

## 输流管道动力学与控制的最新进展\*

唐冶<sup>1,2</sup> 高传康<sup>2</sup> 丁千<sup>1</sup> 杨天智<sup>3†</sup>

(1.天津大学 力学系,天津 300350)(2.安徽工程大学 机械工程学院,芜湖 241000)

(3.东北大学 机械工程与自动化学院,沈阳 110819)

**摘要** 管道系统在航空航天、石油输送、深海探测、核能工程等工程领域发挥着输送流体的作用.由复杂结构功能设计、支承条件、内部流体和外部环境等因素引起输流管道中的流体和管道发生强烈地耦合,导致的动力学问题严重限制了输流管道在各种领域中的工程应用.因此,输流管道的复杂动力学行为引起了工程和科学领域学者们的广泛关注,本文综述和讨论了最新的输流管道振动控制的研究和进展.

**关键词** 输流管道, 动力学, 振动控制, 最新进展

中图分类号:O324;O322

文献标志码:A

## Review on Dynamic and Control of Pipes Conveying Fluidan\*

Tang Ye<sup>1,2</sup> Gao Chuankang<sup>2</sup> Ding Qian<sup>1</sup> Yang Tianzhi<sup>3†</sup>

(1. Department of Mechanics, Tianjin University, Tianjin 300350, China)

(2. School of Mechanical Engineering, Anhui Polytechnic University, Wuhu 241000, China)

(3. School of Mechanical Engineering and Automation, Northeastern University, Shenyang 110819, China)

**Abstract** Pipelines are used to convey fluid in the engineering fields such as aerospace, oil transportation, deep-sea exploration, nuclear power engineering and so on. The strong coupling between the pipes and fluid is induced by the complex structural and functional design, support conditions, internal fluid and external environment, resulting in dynamic problems which severely limit the engineering application of the pipes conveying fluid in various fields. Therefore, the complex dynamic behavior of pipes conveying fluid has been attracted wide attention of scholars in engineering and science. The latest research and progress in vibration and control of pipe conveying fluid are reviewed and discussed in this paper.

**Key words** pipes conveying fluid, dynamic, vibration control, recent development

### 引言

输流管道通常是指输送流体的管状结构,作为各种工程系统中的一种重要的基本单元,被广泛应用于航空航天、机械、土木、海洋、生物、核能、石油能源和动力水能等工程领域,如大型水利工程的

压力管道,石油工程中的输油和输气管道,飞机和液体火箭中的输送推进剂管道,海洋钻探中的输油管道,以及核电工业的热交换管道等.由于内部流体和外部环境的作用,管道在传输流体过程中不可避免地出现许多动力学和稳定性问题.在工程中,失稳、大幅振动和混沌等复杂行为往往会使输流管

2023-03-22 收到第 1 稿,2023-05-06 收到修改稿.

\* 国家自然科学基金资助项目(11902001,12072221,12132010), National Natural Science Foundation of China (11902001,12072221,12132010).

† 通信作者 E-mail:yangtianzhi@me.neu.edu.cn

结构破坏、精度下降和寿命降低。随着科学技术的发展和进步,各种工程结构、机械和传输设备对振动环境、稳定性和抗振能力的要求越来越高。因此,研究输流管道振动及其控制问题具有重要的工程意义。

输流管系统的振动问题研究可以追溯到十九世纪末,Marvel Brillouin 在观察给草坪浇水的橡皮管时,发现流体高速流动引起管道自由端产生一些奇怪的运动,这一现象引起了他的学生 Bourrière 的兴趣<sup>[1]</sup>,并在 1939 年建立了输流管道的线性方程。但是,二次世界大战使相关研究工作遭遇停滞。直到 1950 年,Ashley 和 Haviland<sup>[2]</sup>分析了横跨阿拉伯工程管道的弯曲振动问题。随后,众多学者开始关注输流管系统的固有频率、振动波传播、稳定性和响应振幅等动力学行为<sup>[3-5]</sup>。1987 年,Païdoussis<sup>[6]</sup>精辟地阐述了输流直管的线性振动问题,指出了当流速超过临界值时,悬臂输流管会发生颤振失稳,两端支承管道更容易屈曲失稳。随着研究的不断深入,学者们对输流管道动力学的研究考虑更为一般的三维模型,探索更为复杂的非线性现象。Holmes<sup>[7]</sup>在 Païdoussis 的输流管线性振动模型中引入了几何非线性,从而建立了系统的非线性运动方程,开启了非线性动力学的研究热潮。Meng 等<sup>[8]</sup>基于 Kane 方程和 Ritz 方法,建立了输流管系统全局运动的三维非线性动力学模型,并利用增量谐波平衡方法研究了系统的非线性时域响应。Ghayesh 等<sup>[9]</sup>提出了悬臂输流管的非线性平面运动模型,应用伪弧长和直接积分方法构造系统的分岔图、时间历程图和相图,并指出了随着流速的增加,系统经历超临界的 Hopf 分岔后而进入颤振失稳。Chang 和 Modarres-Sadeghi<sup>[10]</sup>利用有限差分方法讨论了悬臂输流管在基础激励下二维、三维概周期运动和混沌运动的流速条件。Lü 等<sup>[11]</sup>应用 Galerkin 截断和数值技术研究了具有非线性弹簧耦合的两输流管系统的分岔和同步振动。Zhang 等<sup>[12]</sup>数值地分析了在一般边界条件下具有附加质量弹簧约束输流管道的三维动力学,并通过分岔图、相图、功率谱密度图和庞加莱映射图等手段考察了系统分岔和混沌等复杂动力学行为。

由于管道内液体流动的特殊性以及控制方程引入非线性后,系统固有频率之间可能存在一定的比例关系,这时模态的相互影响不容忽视,出现了

内共振现象<sup>[13]</sup>。同济大学的徐鉴教授<sup>[14,15]</sup>采用多尺度方法研究悬臂输流管的内共振,分别推导了 3 : 1、2 : 1 和 1 : 1 内共振的条件,并用数值方法模拟了 3 : 1 内共振下系统的非线性动力学行为。上海大学的陈立群教授<sup>[16]</sup>考虑管内流速处于超临界区域,进一步研究了输流管的主共振和 2 : 1 内共振,并解释了在稳态响应中发生双跳跃现象的机理。Mao 等<sup>[17]</sup>关注了超临界输流管在 3 : 1 内共振情况下的强迫振动响应,研究发现了跳跃、饱和与滞后等现象,并通过数值方法检验了曲线平衡附近的局部分岔行为。

管道所载流体经常由泵等装置提供动力,流体流速不可避免地带有脉动。当这种脉动频率和输流管系统的固有频率满足一定关系时,即使是小的脉动激励,也可能引起大的系统响应。因此,脉动流速所引起的参数振动是输流管系统的另一个重要的动力学问题。Panda 和 Kar<sup>[18,19]</sup>采用多尺度方法分析了 3 : 1 内共振条件下脉动输流管系统的主、组合参数共振,并发现了鞍结分岔及 Hopf 分岔。北京工业大学的杨晓东教授<sup>[20]</sup>讨论了脉动输流黏弹性管道在次谐波共振和组合谐波共振条件下的稳定性。华中科技大学的王琳教授课题组<sup>[21]</sup>提出了一种脉动输流管的涡激动力学模型,并采用直接多尺度方法讨论了锁频条件下脉动参数共振对输流管系统涡激振动的影响,研究结果表明,只有锁频效应和脉动参数共振发生在同一阶模态上时,脉动参数共振才会对响应幅值产生明显的影响。北京工业大学的张伟教授课题组<sup>[22]</sup>考察了超临界脉动输流管在 1 : 2 内共振条件下的超谐波全局动力学,并通过辨别相空间中的多脉冲跳跃轨道说明发生混沌运动的条件。

目前,随着解析方法<sup>[23]</sup>、数值仿真<sup>[24,25]</sup>和实验手段<sup>[26]</sup>的不断成熟,学者们更加关注工程实际情形下的输流管道振动问题,如海洋石油天然气钻井系统、盐矿卤水输送管路系统。为了满足不同的工程应用,不同形状,复杂约束,输送多相流体的,恶劣的工作环境下的管道动力学行为被大量地研究。同时,引进复合材料如功能梯度材料构造管道调控输流管道振动特性来增强输流管道的强度和提高系统的可靠性,也是另外一个重要的研究方向。此外,虽然振动抑制在工程应用中的需求越来越大,但是,关于输流管道振动控制的研究还是相对较

少.

