变撑臂渐硬性钢板弹簧非线性分析*

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摘要 采用单位荷载法推导变撑臂渐硬性钢板弹簧变形和外力之间的关系式,进而推导出非线性动力学方程,然后采用多尺度法对方程进行求解,得到了变撑臂渐硬性钢板弹簧的频响分岔方程和近似解表达式. 通过实例计算,得到了荷载和加载点之间的关系图、频率和加载点之间的关系图以及频响曲线图.

关键词 变撑臂,渐硬性钢板弹簧,频响分岔方程

引言

目前,钢板弹簧广泛应用于各种车辆的悬挂系统中,其结构紧凑,能对车辆很好地起隔振、减振作用. 但钢板弹簧的刚度K是固定不变的. 因此,系统的自振频率随外荷载质量m的变化而变化,对系统的隔振效果影响很大.

近些年为解决这一问题,对渐硬性钢板弹簧有过一些研究[1~3]. 如少片渐硬性钢板弹簧是利用数片钢板的不同曲率,随着外载加大,钢板变形,各片钢板逐步接触,渐次参与工作,达到渐硬目的. 这种渐硬弹簧可使弹性系统的自振频率变化相对减小,但对自振频率的调整范围较小.

为了改进传统型钢板弹簧的性能,本文设计了新型渐硬性钢板弹簧,能够适应载荷变化,保证弹性系统的自振频率ω基本不变或在限定范围内($\omega=C\pm\Delta$)变化. 这种弹簧在工程实际中有着广泛的应用前景,特别在车辆和坦克的弹性减振系统中将得到应用.

1 变撑臂渐硬性钢板弹簧模型[4]

本设计有着与现有板簧相同的悬臂梁结构模型,如图1. 设有半径为R的圆弧形钢板悬臂梁,A端自由,B端固支(与垂直面夹角为β). 如果使用一刚性水平平板与钢板在C点水平光滑接触,然后在刚性平板施加垂直载荷P(刚性平板始终保持水平),钢板在受力后将变形,接触点C将自动跟随载荷由小到大,自悬臂端部向固支点B方向靠近. 这样,随载荷增大,缩短了悬臂梁的支撑力臂,钢板弹簧相应变硬,刚度系数K增加;相反,P减小,接触点C将反方向移动,增加了悬臂梁的支撑力臂,钢板弹簧相应变软,刚度系数K减小,因而成为一种

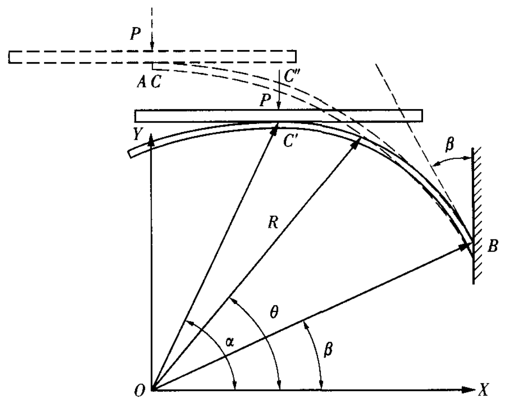

图1 变撑臂渐硬性钢板弹簧模型

Fig. 1 The model of progressive rate springs with variable arm length

可称之(定义)为“变撑臂”的渐硬性钢板弹簧. 载荷P作用下,钢板加载点C从C点变化到C'点,可等价于在垂直载荷P作用下从C''点变化到C'点,如图1所示. 本文采用单位荷载法求解钢板的变形规律.

