat{H}_D(h,\gamma+n\Delta\gamma)-\hat{H}_D(h,\gamma)-n\iint_A\left[\frac{\mathrm{d}}{\mathrm{d}u_1}g^{u_1}(u_1,u_2,I,\gamma)+\frac{\mathrm{d}}{\mathrm{d}u_2}g^{u_2}(u_1,u_2,I,\gamma)\right]\mathrm{d}u_1\mathrm{d}u_2-n\int_{\partial A_1}g^I\mathrm{d}\gamma \quad (14)$$

经计算得到

$$\Delta^n\hat{H}_D(\gamma)=\bar{f}_2[\cos(\gamma+n\Delta\gamma)-\cos\gamma]-\frac{2n\mu_2\varepsilon_1\Delta\gamma}{3\beta_1} \quad (15)$$

求 $\gamma\in[-\pi/2,3\pi/2]$并且满足集合 $Z_{-}^{n}=\{(h,\gamma)\mid\Delta^{n}\hat{H}_{D}(\gamma)=0,D_{\gamma}\Delta^{n}\hat{H}_{D}(\gamma)\neq0\}$条件的能量差分函数的零点，得

$$\gamma_{0,1}^{n}=\frac{3\pi}{2}-\left(\frac{n\Delta\gamma}{2}+\alpha\right)\bmod 2\pi,$$

$$\gamma_{0,2}^{n}=\frac{3\pi}{2}-\left(\pi+\frac{n\Delta\gamma}{2}-\alpha\right)\bmod 2\pi,\quad(16)$$

其中

$$\alpha=\arcsin\frac{-nd\varepsilon_{1}\Delta\gamma}{3\beta_{1}\sin\frac{n\Delta\gamma}{2}}\quad(17)$$

因为每一个脉冲轨道对应着一个能量函数而且是唯一的，所以我们规定连接同宿轨线的能量函数为 $\bar{h}_{0}$，连接中心点的能量函数为 $\bar{h}_{\infty}$，连接周期轨线的能量函数为 $\bar{h}_{n}$，这样就得到如下的能量函数序列

$$\bar{h}_{0}=\hat{H}_{D}(0,\gamma^{s})=\frac{1}{2}\bar{f}_{2}\cdot\left[-dI_{r}(\pi+\arcsin\frac{1}{2}dI_{r})-\sqrt{4-(dI_{r})^{2}}\right],\quad(18a)$$

$$\bar{h}_{n}=\min[\hat{H}_{D}(0,\gamma_{0,1}^{n}),\hat{H}_{D}(0,\gamma_{0,2}^{n})],\quad(18b)$$

$$\bar{h}_{\infty}=\hat{H}_{D}(0,\gamma^{c})=\frac{1}{2}\bar{f}_{2}\left[dI_{r}\arcsin\frac{1}{2}dI_{r}+\sqrt{4-(dI_{r})^{2}}\right].\quad(18c)$$

我们画出 3 个脉冲的示意图如图 1 所示.