本文从一般输流直管/曲管、不同外形输流管道、复杂支承和约束输流管道、运动输流管道、内流和外流作用下输流管道、多相流输流管道、复合材料输流管道动力学特性及输流管道的振动控制等方面进行综述,全面地给出输流管道动力学与控制的最新研究进展.

## 1 一般输流直管/曲管

普通直曲输流管道的研究较为简单,计算工作量小,这种研究模型通常从工程实际中合理假设而得到的.在输流管道系统设计初始阶段,对精度要求不高的动特性预估是可行的.对于普通输流管道,边界条件通常被假设为两端支承和悬臂.

Tan 等<sup>[27]</sup>考虑了 Timoshenko 模型,建立了纵横耦合振动模型,利用有限差分法和离散傅立叶变换方法,研究了初始幅值、外激振力和流速对系统的非线性频率和强迫响应特性的影响.并讨论了 Timoshenko 输流管道模型的优势. Sazesh 和 Shams<sup>[28]</sup>研究了高斯白噪声随机激励下悬臂输流管道的动力学,通过随机时间历程和概密度函数探索管道在颤振点附近的随机行为. Giacobbi 等<sup>[29]</sup>针对输流管道应用于海洋平台砖井开采甲烷晶体的工程问题,考虑管道传输高速的气体和沿管长方向变化的热环境,研究了轴向变密度输流管道的动力学,得出管道入口和出口的密度差对系统稳定性影响较大. Higuchi 等<sup>[30]</sup>提出识别悬臂输流管道自激振动的复模态实验技术,构造了输流管道系统发生颤振时的特征模态. Li 等<sup>[31]</sup>利用谱不变流形方法,对悬臂输流管道的非线性动力学模型进行降维,通过比较降维前后的系统自由振动、强迫振动响应、周期和概周期分岔以及同宿和异宿轨道等复杂的动力学行为,说明所提出的不变流形降维方法的有效性. Zhang 和 Chen<sup>[32]</sup>利用 Galerkin 截断和多尺度方法,结合规范性理论和能量相方法,研究了悬臂输流管道在脉动流和外激励作用下的多脉冲跳跃轨道和混沌动力学.

当管道内流体增加到临界值的,两端支承输流管道发生屈曲,由原来的绕直线平衡位置运动过渡到绕曲线平衡位置进行运动.目前,传播高速流下管道振动越来越普遍,也成为研究重点之一. Tan 等<sup>[33,34]</sup>针对高速流管道常常产生严重的振动问

题,讨论了 Timoshenko 输流管道在超临界情况下的主共振、超谐波共振和参数振动行为,发现超临界情况下管道动力学行为比亚临界情况下更加复杂. Lu 等<sup>[35]</sup>研究了输流管道在超临界流体作用下发生 3 : 1 内共振和应力分布情况,并揭示了抑制内共振提高管道的疲劳生命的机理. Zhu 等<sup>[36]</sup>考虑黏弹性输流管道的面内和面外耦合作用和欧拉梁理论,利用频响图、力幅图、吸引盆、时间历程图和相图,研究了输流管道在亚临界和超临界情况下三维耦合动力学,说明了当出现 2 : 1 内共振时,系统展现典型的跳跃、滞后并出现双峰响应.

为适应工程操作和适应环境变化的安装,曲率比较大输流曲管在工程中应用比较广泛而被研究者重视,以便实现更加灵活性设计. Oyelade 和 Oyediran<sup>[37]</sup>考虑两端简支、两端固支和一端固支一端简支边界条件以及轻微弯曲输流曲管的纵横耦合特性,分析了系统在热载荷作用下的非线性动力学. Zhou 等<sup>[38]</sup>利用绝对节点坐标方法,建立了悬臂轻微输流曲管的非线性控制方法,经过研究发现即使初始几何形变较小,管道流体引起的静态变形也是非常大的,系统的颤振失稳临界流速依赖于静态平衡构造,同时,关注了后屈曲非线性行为. Czerwinski 和 Luczko<sup>[39]</sup>引入轴向力的非线性成分,利用理论和实验方法分析了系统时间历程图、相图、运动轨迹、振动模态和分岔演化规律. 研究了脉动流频率和幅值对输流曲管的各种参数振动影响. Chen 等<sup>[40]</sup>以柔性机器人和生物医学为背景,利用绝对节点坐标方法,对具有任意初始构型的柔性输流曲管进行了几何精确性建模,预估了输流曲管的静态变形及稳定性和非线性振动等大变形行为. Xu 等<sup>[41]</sup>研究了轻微输流曲管涡激振动,他们发现了在稳定流情况下,系统存在针对绕流一三阶模态振动的锁频区域,其振动幅值随着外流体速度增加而增大;在脉动流作用下,系统展现出更加复杂的动力学行为. Yan 等<sup>[42]</sup>研究了两端固支输流曲管的静态平衡构型的分岔和稳定性行为,分析了外力、流速和弧角对系统非线性响应的影响.

## 2 复杂支承和约束管道

在工程中,学者们在输流管道系统中增加复杂支承和约束,试图改善系统的动力学环境. Yamashita 等<sup>[43]</sup>研究了具有弹性支承和端部质量的



输流管道动力学,通过理论结合试验的方法关注了 Hopf-Hopf 耦合和两不稳定模态幅值的演化,在一定的参数区间,存在由 Hopf-Hopf 耦合而产生的混合模态自激振动.Guo 等<sup>[44]</sup>运用传递矩阵方法和实验技术,研究了并行输流管道系统在局部位置受到外激励干扰时振动传递问题,并分析了约束、流速和压力对振动传递特性的影响.Peng 等<sup>[45]</sup>应用哈密顿变分原理,建立了含有运动约束倾斜输流管道的三维非线性运动微分方程,通过数值技术获取系统的相图和振动轨线说明运动规律.ElNajjar 和 Daneshmand<sup>[46]</sup>关注了沿着管长方向增加质量和弹簧提高横向和纵向管道的临界流速的可能行.Askarian 等<sup>[47]</sup>讨论了端部线性弹簧和扭转弹簧约束性输流管道在分数阶黏弹性地基支承情况下的稳定性.Kheiri<sup>[48]</sup>分析了两端强非线性横向和扭转弹簧约束下输流管道的非线性动力学,与悬臂输流管道相比,复杂约束输流管道展现更低的 Hopf 分岔流速,更高的振动位移幅值.在高流速下,存在概周期和混沌运动.Mao 等<sup>[49]</sup>利用模态校正结合投影方法,提出了处理具有非线性和非均匀边界的输流管道振动的近似解析方法,该方法利用谐波平衡法将边界非线性和非均匀项进行描述,通过更多谐波判别响应解的收敛性.与多尺度方法的解对比,说明提出的方法有效性.Zhou 等<sup>[50]</sup>考虑几何大变形和弹性边界条件,计算了复合材料输流管道失稳临界流速和非线性频率,结果表明平移弹簧的变化对临界流速具有轻微影响,旋转弹簧能提高系统的稳定性.Zhou 等<sup>[51]</sup>提出了具有局部刚化的悬臂输流直管和曲管的非线性模型,探索了局部刚段位置和长度对系统非线性静平衡构造和动力学特性的影响.结果表明局部刚段的出现影响两种管道的振动模态,曲管中出现周期 1 和周期 2 的运动,而直管中仅出现周期 1 的运动.Peng 等<sup>[52]</sup>研究脉动流输流管道在运动约束作用下的横纵耦合非线性振动,通过相平面图、庞加莱映射图和功率谱密度图展现如概周期和混沌运动等复杂的运动规律.Liu 等<sup>[53]</sup>提出了悬臂输流管道在松散约束下的涡激振动模型,通过分岔图和 Argand 图及振型图说明了锁频现象及复杂动力学行为.