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从 C' 点到 C'' 点转角变化为

$$\Delta_a = \int \frac{M\cdot 1}{EI}ds = \int_\beta^\alpha \frac{PR(\cos\theta-\cos\alpha)}{EI}Rd\theta =$$

$$\frac{PR^2}{E}\int_\beta^\alpha \frac{(\cos\theta-\cos\alpha)}{I}d\theta =$$

$$\frac{\pi}{2}-\alpha \qquad (1)$$

从 C' 点到 C'' 点转角垂直位移为

$$\Delta = \int \frac{M\cdot\overline{M}}{EI}ds =$$

$$\frac{PR^3}{E}\int_\beta^\alpha \frac{(\cos\theta-\cos\alpha)^2}{I}d\theta \qquad (2)$$

若钢板为三角形板，设钢板宽度为 $b=cx$，则有 $I=\frac{1}{12}vxt^3=\frac{cRt^3}{12}\cos\theta$，代入方程(1)和(2)，解得

$$P = R\left(\frac{\pi}{2}-\alpha\right)/F_1C_0 \qquad (3)$$

$$\Delta = PC_0F \qquad (4)$$

其中参数

$$C_0 = \frac{12R^2}{Ect^3},$$

$$F_1 = \alpha-\beta+\cos\alpha\left(\ln\left(\frac{1-\sin\alpha}{\cos\alpha}\right)+\ln\left(\frac{1+\sin\beta}{\cos\beta}\right)\right),$$

$$F = \frac{1}{2}(-4(\alpha-\beta)\cos\alpha+(1+\cos2\alpha)\cdot(-\ln\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)+\ln\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)+\ln\left(\cos\frac{\beta}{2}-\sin\frac{\beta}{2}\right)-\ln\left(\cos\frac{\beta}{2}+\sin\frac{\beta}{2}\right)+2\sin\alpha-2\sin\beta)=$$

$$-\cos\alpha((\alpha-\beta)+F_1)+\sin\alpha-\sin\beta$$

2 非线性动力学控制方程

载荷垂直位移即水平板位移为

$$s = R(\sin\alpha-\sin\alpha_0)-(\Delta-\Delta_0) \qquad (5)$$

其中 Δ_0 为静止荷载作用下垂直位移，设钢板弹簧上的载荷 $P_0=mg$，那么，在任意位置，其加速度为

$$a = \frac{P-P_0}{m} = \frac{P}{m}-g = \ddot{s} \qquad (6)$$

即

$$\frac{R}{mC_0}\frac{\left(\frac{\pi}{2}-\alpha\right)}{F_1} - R(\sin\alpha)'' + \Delta'' = g \qquad (7)$$

将方程(4)代入方程(7)，有

$$\frac{1}{mC_0}\frac{\left(\frac{\pi}{2}-\alpha\right)}{F_1} + (\alpha'^2\sin\alpha-\alpha''\cos\alpha) + \left(\frac{\frac{\pi}{2}-\alpha}{F_1}F\right)'' = g/R \qquad (8)$$

令 $\alpha=\alpha_0+u$ 代入(11)式并将其展开，得

$$A_1u'' + A_2u'^2 + A_3u + A_4u^2 + A_5u^3 + A_6 - g/R = 0 \qquad (9)$$

其中 α_0 为静荷载作用下加载点位置. 其中参数 $A_1\sim A_6$ 为

$\xi=\log[\sec(\alpha_0)-\tan(\alpha_0)]$

$\eta=\log[\sec(\beta)+\tan(\beta)]$

$A_1=(2\pi\eta-4\beta\eta+2\pi\xi-4\beta\xi+4(\eta+\xi)\sin(\beta-\alpha_0)+4(\eta+4\xi)\sin(\beta+\alpha_0)-2(\eta+\xi)\cdot\cos(2\alpha_0)(\pi+2\beta-4\alpha_0)-4(\eta+\xi)\cdot\sin(\beta)\cdot\sin(\alpha_0)(\pi-2\alpha_0)^2+(\eta+\xi)^2\cdot\sin(3\alpha_0)(\pi-2\alpha_0)+8\cos(\alpha_0(\beta-a_0)^2+8\sin(\beta)(-\beta+\alpha_0)+4\sin(\alpha_0)(-\eta-\pi\beta\eta-\pi\beta\xi-\xi\sin(2\alpha_0)+(\pi\eta+2\beta\eta+2\beta\xi+\pi\xi\sin(2\alpha_0)\alpha_0-2(\eta+\xi)\alpha_0^2)+\sin(\alpha_0)(8\beta+8\pi\beta^2+\pi(\eta+\xi)^2-2(4+8\pi\beta+8\beta^2+\eta^2+2\eta\xi+\xi^2)\alpha_0+8(\pi+4\beta))\alpha^2-16\alpha_0^3))/(8(-\beta+(\eta+\xi))\cos(\alpha_0)+\alpha_0)^2)$