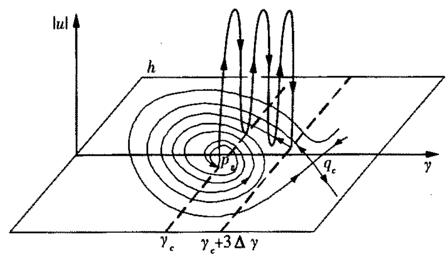

图 1　3 脉冲示意图

Fig. 1　The three-pulse homoclinic orbits

5　数值计算

为了进一步说明悬臂梁非平面运动中存在着由多脉冲跳跃而导致的混沌现象，我们用 Dynamics

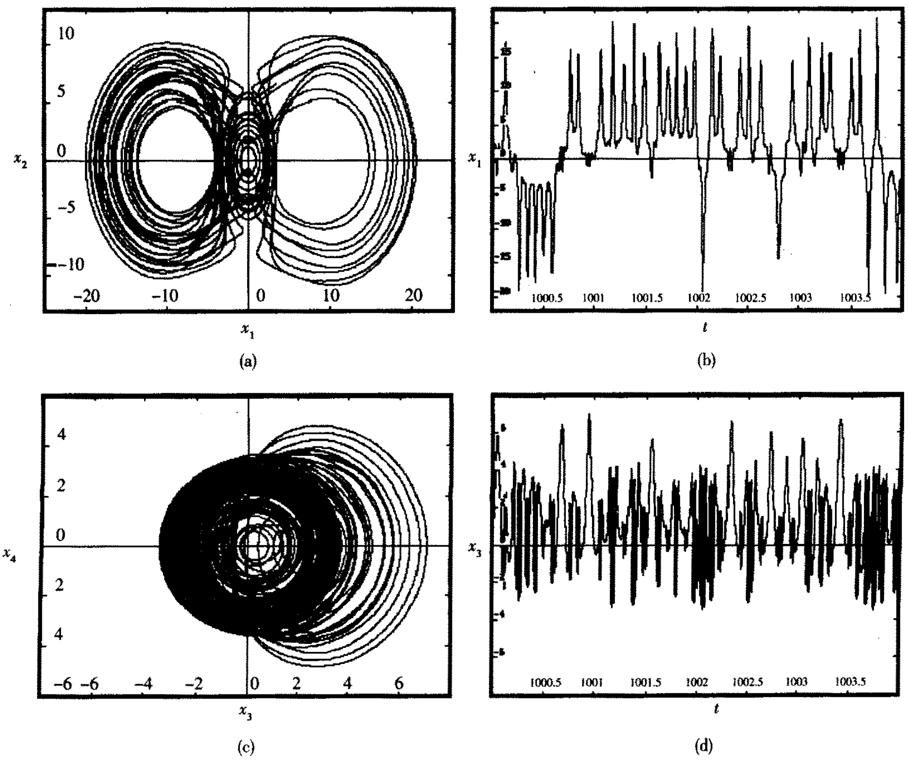

图 2　混沌的数值结果

$x_{10}=0.1385,x_{20}=0.055,x_{30}=0.35,x_{40}=0.180,c=0.1,\sigma_{1}=2.0,\sigma_{2}=6.5,\alpha_{2}=-4.2,\alpha_{3}=0.01,\beta_{1}=-5.1,\beta_{2}=2.3,f_{1}=49.8,f_{2}=216.8$

Fig. 2　The chaotic results of numerical simulation for

$x_{10}=0.1385,x_{20}=0.055,x_{30}=0.35,x_{40}=0.180,c=0.1,\sigma_{1}=2.0,\sigma_{2}=6.5,\alpha_{2}=-4.2,\alpha_{3}=0.01,\beta_{1}=-5.1,\beta_{2}=2.3,f_{1}=49.8,f_{2}=216.8$

软件对平均方程（2）进行数值计算，得出相图和波形图如图2所示.

6 结论

从上面的理论分析和数值计算，我们得出以下几个结论：首先，对于一个非线性动力系统来讲，是否能够出现多脉冲现象主要依赖系统的参数；第二，当时间趋向无穷大时，多脉冲跳跃现象会一致持续下去，最终导致系统出现 Smale 马蹄型混沌；第三，平面内和平面外之间是通过多脉冲来传递能量的.

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MULTI-PULSE ORBITS IN NONLINEAR NON-PLANAR MOTION OF A CANTILEVER BEAM*

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Abstract First we formulated a set of integral-partial differential governing equations, which describes the non-linear non-planar oscillations of a cantilever beam. Then by applying the Galerkin procedure and the multi-scale method, we obtained the averaged equations; From the partial differential governing equations and from the averaged equation and by using the theory of normal form, we found the explicit formulas of normal form. Based on the normal form obtained above, the dissipative version of the energy-phrase method was utilized to analyze the multi-pulse global bifurcations and chaotic dynamics in the nonlinear nonplanar oscillations of the cantilever beam, which predicted that there are some multi-pulse Shilnikov type orbits. The numerical simulations shows that the multi-pulse Shilnikov type orbits do exist in the nonlinear nonplanar oscillations of the cantilever beam.

Key words cantilever beam, multi-pulse Shilnikov orbits, nonlinear non-planar oscillations, chaotic dynamics

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