### 3 运动有大运动叠加的输流管道

学者们从工程领域中简化出三种运动管道模

型:沿着管道轴线平动抽吸输流管道、绕管道轴线旋转的输流管道、绕管道径向旋转的输流管道.Yan 等<sup>[54]</sup>建立了沿轴向时变滑动输流管道的非线性动力学模型,研究了系统动力学稳定性和非线性行为,结果表明当流速超过临界值时,颤振幅值随着时间变化而改变,随着滑动率的增加,管道系统更容易失稳,而质量比和重力的增加能提高系统的稳定性.Liang 等<sup>[55]</sup>考虑旋转速度和流速脉动情形,提出了绕管道轴线旋转的输流管道非线性参数振动模型,利用多尺度方法分析了系统稳定性,通过数值方法模拟了系统非线性响应和空间振动形态.Liang 等<sup>[56]</sup>研究了绕管道轴线旋转的多跨功能梯度输流管道的动力学,结果表明引入中间支承可提高系统的稳定性,不同跨度的模态特征能确定管道振动幅值位置.Ebrahimi 和 Ziaei-Rad<sup>[57]</sup>提出了绕管道轴线旋转的悬臂压电输流管道振动模型,考察了流速、电阻、旋转速度和压电层覆盖角对系统的动力学轨线和稳定性影响.Liang 等<sup>[58]</sup>研究了绕管道轴线旋转的两端支承输流管道在内外流共同作用下的三维动力学.Abdollahi 等<sup>[59]</sup>进行了绕管道径向旋转的输流管道在环流液体媒介中的稳定性分析,考虑双陀螺力的影响,通过解析和半解析解获取稳定性解.

### 4 内流和外流作用下输流管道

在工业领域,涉及输流管道同时受到内外流共同影响的系统也是非常常见的,例如,热交换器、钻井作业的钻柱和石油勘探,以及在“盐井洞穴”中提取碳氢化合物等.Paidoussis 等<sup>[60]</sup>综述了悬臂输流管道在内和反向外流作用下动力学问题.Abdelbaki 等<sup>[61]</sup>提出了悬臂输流管系在内外轴向流作用下的全局非线性模型,利用伪弧长延拓方法结合直接数值积分方法计算系统的运动微分方程,预估了不稳定性引起的颤振、超临界下的极限环振动和频率受外流的限制程度、重力、质量比等参数的影响.Zhou 等<sup>[62,63]</sup>利用能量方法推导了悬臂输流管道在轴向激励下的非线性三维控制方程,通过非线性数值方法预测了系统的非线性响应,流速在亚临界条件下,轴向激励能够产生共振响应,超临界情况下,轴向激励在某些特定区间能使系统稳定,非平面周期自激振动演化成平面概周期或周期运动.Jiang 等<sup>[64]</sup>研究了两端支承输流管道在轴向内外流作用

下的稳定性和三维非线性动力学,揭示了一些有趣动力学现象如周期、概周期和混沌运动等。Abdelbaki等<sup>[65]</sup>提出了悬臂输流管系在部分限制外流作用下的非线性理论模型,探索了环向区域的参数对动力学行为影响,通过实验验证了提出模型的有效性。Minas和Paidoussis等<sup>[66]</sup>搭建了悬臂管系在内外流作用下实验平台,上部环绕部分由一个同心圆的刚性圆柱形管组成,并安装在一个装满水的水箱中。水从管道上部流入,在其自由端排入水箱,理论和实验预测了环状流强烈地影响管系失稳。Butt等<sup>[67]</sup>提出了悬挂吸水管系统在内流作用下线性模型,研究了一定内外流速比下的系统复模态,结果表明在足够高外流速下,系统失稳主要由一阶颤振引起。Chehreghani<sup>[68]</sup>利用实验方法研究了悬臂输流管道在反向环流作用下动力学问题,当外内流速比较低时,系统失稳由二阶模态颤振引起;当外内流速比较高时,管道经历静变形,伴随着高速内流引起的周期和混沌等复杂动力学行为。Dane-shmand等<sup>[69]</sup>执行了部分限制悬臂管道在内和反向外流作用下的耦合双向流固耦合分析。Zhou等<sup>[70]</sup>建立了端部锥形悬臂输流管道在内外轴向流作用下的动力学模型,研究了系统失稳边界和模态及非线性振动幅值和形态。

## 5 多相流输流管道

在石油和天然气开采领域,存在石油和天然气混合油气井,为降低成本,通常采用油气混合输送模式,这是典型的气液两相流。较普通的单相流、多相流造成许多缺点,如承载能力的降低、流体物理性质的变化、流体流动的中断和系统效率的降低等<sup>[71]</sup>。这些缺陷为动力学设计带来相应的挑战,如空间密度变化引起流速的暂态变化,进而引起参数振动等。因此,研究多相流输流管道的振动特性具有重要的设计意义。Ebrahimi-Mamaghani等<sup>[72]</sup>提出了立管传输气液两相流的数学模型,应用 Galerkin 截断和特征值分析得到了如气体体积分数、流速、结构阻尼和重力参数对系统稳定性的影响。Guo等<sup>[73]</sup>研究了管中管系统在热环境和二相流作用下的动力学特性和稳定性,通过 Argand 图、稳定图、时间历程图说明了在超临界流作用下,不同于单相流,二相流系统展现了二管道耦合颤振失稳。Ma和Srinil<sup>[74]</sup>数值研究了传递气液两相流倾斜弯曲输流

管道的平面动力学。Liu等<sup>[75]</sup>采用绝对节点坐标方法,建立了海洋立管在内部二相流作用下的非线性数学模型。通过时域和频域的变化,说明了提出建模方法的有效性。Oyelade和 Oyediran<sup>[76]</sup>利用哈密顿变分原理,建立了传递二相流水平管道的非线性控制方程,分析了初始变形、体积分数和质量比对系统的频率、临界流速、响应位移分岔和混沌的影响。Zhou等<sup>[77]</sup>研究了传递气液两相流倾斜输流管道的自由振动和稳定性,结果表明,倾斜引起的重力对系统振动特性具有重要的影响,随着倾斜角度加大,系统临界气体速度和振动频率降低,系统更容易失稳;对于给定倾斜角,系统对表面气体/液体速度的动态响应与质量比直接相关。Ebrahimi-Mamaghani等<sup>[78]</sup>通过特征值求解,研究了两端支承输送二相流管道系统的稳定性,同时给出了非线性振动频率闭式解,结果表明系统稳定性随着液相密度的降低而提高,提高气体分数和流体混合速度可增大系统非线性振动频率。Chang等<sup>[79]</sup>研究了含气液两相流输流管道在洋流作用下的涡激锁频特性,利用哈密顿变分原理,建立了管道系统平面运动微分方程,范德波尔振动模型模拟洋流的涡激力,Newmark- $\beta$ 和四阶龙哥库塔法求解系统动力学响应,得到了锁频区间随着液相速度、纤维方向角的增大而向右移动,而轴向张力增大使锁频区域向左移动。Xie等<sup>[80]</sup>针对多相流引起的流体变密度,研究了输流管道经历涡激振动情况下的非线性参数振动行为,结果表明,当内部流体密度在不同系统固有频率附近波动时,管道的振动会变得不均匀或非周期性,位移量会增加或减少。随着内部流体波动幅度的增大,密度较大时,这些现象就会变得更加明显。随后,Xie等<sup>[81]</sup>将变密度输流管道扩展到系统经历 cross-flow 和 in-line 耦合涡激振动的动力学响应分析,给出了在不同参数共振下的管道系统不同模态被激励的动力学特性。