$A_2=(\sec(\alpha_0)(8\pi\beta-16\pi\beta^3-8\eta^2-18\pi\beta\eta^2-16\eta\xi-36\pi\beta\eta-8\xi^2-18\pi\beta\xi^2+8(\eta+\xi)^2\cdot\cos(\beta-3\alpha_0)-8(\eta+\xi)^2\cos(\beta+3\alpha_0)-2(\eta+\xi)^3\sin(5\alpha_0)+(\eta+\xi)^3\cos(5\alpha_0)(\pi-2\alpha_0)+4(\eta+\xi)^2(-5\cos(\alpha_0)+\cos(3\alpha_0))\sin(\beta)(\pi-2\alpha_0)+12(\eta+\xi)^2\sin(4\alpha_0)(\beta-\alpha_0)+8(\eta+\xi)\sin(\beta)(\pi-2\alpha_0)(\beta-\alpha_0)+48\pi\beta^2\alpha_0+32\beta^3\alpha_0+18\pi\eta^2\alpha_0+36\beta\eta^2\alpha_0+36\pi\eta\xi\alpha_0+72\beta\eta\xi\alpha_0+18\pi\xi^2\alpha_0+36\pi\beta^2\alpha_0+16\alpha_0^2-48\pi\beta\alpha_0^2+96\beta^2\alpha_0^2-36\eta^2\alpha_0^2-72\eta\xi\alpha_0^2-36\xi^2\alpha_0^2+16\pi\alpha_0^3+96\beta\alpha_0^3-32\alpha_0^4+8(\eta+\xi)\cdot\sin(\beta)\sin(2\alpha_0)(\pi-4\beta+2\alpha_0)+2(\eta+\xi)^2\cos(4\alpha_0)(4-3\pi\beta+3(\pi+2\beta-2\alpha_0)\alpha_0)-16\sin(\beta)\sin(\alpha_0)(\pi\beta-(\eta+\xi)^2-(\pi+2\beta)\alpha_0+2\alpha_0^2)-4(\eta+\xi)\cdot\sin(\alpha_0)(4\pi\beta+10\beta^2+(\eta+\xi)^2-4(\pi+7\beta)\alpha_0+$

$18\alpha_0^2(-2(\eta+\xi)\sin(3\alpha_0)(4\beta(2\pi+5\beta)+3(\eta+\xi)^2-8(\pi+7\beta)\alpha_0+36\alpha_0^2+8\cos(2\alpha_0)\cdot(\pi-2\alpha_0)(\alpha_0-\beta)(1+2\beta^2+3(\eta+\xi)^2+2\alpha_0(\alpha_0-2\beta))+(\eta+\xi)\cos(3\alpha_0)\cdot(-16\beta+\pi(4+8\beta^2+5(\eta+\xi)^2+2\alpha_0(4-8\beta(\pi+\beta)-5(\eta+\xi)^2+4(\pi+4\beta-2\alpha_0)\alpha_0))+2(\eta+\xi)\cos(\alpha_0)\cdot(8\beta+\pi(-2+28\beta^2+5(\eta+\xi)^2)+2\alpha_0\cdot(-2-28\beta(\pi+\beta)-5(\eta+\xi)^2+14(\pi+4\beta-2\alpha_0)\alpha_0))+8\sin(2\alpha_0)(2\pi(\beta^2+(\eta+\xi)^2)+3\beta(2\beta^2+(\eta+\xi)^2)-\alpha_0(4\pi\beta+22\beta^2+7(\eta+\xi)^2-2(\pi+13\beta)\alpha_0+10\alpha_0^2))))/(32(-\beta+(\eta+\xi)\cos(\alpha_0)+\alpha_0)^3)$

