

车-简支梁桥耦合振动系统中全局最大动力响应研究^{*}

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摘要 本文采用四分之一车辆模型和等截面简支梁模型,建立了车-桥耦合振动计算模型,分析了车辆匀速驶过桥梁时,桥梁动挠度、动弯矩、动剪力的全局最大值及发生的位置.进一步分析计算了车速、车距和桥梁模态截断阶数对桥梁动挠度、动弯矩、动剪力全局最大值的影响.结果表明,车速对桥梁动挠度、动弯矩、动剪力全局最大值影响较大,全局最大动挠度和全局最大动弯矩均出现在跨中附近,车速不同位置也不同,而全局最大动剪力均出现在车辆下桥的梁端;两车同时上桥时,前后车车距越大,桥梁动挠度、动弯矩、动剪力全局最大值越小,当达到一定车距时,三者不再减小且与单车情况相同;为提高桥梁动挠度、动弯矩、动剪力全局最大值计算精度,桥梁模态截断阶数宜分别大于 3 阶、6 阶、7 阶.

关键词 车-桥耦合振动, 全局最大值, 车速, 车距, 桥梁模态截断阶数

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The Global Maximum Dynamic Responses of Vehicle-Simply Supported Bridge Coupling System^{*}

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Abstract In this paper, a quarter vehicle model and a uniform cross-section simply supported beam model are used to establish a vehicle-bridge coupling system. The global maximum values and its location on the bridge of dynamic deflection, dynamic bending moment and dynamic shear force are analyzed when the vehicle travelling through the bridge at a constant speed. The effects of vehicle speed, distance between front and rear vehicles, and bridge modal truncation order on the global maximum value are further analyzed and calculated. The results show that the vehicle speed has a greater impact on the global maximum value of dynamic deflection, dynamic bending moment, and dynamic shear force of the bridge. Both the global maximum dynamic deflection and the global maximum dynamic bending moment appear in the mid-span range, the locations are different with different vehicle speeds. The global maximum dynamic shear force appears at the beam end of the vehicle exiting the bridge. when two vehicles are on the

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bridge at the same time, the greater the distance between the front and rear vehicles, the smaller the global maximum value of dynamic deflection, dynamic bending moment, and dynamic shear force of the bridge. When a certain distance is reached, the global maximum values are no longer reduced and are the same as the case of a single vehicle. In order to improve the calculation accuracy of the dynamic deflection, dynamic bending moment, and dynamic shear force of the bridge, the bridge modal truncation order should be greater than 3 orders, 6 orders and 7 orders, respectively.

Key words vehicle-bridge coupling vibration, global maximum, vehicle speed, distance between front and rear vehicles, bridge modal truncation order

引言

为了提升车辆的动力性、稳定性、平顺性和安全性,车辆动力学与控制已经成为近年来相关学者研究的重点^[1]并取得了一定研究成果.而车辆行驶的过程中通过桥梁时,会引起桥梁振动,桥梁的振动反过来会影响车辆的振动,这种车-桥之间相互影响的振动,称为车-桥耦合振动^[2].车、桥之间的耦合振动会加剧对桥梁结构的损伤和破坏,也会影响车辆的安全性和舒适性.因此,车-桥耦合振动^[3-6]问题长期以来也一直被国内外学者和工程师广泛关注.在众多的研究中发现,车速对车-桥耦合系统中桥梁的跨中动挠度、动应力、动弯矩等响应影响较大,且车速越快,越易激发桥梁的高阶模态;桥面不平顺对桥梁结构的冲击系数影响较大^[7-13].还有学者指出在多车同时驶过桥梁时的车-桥耦合振动系统中^[14-16],车辆行驶间距接近桥梁单孔跨径时产生的冲击系数均较大,桥面不平顺度对动弯矩、动剪力的影响较大,整座桥梁最大动挠度发生在跨中附近,并且当车辆位于跨中附近时出现最大动挠度.

目前的研究大多集中于对桥梁跨中动态响应进行分析,而实际上桥梁各点的振动响应有较大差别.因此,本文拟建立车-桥耦合系统振动方程,计算桥梁各点动态响应,及不同车速下桥梁动挠度、动弯矩、动剪力全局最大值及最大值发生的位置,分析车速、车距、模态截断阶数对桥梁动力响应的影响情况.计算结果可为车-桥耦合振动问题的数值计算及实际桥梁的安全运营提供理论依据.

1 车-桥耦合系统计算模型

图1所示为考虑桥面不平顺的车-桥耦合系统模型,车辆简化为二自由度四分之一车模型,桥梁

为简支梁.梁的跨度为 l ;梁的单位长度质量为 m ;梁截面抗弯刚度为 EI ;梁竖向动挠度为 $w(x, t)$;桥面不平顺函数为 $r(t) = 0.001 \sin(2\pi vt/10)$; m_1 , m_2 表示轮胎质量和车体质量; k_1 , k_2 表示悬架刚度和轮胎刚度; c_1 , c_2 表示悬架阻尼系数和轮胎阻尼系数; z_1 , z_2 表示轮胎动位移和车体动位移.