## 6 复合材料输流管道动力学特性

复合材料是由两种或两种以上的材料复合而成的新材料,从而能改善材料的力学性能。比较典型的功能梯度材料由体积含量在空间位置上可连续变化的两组分材料组成。在制造这种材料时,通过改变组分的体积指数率而使该材料具有物理属性分布沿着某一方向连续梯度变化的性质。与传统

材料相比,功能梯度材料拥有很多优点,例如,缓解或消除材料的应力集中,提高结构连接强度和增强结构抗热腐蚀性能等<sup>[82]</sup>.因此,在工程领域中制造重要部件广泛采用这样的材料.最近,学者们为了优化动力学特性而将功能梯度材料引入输流管系统.Reddy等<sup>[83]</sup>提出应用功能梯度材料构造管道来提高传递热流体的稳定性,采用谐波平衡法和龙格库塔法获取系统时域和频域响应.在前屈曲构造,一阶和二阶主参数振动不稳定区域发生偏差;在后屈曲状态下,通过间歇过渡路径、循环折叠分岔、周加倍分岔和亚临界分岔而产生混沌运动.Lu等<sup>[84]</sup>分析了内共振和轴向功能梯度材料对输流管道疲劳寿命的影响,应用 Galerkin 截断和直接多尺度方法得到主共振和 3:1 内共振情况下的可解性条件,研究结果表现内共振缩短轴向功能梯度管道的疲劳寿命,降低功能梯度分布系数有利于降低系统共振响应和最大应力.Zhu等<sup>[85-87]</sup>研究了多孔功能梯度输流管道在初始变形下后屈曲静态和动态特性,在弹性地基下的非线性自由和强迫振动,以及三维非线性动力学.Guo等<sup>[88]</sup>提出了随机轴向功能梯度材料构造输流管道系统的有效统计性固有频率分析方法.Liu和Li<sup>[89]</sup>考虑高阶圆柱梁模型和几何非线性,建立了功能梯度输流管道在弹性地基下的非线性控制方法和边界条件,采用微分求积方法确定系统的非线性频率和幅频响应,并揭示了几何和物理参数对系统动力学行为影响.Chang等<sup>[90]</sup>预估了具有初始变形下弹性地基功能梯度输流管道静态屈曲和后屈曲动力学特性.除此之外,Babaei<sup>[91]</sup>利用二步扰动法,研究了功能梯度碳纳米管增强输流管道的热前屈曲和后屈曲的频率响应受几何特性、地基刚度、碳纳米管增强分布形式和体积分数的影响.Ghadirian等<sup>[92]</sup>基于 Timoshenko 梁模型,研究了功能梯度碳纳米管增强输流管道的非线性自由振动和稳定性.Ren等<sup>[93]</sup>针对工程中飞机中输流管道存在内流和外载荷联合作用下流固耦合振动,研究了功能梯度石墨烯增强输流管道在前屈曲和后屈曲情况下碰撞动力学和突跳行为受流速、碰撞速度、结构材料和几何因子的影响.结果表明,随着碰撞能量提高,流体促使后屈曲管道展现对称的双稳态特征,结构阻尼对响应影响较大.Li和Liu<sup>[94]</sup>考虑高阶剪切变形梁模态,建立了各项异性复合材料输流管道在弹性地基下的非线性振动模

型,利用微分求积方法和迭代算法确定了非线性频率和幅频响应.Guo等<sup>[95]</sup>关注了悬臂弹性连接双复合材料输流管道系统的流固耦合失稳和分岔特性,通过 Argand 图分析了颤振失稳,利用分岔图、时间历程和相图等非线性动力学分析手段,研究了系统在后屈曲情况下的周期和概周期等复杂现象,并发现了二管道的纤维排布方法能打破系统对称稳定性区间和分岔行为.Guo等<sup>[96,97]</sup>研究了具有时变张力复合材料输流管道在亚临界和超临界下的非线性动力学,以及在热环境下的屈曲和后屈曲行为.

## 7 输流管道的振动控制

输流管道动力学分析的最终目的是振动控制,减少管道振动幅值,改善其工作的动力学环境,提高机械系统运转的可靠性.目前,对于输流管道的振动控制研究主要分为优化设计结构的控制、被动控制和主动控制.

对于工程中应用的输流管道,安装和设计是固定的,因此外激励频率的变化范围也是一定的.通过理论分析和实验方法,调谐管道参数、支承位置、材料和结构布置方式等,可实现系统固有频率和模态的避免共振的调控,实现管道的振动控制.Shoaib等<sup>[98]</sup>通过对带隙的实验分析,利用周期性惯性放大机构来减弱输流管道的振动.通过考虑轴向运动和旋转运动,Liang等<sup>[99]</sup>提出了一种新的输送流体声子晶体(PC)管模型,并发现了耦合区域的振动自抑制行为.Lyu等<sup>[100]</sup>根据带隙产生的机理提出了一种超薄压电晶格来抑制输流管道的振动.在旋转局部共振输流管道的基础上,Liang等<sup>[101]</sup>人提出了一种新的动态超材料结构.结果表明,局部谐振管更容易形成低频带隙,有利于振动抑制.

以往的研究主要集中在通过系统本身的优化设计来抑制输流管道的振动.通过引入特定的阻尼力,提出被动控制,以达到更好的减振效果.被动控制方法由于结构设计简单,不需要外部能源,能有效地减小结构在高频段的振动,已被广泛应用于结构振动的抑制.Khazaei等<sup>[102]</sup>提出了一种由两个线性弹簧、一个轻质量块和一个线性阻尼器组成的被动非线性吸振器,它以接地和非接地的形式与管道连接,以实现输流管道的振动抑制.Ding



等<sup>[103]</sup>使用由三个线性弹簧组成的准零刚度系统作为非线性隔振器,研究了流体流速和隔振器参数对输流管道隔振性能的影响.基于非线性能量汇(NES),Zhou等<sup>[104]</sup>引入了一种被动控制方法来讨论对管道振动幅度的控制效果,发现三个抑制区的变化与管道的动态特性有关.Mamaghani和Khadem<sup>[105]</sup>提出了一种光滑NES来抑制在外部简谐载荷激励下固定输流管道的振动.Mao等<sup>[106]</sup>采用了一种非线性减振器,通过吸收振动能量成功地抑制了管道的弯曲振动.El-Borgi等<sup>[107]</sup>基于有限元模型,结合沿管道长度分布的调谐局部谐振器,研究了管道系统在共振频率下的振动衰减.Ishikawa等<sup>[108]</sup>利用圆盘状黏弹性阻尼器设计了一种动力吸振器,并研究了附加吸振器的圆柱管的振动控制.由于传统NES的局限性,Khazaee等<sup>[109]</sup>提出了多个NES的并联和串联结构,以控制输流管道的振动.KWAG等<sup>[110]</sup>采用调谐质量阻尼器来降低实际核管道系统的地震反应.Duan等<sup>[111]</sup>提出了一种惰性增强型非线性能量汇来抑制高亚临界流速下输流管道的振动.鉴于传统的动力吸振器在较窄的频率范围内有效工作,Wu等<sup>[112]</sup>提出将多个动力吸振器周期性地连接到一个充液管道上,以拓宽减振的频带.Yang等<sup>[113]</sup>提出了一种结合负刚度单元的增强型吸振器,并研究了输流管道的自适应减振问题.