$A_3=(-2\alpha_0+2\beta-2(\eta+\xi)\cos(\alpha_0)+(\pi-2\alpha_0)(\eta+\xi)\sin(\alpha_0))/(2c_0m(\alpha_0-\beta+(\eta+\xi)\cos(\alpha_0))^2)$

$A_4=-((\eta+\xi)\sin(\alpha_0)-((\pi-2\alpha_0)(2\alpha_0\eta-2\beta\eta+2\alpha_0\xi-2\beta\xi+5(\eta+\xi)^2\cos(\alpha_0)+2(\alpha_0-\beta)(\eta+\xi)\cos(2\alpha_0)-\eta^2\cos(3\alpha_0)-2\eta\xi\cos(3\alpha_0)-\xi^2\cos(3\alpha_0)-4(\alpha_0-\beta)\cdot\sin(2\alpha_0)-2(\eta-\xi)\sin(2\alpha_0)))(16(\alpha_0-\beta)+(\eta+\xi)\cos(\alpha_0))\cos(\alpha_0)))/(c_0m(\alpha_0-\beta+(\eta+\xi)\cos(\alpha_0))^2)$

$A_5=-192(\eta+\xi)^2\sin(\alpha_0)^2(-\beta+(\eta+\xi)\cdot\cos(\alpha_0)+\alpha_0+48\sec(\alpha_0)(\eta+\xi(1+\cos(2\alpha_0)-2\sin(\alpha_0)(-\beta+(\eta+\xi)\cos(\alpha_0)+\alpha_0)^2+\sec(\alpha_0)^2(\pi-2\alpha_0)(24\beta^2+26\eta^2-10(\eta+\xi)^2\cos(4\alpha_0)+(\eta+\xi)^3\sin(5\alpha_0)+16(\eta+\xi)\cos(3\alpha_0)(\beta-\alpha_0)+16(\eta+\xi)^2\cdot\sin(2\alpha_0)(\beta-\alpha_0)+8(\eta+\xi)^2\sin(4\alpha_0)(\beta-\alpha_0-48(\beta\alpha_0+24\alpha_0^2+80(\eta+\xi)\cos(\alpha_0)(\beta-\alpha_0)-2(\eta+\xi)\sin(3\alpha_0)(4\beta^2+21(\eta+\xi)^2+8\beta\alpha_0-4\alpha_0^2-2(\eta+\xi)\sin(\alpha_0)(-2\beta^2+11(\eta+\xi)^2+4\beta\alpha_0-2\alpha_0^2)+8\cos(2\alpha_0)\cdot(\beta^2+2(\eta+\xi)^2-2\beta\alpha_0+\alpha_0^2)))/(192c_0m(\alpha_0-\beta+(\eta+\xi)\cos(\alpha_0))^4)$

$$A_6=\frac{\pi/2-\alpha_0}{c_0m(\alpha_0-\beta+(\eta+\xi)\cos(\alpha_0))} \tag{10}$$

方程(9)是变撑臂渐硬性钢板弹簧的运动控制方程.

3 摄动分析

变撑臂渐硬性钢板弹簧的运动控制方程可化为

$$u''+\hat{\mu}u'^2+\omega_0^2u+\delta_1u^2+\delta_2u^3+\delta_3=0 \tag{11}$$

其中参数

$$\begin{aligned}\hat{\mu}&=A_2/A_1\\ \omega_0^2&=A_3/A_1\\ \delta_1&=A_4/A_1\\ \delta_2&=A_5/A_1\\ \delta_3&=(A_6-g/R)/A_1\end{aligned} \tag{12}$$

应用多尺度法,令 $\hat{\mu}=\varepsilon\mu$,ε 为小参数,方程(11)的近似解可以写成

$$u=\varepsilon u_1(T_0,T_1,T_2)+\varepsilon^2u_2(T_0,T_1,T_2)+\varepsilon^3u_3(T_0,T_1,T_2)+\cdots \tag{13}$$