采用被动控制方法对低频振动进行抑制是相当有限的.为了进一步拓宽频段,学者们也开始关注主动控制方法.例如,Li等<sup>[114]</sup>提出了一种前馈阻尼方法,用于降低输流悬臂管自由端的响应幅度.Chen等<sup>[115]</sup>采用硬磁软材料构建输流管道,探索动力学调节机制,研究强几何非线性和磁力耦合下管道的受控运动.在电磁驱动装置的基础上,Pisarski等<sup>[116]</sup>提出了悬臂输流管的优化控制,并验证了控制器在三个不同速度区域的有效性.Amiri等<sup>[117]</sup>开发了一种智能控制方法,通过选择智能磁致伸缩层作为作动器来减弱悬臂输流微管的振动.Szmidt等<sup>[118]</sup>利用一种涡流阻尼器来提高输流管道的稳定性,并提出了一种用于优化控制的状态反馈参数化方法.另外,纯主动控制的效果在高频段并不明显,从而提出主被动控制一体化以弥补单一控制方式的缺陷.针对这一问题,Paknejad等<sup>[119]</sup>设计了一种新型鲁棒混合控制系统,通过将被动电磁分流阻尼

器与负反馈控制方法相结合来实现振动控制.Lu等<sup>[120]</sup>研究了输流管道在大变形情况下的压电能量采集问题,有利于输流管道的振动控制.Tang等<sup>[121]</sup>提出了抑制悬臂输流管道振动的主动被混合压电网络控制方法,与纯主动控制相比,具有显著的宽频减振优势,也解决了传动被动控制低频失效和频带过窄的问题.

## 8 总结与展望

本文从输流管道动力学和振动控制的研究方面,对最新的相关文献进行回顾,展望如下:

(1)高维、大变形输流管道的非线性振动,随着工程结构向着高速运转、大型化方向发展,需精确地考虑三维和强非线性影响等因数,这势必增加了输流管道的建模和求解难度,需发展准确模型降维和非线性动力学求解方法,实现输流管道振动特性的准确预估.

(2)复杂管道系统的流固耦合机理研究,为了实现工程应用,对一些复杂外形如螺旋线型以及管中管系统进行整合设计,与传统模型相比,流固耦合机理更加复杂,发展非线性动力学理论和实验分析手段,也将是一个非常重要的研究趋势之一.

(3)宽频、高效的输流管道振动控制,会是输流管道研究的重要课题之一,如何引入智能控制方法,克服主动和被动控制各自的劣势,达到输流管道振动抑制的目的,是输流管道振动控制易于工程应用的重要方向.

综上所述,我们对输流管道的建模,动力学分析和振动控制的最新进展进行了回顾,给出了未来发展趋势的建议.希望能为输流管道的工程设计和应用提供建设性的研究思路.

## 参考文献

- [1] BOURRIÈRES F J. Sur un phénomène d'oscillation autoentretenu en mécanique des fluides reels [J]. Publications Scientifiques et Techniques du Ministère de l'Air, 1939,147: 57-65.
- [2] ASHLEY H, HAVILAND G. Bending vibrations of a pipe line containing flowing fluid [J]. Journal of Applied Mechanics, 1950, 17(3): 229-232.
- [3] PLAUT R H, HUSEYIN K. Instability of fluid-conveying pipes under axial load [J]. Journal of Ap-

- plied Mechanics, 1975, 42(4): 889—890.
- [4] SINGH K, MALLIK A K. Wave propagation and vibration response of a periodically supported pipe conveying fluid [J]. Journal of Sound and Vibration, 1977, 54(1): 55—66.
- [5] PAÏDOUSSIS M P, ISSID N T. Dynamic stability of pipes conveying fluid [J]. Journal of Sound and Vibration, 1974, 33(3): 267—294.
- [6] PAÏDOUSSIS M P. Flow-induced instabilities of cylindrical structures [J]. Applied Mechanics Reviews, 1987, 40(2): 163—175.
- [7] HOLMES, P J. Bifurcations to divergence and flutter in flow-induced oscillations: a finite dimensional analysis [J]. Journal of Sound and Vibration, 1977, 53(4): 471—503.
- [8] MENG D, GUO H Y, XU S P. Non-linear dynamic model of a fluid-conveying pipe undergoing overall motions [J]. Applied Mathematical Modelling, 2011, 35(2): 781—796.
- [9] GHAYESH M H, PAÏDOUSSIS M P, AMABILI M. Nonlinear dynamics of cantilevered extensible pipes conveying fluid [J]. Journal of Sound and Vibration, 2013, 332(24): 6405—6418.
- [10] CHANG G H, MODARRES-SADEGHI Y. Flow-induced oscillations of a cantilevered pipe conveying fluid with base excitation [J]. Journal of Sound and Vibration, 2014, 333(18): 4265—4280.
- [11] LÜ L, HU Y, WANG X, et al. Dynamical bifurcation and synchronization of two nonlinearly coupled fluid-conveying pipes [J]. Nonlinear Dynamics, 2015, 79(4): 2715—2734.
- [12] ZHANG T, OUYANG H, ZHANG Y O, et al. Nonlinear dynamics of straight fluid-conveying pipes with general boundary conditions and additional springs and masses [J]. Applied Mathematical Modelling, 2016, 40(17-18): 7880—7900.
- [13] NAYFEH A H, MOOK D T. Nonlinear oscillations [M]. Wiley, New York, 1979.
- [14] XU J, YANG Q B. Flow-induced internal resonances and mode exchange in horizontal cantilevered pipe conveying fluid (I) [J]. Applied Mathematics and Mechanics, 2006, 27(7): 935—941.
- [15] XU J, YANG Q B. Flow-induced internal resonances and mode exchange in horizontal cantilevered pipe conveying fluid (II) [J]. Applied Mathematics and Mechanics, 2006, 27(7): 943—951.
- [16] CHEN L Q, ZHANG Y L, ZHANG G C, et al. Evolution of the double-jumping in pipes conveying fluid flowing at the supercritical speed [J]. International Journal of Non-Linear Mechanics, 2014, 58: 11—21.
- [17] MAO X Y, DING H, CHEN L Q. Steady-state response of a fluid-conveying pipe with 3 : 1 internal resonance in supercritical regime [J]. Nonlinear Dynamics, 2016, 86(2): 795—809.
- [18] PANDA L N, KAR R C. Nonlinear dynamics of a pipe conveying pulsating fluid with parametric and internal resonances [J]. Nonlinear Dynamics, 2007, 49(1-2): 9—30.
- [19] PANDA L N, KAR R C. Nonlinear dynamics of a pipe conveying pulsating fluid with combination, principal parametric and internal resonances [J]. Journal of Sound and Vibration, 2008, 309(3-5): 375—406.
- [20] YANG X D, YANG T Z, JIN J D. Dynamic stability of a beam-model viscoelastic pipe for conveying pulsative fluid [J]. Acta Mechanica Solida Sinica, 2007, 20: 1—7.
- [21] DAI H L, WANG L, QIAN Q, et al. Vortex-induced vibrations of pipes conveying pulsating fluid [J]. Ocean Engineering, 2014, 77: 12—22.
- [22] ZHOU S, YU T J, YANG X D, et al. Global dynamics of pipes conveying pulsating fluid in the supercritical regime [J]. International Journal of Applied Mechanics, 2017, 9(02): 1750029.
- [23] MAO X Y, SUN J Q, DING H, et al. An approximate method for one-dimensional structures with strong nonlinear and nonhomogenous boundary conditions [J]. Journal of Sound and Vibration, 2020, 469: 115128.
- [24] GAO P X, ZHANG Y L, LIU X F, et al. Vibration analysis of aero parallel-pipeline systems based on a novel reduced order modeling method [J]. Journal of Mechanical Science and Technology, 2020, 34: 3137—3146.
- [25] ZHOU K, YI H R, DAI H L, et al. Nonlinear analysis of L-shaped pipe conveying fluid with the aid of absolute nodal coordinate formulation [J]. Nonlinear Dynamics, 2022, 107: 391—412.
- [26] YAMASHITA K, YAGYU T, YABUNO H. Nonlinear interactions between unstable oscillatory modes in a cantilevered pipe conveying fluid [J]. Nonlinear Dynamics, 2019, 98: 2927—2938.
- [27] TAN X, DING H, CHEN L Q. Nonlinear frequen-



- cies and forced responses of pipes conveying fluid via a coupled Timoshenko model [J]. *Journal of Sound and Vibration*, 2019, 455: 241–255.
- [28] SAZESH S, SHAMS S. Vibration analysis of cantilever pipe conveying fluid under distributed random excitation [J]. *Journal of Fluids and Structures*, 2019, 87: 84–101.
- [29] GIACOBBI D B, SEMLER C, PAÏDOUSSIS M P. Dynamics of pipes conveying fluid of axially varying density [J]. *Journal of Sound and Vibration*, 2020, 473: 115202.
- [30] HIGUCHI E, YABUNO H, YAMASHITA K. Method of experimentally identifying the complex mode shape of the self-excited oscillation of a cantilevered pipe conveying fluid [J]. *Nonlinear Dynamics*, 2022, 109: 589–604.
- [31] LI M W, YAN H, WANG L. Nonlinear model reduction for a cantilevered pipe conveying fluid: A system with asymmetric damping and stiffness matrices [J]. *Mechanical Systems and Signal Processing*, 2023, 188: 109993.
- [32] ZHANG L, CHEN F Q. Multi-pulse jumping orbits and chaotic dynamics of cantilevered pipes conveying time-varying fluid [J]. *Nonlinear Dynamics*, 2019, 97: 991–1009.
- [33] TAN X, DING H, SUN J Q, CHEN L Q. Primary and super-harmonic resonances of Timoshenko pipes conveying high-speed fluid [J]. *Ocean Engineering*, 2020, 203: 107258.
- [34] TAN X, DING H. Parametric resonances of Timoshenko pipes conveying pulsating high-speed fluids [J]. *Journal of Sound and Vibration*, 2020, 485: 115594.
- [35] LU Z Q, ZHANG K K, DING H, et al. Internal resonance and stress distribution of pipes conveying fluid in supercritical regime [J]. *International Journal of Mechanical Sciences*, 2020, 186: 105900.
- [36] ZHU B, ZHANG X L, ZHAO T Y. Nonlinear planar and non-planar vibrations of viscoelastic fluid-conveying pipes with external and internal resonances [J]. *Journal of Sound and Vibration*, 2023, 548: 117558.
- [37] OYELADE A O, OYELADE A A. The effect of various boundary conditions on the nonlinear dynamics of slightly curved pipes under thermal loading [J]. *Applied Mathematical Modelling*, 2020, 87: 332–350.
- [38] ZHOU K, NI Q, CHEN W, et al. Static equilibrium configuration and nonlinear dynamics of slightly curved cantilevered pipe conveying fluid [J]. *Journal of Sound and Vibration*, 2021, 490: 115711.
- [39] CZERWINSKI A, ŁUCZKO J. Nonlinear vibrations of planar curved pipes conveying fluid [J]. *Journal of Sound and Vibration*, 2021, 501: 116054.
- [40] CHEN W, ZHOU K, WANG L, et al. Geometrically exact model and dynamics of cantilevered curved pipes conveying fluid [J]. *Journal of Sound and Vibration*, 2022, 534: 117074.
- [41] XU A D, CHAI Y Y, LI F M, et al. Nonlinear vortex-induced vibrations of slightly curved pipes conveying fluid in steady and oscillatory flows [J]. *Ocean Engineering*, 2023, 270: 113623.
- [42] YAN H, LI M W, WANG L. Bifurcation and stability analysis of static equilibrium configuration of curved pipe conveying fluid [J]. *European Journal of Mechanics-A/Solids*, 2023, 97: 104813.
- [43] YAMASHITA K, KITaura K, NISHIYAMA N, et al. Non-planar motions due to nonlinear interactions between unstable oscillatory modes in a cantilevered pipe conveying fluid [J]. *Mechanical Systems and Signal Processing*, 2022, 178: 109183.
- [44] GUO X M, GE H, XIAO C L, et al. Vibration transmission characteristics analysis of the parallel fluid-conveying pipes system: Numerical and experimental studies [J]. *Mechanical Systems and Signal Processing*, 2022, 177: 109180.
- [45] PENG G, XIONG Y M, LIU L M, et al. 3-D nonlinear dynamics of inclined pipe conveying fluid, supported at both ends [J]. *Journal of Sound and Vibration*, 2019, 449: 405–426.
- [46] ELNAJJAR J, DANESHMAND F. Stability of horizontal and vertical pipes conveying fluid under the effects of additional point masses and springs [J]. *Ocean Engineering*, 2020, 206: 106943.
- [47] ASKARIAN A R, PERMOON M R, SHAKOURI M. Vibration analysis of pipes conveying fluid resting on a fractional Kelvin-Voigt viscoelastic foundation with general boundary conditions [J]. *International Journal of Mechanical Sciences*, 2020, 179: 105702.
- [48] KHEIRI M. Nonlinear dynamics of imperfectly-supported pipes conveying fluid [J]. *Journal of Fluids and Structures*, 2020, 93: 102850.
- [49] MAO X Y, SHU S, FAN X, et al. An approximate