其中 $T_0=t,T_1=\varepsilon t,T_2=\varepsilon^2t$ 将方程(13)代入方程(11),令的同次幂系数相等,有

$$\begin{aligned}&D_0^2u+\omega_0^2u_1=0\\ &D_0^2u_1+\omega_0^2u_2=-2D_0D_1u_1-\delta_1u_1^2\\ &D_0^3u_3+\omega_0^2u_3=-\mu(D_0u_1)^2-D_0D_1u_2-D_1^2u_1-2D_0D_2u_1-\delta_2u_1^3-2\delta_1u_1u_2\end{aligned} \tag{14}$$

方程(14)第1式的解为

$$u_1=Ae^{iT_0\omega_0}+\overline{A}e^{-iT_0\omega_0} \tag{15}$$

其中 A 为 T_1,T_2 的函数,将方程(15)代入方程(14)第2式,令永年项系数为零,有

$$\frac{\partial}{\partial T_1}A(T_1,T_2)=0 \tag{16}$$

A 仅为 T_2 的函数.

$$D_0^2u_2+\omega_0^2u_2=-A^2e^{2iT_0\omega_0}\delta_1-2A\delta_1\overline{A}-e^{-2iT_0\omega_0}\delta_1\overline{A}^2 \tag{17}$$

方程(17)的解可写为

$$u_2=\frac{A^2e^{2iT_0\omega_0}\delta_1}{3\omega_0^2}-\frac{2A\delta_1\overline{A}}{\omega_0^2}+\frac{e^{-2iT_0\omega_0}\delta_1\overline{A}^2}{3\omega_0^2} \tag{18}$$

将方程(15)和方程(18)代入方程(14)第3式,令永年项系数为零,有

$$-3A^2\delta_2+\frac{10A^2\delta_1^2\overline{A}}{3\omega_0^2}-D_1^2A-2i\omega_0D_2A=0 \tag{19}$$

令 $A=\frac{a}{2}e^{i\theta}$,代入上式,实部和虚部分离,得到系统的解调方程组.

$$\left(\frac{5a^3\delta_1^2}{12\omega_0^2}-\frac{3}{8}a^3\delta_2\right)\cos\theta+a'\omega_0\sin\theta+a\theta'\omega_0\cos\theta=0$$

$$\left(\frac{5a^3\delta_1^2}{12\omega_0^2}-\frac{3}{8}a^3\delta_2\right)\sin\theta-a'\omega_0\cos\theta+a\theta'\omega_0\sin\theta=0 \tag{20}$$

解得

$$a'=0$$
$$\left(\frac{5a^3\delta_1^2}{12\omega_0^2}-\frac{3}{8}a^3\delta_2\right)+a\theta'\omega_0=0 \tag{21}$$

对方程(21)进行积分,有

$$a=a_0$$
$$\theta=-\frac{a^2}{\omega_0}\left(\frac{5a^3\delta_1^2}{12\omega_0^2}-\frac{3}{8}\delta_2\right)T_2+\theta_1= \tag{22}$$
$$\theta_0 t+\theta_1$$

其中 a_0 为常数,由初始条件确定

$$\theta_0=-\frac{a^2\varepsilon^2}{\omega_0}\left(\frac{5a^3\delta_1^2}{12\omega_0^2}-\frac{3}{8}\delta_2\right)=$$
$$-\frac{(a_0\varepsilon)^2}{\omega_0}\left(\frac{5a^3\delta_1^2}{12\omega_0^2}-\frac{3}{8}\delta_2\right)$$

所以变撑臂渐硬性钢板弹簧运动控制方程的二阶近似摄动解可写成

$$u=\varepsilon u_1(T_0,T_1,T_2)+\varepsilon^2u_2(T_0,T_1,T_2)+\cdots=$$
$$\varepsilon\left(\frac{a}{2}e^{i((\varepsilon_0+\theta_0)t+\theta_1)}+\frac{a}{2}e^{-i((\omega_0+\theta_0)t+\theta_1)}\right)+$$
$$\varepsilon^2\left(\frac{a^2e^{2i((\omega_0+\theta_0)t+\theta_1)}\delta_1}{12\omega_0^2}-\frac{\delta_1a^2}{2\omega_0^2}+\right.$$
$$\left.\frac{\delta_1a^2e^{-2i((\omega_0+\theta_0)t+\theta_1)}}{12\omega_0^2}\right)+\cdots \tag{23}$$

因此,系统的频响方程为

$$\omega=\omega_0-\frac{(a_0\varepsilon)^2}{\omega_0}\left(\frac{5\delta_1^2}{12\omega_0^2}-\frac{3}{8}\delta_2\right) \tag{24}$$

4 算 例

设 $R=0.6\text{m},\beta=15^\circ,c=1,\varepsilon=0.1$,钢板厚度 $t=10\text{mm}$,得到图2~图4.