- method for pipes conveying fluid with strong boundaries [J]. *Journal of Sound and Vibration*, 2021, 505: 116157.
- [50] ZHOU J, CHANG X P, XIONG Z J, et al. Stability and nonlinear vibration analysis of fluid-conveying composite pipes with elastic boundary conditions [J]. *Thin-Walled Structures*, 2022, 179: 109597.
- [51] ZHOU K, NI Q, GUO Z L, et al. Nonlinear dynamic analysis of cantilevered pipe conveying fluid with local rigid segment [J]. *Nonlinear Dynamics*, 2022, 109: 1571–1589.
- [52] PENG G, XIONG Y M, GAO Y, et al. Non-linear dynamics of simply supported fluid-conveying pipe subjected to motion-limiting constraints: Two-dimensional analysis [J]. *Journal of Sound and Vibration*, 2018, 435: 192–204.
- [53] LIU Z Y, WANG L, DAI H L, et al. Nonplanar vortex-induced vibrations of cantilevered pipes conveying fluid subjected to loose constraints [J]. *Ocean Engineering*, 2019, 178: 1–19.
- [54] YAN H, DAI H L, NI Q, et al. Nonlinear dynamics of a sliding pipe conveying fluid [J]. *Journal of Fluids and Structures*, 2018, 81: 36–57.
- [55] LIANG F, GAO A, LI X F, et al. Nonlinear parametric vibration of spinning pipes conveying fluid with varying spinning speed and flow velocity [J]. *Applied Mathematical Modelling*, 2021, 95: 320–338.
- [56] LIANG F, GAO A, YANG X D. Dynamical analysis of spinning functionally graded pipes conveying fluid with multiple spans [J]. *Applied Mathematical Modelling*, 2020, 83: 454–469.
- [57] EBRAHIMI R, ZIAEI-RAD S. Nonplanar vibration and flutter analysis of vertically spinning cantilevered piezoelectric pipes conveying fluid [J]. *Ocean Engineering*, 2022, 261: 112180.
- [58] LIANG F, YANG X D, ZHANG W, et al. Vibrations in 3D space of a spinning supported pipe exposed to internal and external annular flows [J]. *Journal of Fluids and Structures*, 2019, 87: 247–262.
- [59] ABDOLLAHI R, FIROUZ-ABADI R D, RAHMANNIAN M. On the stability of rotating pipes conveying fluid in annular liquid medium [J]. *Journal of Sound and Vibration*, 2021, 494: 115891.
- [60] PAÏDOUSSIS M P, ABDELBAKI A R, BUTT M F J, et al. Dynamics of a cantilevered pipe subjected to internal and reverse external axial flow: A review [J]. *Journal of Fluids and Structures*, 2021, 106: 103349.
- [61] ABDELBAKI A R, PAÏDOUSSIS M P, MISRA A K. A nonlinear model for a hanging tubular cantilever simultaneously subjected to internal and confined external axial flow [J]. *Journal of Sound and Vibration*, 2019, 449: 349–367.
- [62] ZHOU K, NI Q, WANG L, et al. Planar and non-planar vibrations of a fluid-conveying cantilevered pipe subjected to axial base excitation [J]. *Nonlinear Dynamics*, 2020, 99: 2527–2549.
- [63] ZHOU K, NI Q, DAI H L, et al. Nonlinear forced vibrations of supported pipe conveying fluid subjected to an axial base excitation [J]. *Journal of Sound and Vibration*, 2020, 471: 115189.
- [64] JIANG T L, DAI H L, WANG L. Three-dimensional dynamics of fluid-conveying pipe simultaneously subjected to external axial flow [J]. *Ocean Engineering*, 2020, 217: 107970.
- [65] ABDELBAKI A R, PAÏDOUSSIS M P, MISRA A K. A nonlinear model for a hanging cantilevered pipe discharging fluid with a partially-confined external flow [J]. *International Journal of Non-Linear Mechanics*, 2020, 118: 103290.
- [66] MINAS S L, PAÏDOUSSIS M P. Dynamics of a shrouded cantilevered pipe subjected to internal and annular flows [J]. *Journal of Sound and Vibration*, 2021, 490: 115729.
- [67] BUTT M F J, PAÏDOUSSIS M P, NAHON M. Dynamics of a confined pipe aspirating fluid and concurrently subjected to external axial flow: A theoretical investigation [J]. *Journal of Sound and Vibration*, 2021, 509: 116148.
- [68] CHEHREGHANI M, ABDELBAKI A R, MISRA A K, et al. Experiments on the dynamics of a cantilevered pipe conveying fluid and subjected to reverse annular flow [J]. *Journal of Sound and Vibration*, 2021, 515: 116480.
- [69] DANESHMAND F, LIAGHAT T, PAÏDOUSSIS M P. A coupled two-way fluid-structure interaction analysis for the dynamics of a partially confined cantilevered pipe under simultaneous internal and external axial flow in opposite directions [J]. *Journal of Pressure Vessel Technology*, 2022, 144 (2): 021401.
- [70] ZHOU K, DAI H L, WANG L, et al. Modeling

- and nonlinear dynamics of cantilevered pipe with tapered free end concurrently subjected to axial internal and external flows [J]. *Mechanical Systems and Signal Processing*, 2022, 169: 108794.
- [71] MONETTE C, PETTIGREW M J. Fluidelastic instability of flexible tubes subjected to two-phase internal flow [J]. *Journal of Fluids and Structures*, 2004, 19: 943–956.
- [72] EBRAHIMI-MAMAGHANI A, SOTUDEH-GHAREBAGH R, ZARGHAMI R, et al. Dynamics of two-phase flow in vertical pipes [J]. *Journal of Fluids and Structures*, 2019, 87: 150–173.
- [73] GUO Y, ZHU B, ZHAO X, et al. Dynamic characteristics and stability of pipe-in-pipe system conveying two-phase flow in thermal environment [J]. *Applied Ocean Research*, 2020, 103: 102333.
- [74] MA B, SRINIL N. Planar dynamics of inclined curved flexible riser carrying slug liquid-gas flows [J]. *Journal of Fluids and Structures*, 2020, 94: 102911.
- [75] LIU D P, AI S M, SUN L P, et al. Numerical modelling of offshore risers conveying slug flow under the ALE-ANCF framework [J]. *Ocean Engineering*, 2021, 235: 109415.
- [76] OYELADE A O, OYEDIRAN A A. Nonlinear dynamics of horizontal pipes conveying two phase flow [J]. *European Journal of Mechanics-A/Solids*, 2021, 90: 104367.
- [77] ZHOU Y L, MI L D, YANG M. Free vibration and stability analysis of inclined pipes conveying gas-liquid slug flow [J]. *Journal of Sound and Vibration*, 2022, 541: 117348.
- [78] EBRAHIMI-MAMAGHANI A, MOSTOUFI N, SOTUDEH-GHAREBAGH R, et al. Vibrational analysis of pipes based on the drift-flux two-phase flow model [J]. *Ocean Engineering*, 2022, 249: 110917.
- [79] CHANG X P, QU C J, SONG Q, et al. Coupled cross-flow and in-line vibration characteristics of frequency-locking of marine composite riser subjected to gas-liquid multiphase internal flow [J]. *Ocean Engineering*, 2022, 266: 113019.
- [80] XIE W D, GAO X F, WANG E H, et al. An investigation of the nonlinear dynamic response of a flexible pipe undergoing vortex-induced vibrations and conveying internal fluid with variable-density [J]. *Ocean Engineering*, 2019, 183: 453–468.
- [81] XIE W D, LIANG Z L, JIANG Z Y, et al. Dynamic responses of a flexible pipe conveying variable-density fluid and experiencing cross-flow and in-line coupled vortex-induced vibrations [J]. *Ocean Engineering*, 2022, 260: 111811.
- [82] TANG Y, YANG T Z. Post-buckling behavior and nonlinear vibration analysis of a fluid-conveying pipe composed of functionally graded material [J]. *Composite Structures*, 2018, 185: 393–400.
- [83] REDDY R S, PANDA S, NATARAJAN G. Nonlinear dynamics of functionally graded pipes conveying hot fluid [J]. *Nonlinear Dynamics*, 2020, 99: 1989–2010.
- [84] LU Z Q, ZHANG K K, DING H, et al. Nonlinear vibration effects on the fatigue life of fluid-conveying pipes composed of axially functionally graded materials [J]. *Nonlinear Dynamics*, 2020, 100: 1091–1104.
- [85] ZHU B, CHEN X C, GUO Y, et al. Static and dynamic characteristics of the post-buckling of fluid-conveying porous functionally graded pipes with geometric imperfections [J]. *International Journal of Mechanical Sciences*, 2021, 189: 105947.
- [86] ZHU B, XU Q, LI M, et al. Nonlinear free and forced vibrations of porous functionally graded pipes conveying fluid and resting on nonlinear elastic foundation [J]. *Composite Structures*, 2020, 252: 112672.
- [87] ZHU B, GUO Y, CHEN B, et al. Nonlinear non-planar dynamics of porous functionally graded pipes conveying fluid [J]. *Communications in Nonlinear Science and Numerical Simulation*, 2023, 117: 106907.
- [88] GUO Q, LIU Y S, CHEN B Q, et al. An efficient stochastic natural frequency analysis method for axially varying functionally graded material pipe conveying fluid [J]. *European Journal of Mechanics/A Solids*, 2021, 86: 104155.
- [89] LIU T, LI Z M. Nonlinear vibration analysis of functionally graded material tubes with conveying fluid resting on elastic foundation by a new tubular beam model [J]. *International Journal of Non-Linear Mechanics*, 2021, 137: 103824.
- [90] CHANG X P, ZHOU J, LI Y H. Post-buckling characteristics of functionally graded fluid-conveying pipe with geometric defects on Pasternak foundation [J]. *Ocean Engineering*, 2022, 266: 113056.



- [91] BABAEI H. On frequency response of FG-CNT reinforced composite pipes in thermally pre/post buckled configurations [J]. *Composite Structures*, 2021, 276: 114467.
- [92] GHADIRIAN H, MOHEBPOUR S, MALEKZADEH P, et al. Nonlinear free vibrations and stability analysis of FG-CNTRC pipes conveying fluid based on Timoshenko model [J]. *Composite Structures*, 2022, 292: 115637.
- [93] REN Y R, LI L Z, JIN Q D. Vibration and snapthrough of fluid-conveying graphene-reinforced composite pipes under low-velocity impact [J]. *AIAA Journal*, 2021, 59(12): 5091—5105.
- [94] LI Z M, LIU T. A new displacement model for nonlinear vibration analysis of fluid-conveying anisotropic laminated tubular beams resting on elastic foundation [J]. *European Journal of Mechanics-A/Solids*, 2021, 86: 104172.
- [95] GUO Y, LI J A, ZHU B, et al. Flow-induced instability and bifurcation in cantilevered composite double-pipe systems [J]. *Ocean Engineering*, 2022, 258: 111825.
- [96] GUO Y, ZHU B, LI Y H. Nonlinear dynamics of fluid-conveying composite pipes subjected to time-varying axial tension in sub- and super-critical regimes [J]. *Applied Mathematical Modelling*, 2022, 101: 632—653.
- [97] GUO Y, ZHU B, YANG B, et al. Flow-induced buckling and post-buckling vibration characteristics of composite pipe in thermal environment [J]. *Ocean Engineering*, 2022, 243: 110267.
- [98] SHOAIB M, PANG W, LI F. Vibration reduction of pipes conveying fluid with periodic inertial amplification mechanisms [J]. *Waves in Random and Complex Media*, 2021, 2021: 1—16.
- [99] LIANG F, CHEN Y, GONG J J, et al. Vibration self-suppression of spinning fluid-conveying pipes composed of periodic composites [J]. *International Journal of Mechanical Sciences*, 2022, 220: 107150.
- [100] LYU X, CHEN F, REN Q, et al. Ultra-thin piezoelectric lattice for vibration suppression in pipe conveying fluid [J]. *Acta Mechanica Solida Sinica*, 2020, 33(6): 770—780.
- [101] LIANG F, CHEN Y, GUAN D, et al. Low-frequency band gap characteristics of a novel spinning metamaterial pipe with Timoshenko model [J]. *Journal of Sound and Vibration*, 2022, 541: 117316.
- [102] KHAZAEE M, KHADEM S E, MOSLEMI A, et al. Vibration mitigation of a pipe conveying fluid with a passive geometrically nonlinear absorber: a tuning optimal design [J]. *Communications in Nonlinear Science and Numerical Simulation*, 2020, 91: 105439.
- [103] DING H, JI J C, CHEN L Q. Nonlinear vibration isolation for fluid-conveying pipes using quasi-zero stiffness characteristics [J]. *Mechanical Systems and Signal Processing*, 2019, 121: 675—688.
- [104] ZHOU K, XIONG F R, JIANG N B, et al. Nonlinear vibration control of a cantilevered fluid-conveying pipe using the idea of nonlinear energy sink [J]. *Nonlinear Dynamics*, 2019, 95(2): 1435—1456.
- [105] MAMAGHANI A E, KHADEM S E. Vibration control of a pipe conveying fluid under external periodic excitation using a nonlinear energy sink [J]. *Nonlinear Dynamics*, 2016, 86(3): 1761—1795.
- [106] MAO X Y, DING H, CHEN L Q. Bending vibration control of pipes conveying fluids by nonlinear torsional absorbers at the boundary [J]. *Science China-Technological Sciences*, 2021, 64(8): 1690—1704.
- [107] EL-BORGI S, ALRUMAIHI A, RAJENDRAN P, et al. Model updating of a scaled piping system and vibration attenuation via locally resonant bandgap formation [J]. *International Journal of Mechanical Sciences*, 2021, 194: 106211.
- [108] ISHIKAWA S, TANAKA K, YANO D, et al. Design of a disc-shaped viscoelastic damping material attached to a cylindrical pipe as a dynamic absorber or Houde damper [J]. *Journal of Sound and Vibration*, 2020, 475: 115272.
- [109] KHAZAEE M, KHADEM S E, MOSLEMI A, et al. A comparative study on optimization of multiple essentially nonlinear isolators attached to a pipe conveying fluid [J]. *Mechanical Systems and Signal Processing*, 2020, 141: 106442.
- [110] KWAG S, EEM S, KWAK J, et al. Mitigation of seismic responses of actual nuclear piping by a newly developed tuned mass damper device [J]. *Nuclear Engineering and Technology*, 2021, 53(8): 2728—2745.
- [111] DUAN N, WU Y H, SUN X M, et al. Vibration control of conveying fluid pipe based on inerter enhanced nonlinear energy sink [J]. *IEEE Transactions on Circuits and Systems I-regular Papers*,

- 2021, 68(4): 1610–1623.
- [112] WU J H, ZHU H Z, SUN Y D, et al. Reduction of flexural vibration of a fluid-filled pipe with attached vibration absorbers [J]. *International Journal of Pressure Vessels and Piping*, 2021, 194: 104525.
- [113] YANG T Z, LIU T, TANG Y, et al. Enhanced targeted energy transfer for adaptive vibration suppression of pipes conveying fluid [J]. *Nonlinear Dynamics*, 2019, 97(3): 1937–1944.
- [114] LI J, DENG H, JIANG W. Dynamic response and vibration suppression of a cantilevered pipe conveying fluid under periodic excitation [J]. *Journal of Vibration and Control*, 2019, 25(11): 1695–1705.
- [115] CHEN W, WANG L, PENG Z. A magnetic control method for large-deformation vibration of cantilevered pipe conveying fluid [J]. *Nonlinear Dynamics*, 2021, 105(2): 1459–1481.
- [116] PISARSKI D, KONOWROCKI R, SZMIDT T. Dynamics and optimal control of an electromagnetically actuated cantilever pipe conveying fluid [J]. *Journal of Sound and Vibration*, 2018, 432: 420–436.
- [117] AMIRI A, MASOUMI A, TALEBITOOTI R. Flutter and bifurcation instability analysis of fluid-conveying micro-pipes sandwiched by magnetostrictive smart layers under thermal and magnetic field [J]. *International Journal of Mechanics and Materials in Design*, 2020, 16(3): 569–588.
- [118] SZMIDT T, PISARSKI D, KONOWROCKI R. Semi-active stabilisation of a pipe conveying fluid using eddy-current dampers: state-feedback control design, experimental validation [J]. *Meccanica*, 2019, 54(6): 761–777.
- [119] PAKNEJAD A, ZHAO G, CHESNE S, et al. Hybrid electromagnetic shunt damper for vibration control [J]. *Journal of Vibration and Acoustics-transactions of the ASME*, 2020, 143(2): 021010.
- [120] LU Z Q, CHEN J, DING H, et al. Energy harvesting of a fluid-conveying piezoelectric pipe [J]. *Applied Mathematical Modelling*, 2022, 107: 165–181.
- [121] TANG Y, GAO C K, LI M M, et al. Novel active-passive hybrid piezoelectric network for vibration suppression in fluid-conveying pipes [J]. *Applied Mathematical Modelling*, 2023, 117: 378–398.