从图中可以看出:①图2反映了静荷载和加载点之间的关系:荷载越大,α_0 越小;荷载越小,α_0 越大. ②图3反映了频率和加载点之间的关系. 从图中可以看出,在强度允许的情况下,频率 ω 保持在一个较小的范围,这正是变撑臂渐硬性钢板弹簧的优点所在(当荷载很小时,频率迅速增加,考虑钢板的自重和实际最小荷载,此时没有实际意义). ③图4反映了频率 ω 和振幅 a_0 之间的关系:频率随振幅的增加而增加.

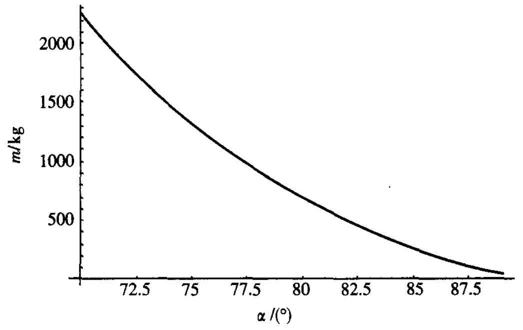

图2 荷载和加载点关系图

Fig. 2 The curves of load and load point

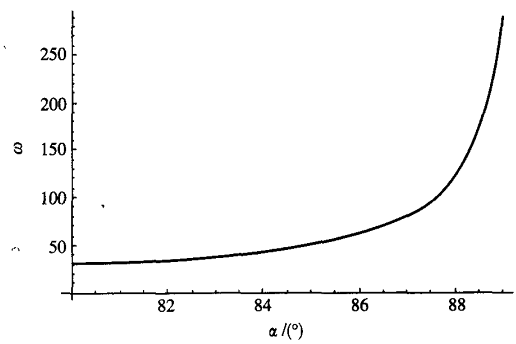

图3 频率和加载点关系图

Fig. 3 The curves of frequency and load point

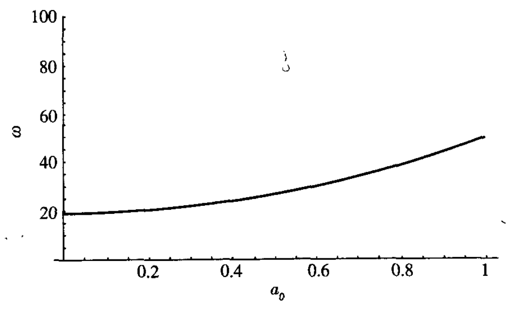

图4 $\varepsilon=0.1$ 时 $\omega\sim a_0$ 关系

Fig. 4 The frequency response curves when $\varepsilon=0.1$

5 结 论

本文建立了变撑臂渐硬性钢板弹簧的运动模型,得到了系统的运动控制方程,通过多尺度法对运

动控制方程进行了深入的分析,获得了变撑臂渐硬性钢板弹簧的频响方程和系统的近似解.同时,通过算例,得到了反映钢板弹簧性质的荷载和加载点关系图、频率和加载点关系图以及频响曲线图.

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THE NON-LINEAR DYNAMICS ANALYSIS OF PROGRESSIVE RATE SPRINGS WITH VARIABLE ARM LENGTH*

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Abstract The expression for the relation between the deform and the external force of progressive rate springs with variable arm length was established by using unit load method, and then the non-linear dynamics equations was given. After solving the equations by means of multiple scales method , the modulation equations and its approximate solution were obtained . The curves of load and load point, frequency and load point and the frequency response curves were given by illustrative examples.

Key words variable arm length, progressive rate springs, modulation equations

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