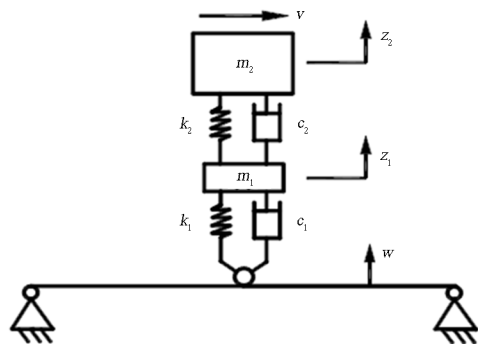


图1 车-桥耦合系统模型
Fig.1 Vehicle-bridge coupling system model

车-桥耦合系统中车辆和梁的振动微分方程为:

$$m_2 \ddot{z}_2 + k_2(z_2 - z_1) + c_2(\dot{z}_2 - \dot{z}_1) = 0 \quad (1)$$

$$m_1 \ddot{z}_1 + k_2(z_1 - z_2) + c_2(\dot{z}_1 - \dot{z}_2) + k_1[z_1 - w - r(t)] + c_1[\dot{z}_1 - \dot{w} - \dot{r}(t)] = 0 \quad (2)$$

$$EI \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} = -\{k_1[w + r(t) - z_1] + c_1[\dot{w} + \dot{r}(t) - \dot{z}_1] + (m_1 + m_2)g\} \delta(x - vt) \quad (3)$$

令 $w(x, t) = \sum_{i=1}^N \varphi_i(x) q_i(t)$, 其中 N 为模态截断阶数, 本文中 N 取 7, 可得到车-桥耦合系统的振动方程为:

$$[\bar{M}] \ddot{U} + [\bar{C}] \dot{U} + [\bar{K}] U = [F] \quad (4)$$

其中:

$$U = [z_2, z_1, q_1, q_2, \dots, q_N]^T$$

$$[\bar{\boldsymbol{M}}]=\begin{bmatrix}m_2\\&m_1\\&&M_1\\&&&M_2\\&&&&\ddots\\&&&&&M_N\end{bmatrix}$$

$$[\bar{\boldsymbol{C}}]=\begin{bmatrix}c_2&&-c_2\\-c_2&c_2+c_1&&-c_1\varphi_1(x)&-c_1\varphi_2(x)&\cdots&-c_1\varphi_N(x)\\&-c_1\varphi_1(x)&&c_1\varphi_1^2(x)\\&-c_1\varphi_2(x)&&c_1\varphi_2(x)\varphi_1(x)&c_1\varphi_2^2(x)\\&\vdots&&\vdots&\vdots&\ddots\\-c_1\varphi_N(x)&c_1\varphi_N(x)\varphi_1(x)&c_1\varphi_N(x)\varphi_2(x)&\cdots&c_1\varphi_N^2(x)\end{bmatrix}$$

$$[\bar{\boldsymbol{K}}]=\begin{bmatrix}k_2&&-k_2\\-k_2&k_2+k_1&&-k_1\varphi_1(x)&-k_1\varphi_2(x)&\cdots&-k_1\varphi_N(x)\\&-k_1\varphi_1(x)&&K_1+k_1\varphi_1^2(x)\\&-k_1\varphi_2(x)&&k_1\varphi_2(x)\varphi_1(x)&K_2+k_1\varphi_2^2(x)\\&\vdots&&\vdots&\vdots&\ddots\\-k_1\varphi_N(x)&k_1\varphi_N(x)\varphi_1(x)&k_1\varphi_N(x)\varphi_2(x)&\cdots&K_N+k_1\varphi_N^2(x)\end{bmatrix}$$

$$[\boldsymbol{F}]=\begin{bmatrix}0\\k_1r(t)+c_1\dot{r}(t)\\-[k_1r(t)+c_1\dot{r}(t)+(m_1+m_2)g]\varphi_1(x)\\-[k_1r(t)+c_1\dot{r}(t)+(m_1+m_2)g]\varphi_2(x)\\\vdots\\-[k_1r(t)+c_1\dot{r}(t)+(m_1+m_2)g]\varphi_N(x)\end{bmatrix}$$

式中：解振动方程式(4)。

$M_i=\int_0^L m\varphi_i(x)\varphi_j(x)dx,i=j,(i,j=1,2,\cdots,N)$

$K_i=\int_0^L EI\varphi'''_i(x)\varphi_j(x)dx,i=j,(i,j=1,2,\cdots,N)$

式(4)中, $[\bar{\boldsymbol{M}}]$ 、 $[\bar{\boldsymbol{C}}]$ 、 $[\bar{\boldsymbol{K}}]$ 、 $[\boldsymbol{F}]$ 矩阵随车辆在桥梁上位置的变化而变化,本文运用 Newmark- β 法求

2 车-桥耦合系统动力响应

本节推导求解车-桥耦合系统振动方程的解析解^[17],将数值解与解析解进行对比,验证解的正确性. 车辆作用下简支梁振动微分方程为:

$$\ddot{q}_i(t)+\omega_n^2q_i(t)=2\sin\left(\frac{i\pi vt}{L}\right)\left\{\frac{-(m_1+m_2)g}{mL}+\frac{(m_1+m_2)}{mL}\frac{k_1}{m_1}[q_i(t)-\omega(x,t)]+\right.$$

$$\left.\frac{(m_1+m_2)}{mL}\frac{c_1}{m_1}[\dot{q}_i(t)-\dot{\omega}(x,t)]-\frac{(m_1+m_2)}{mL}\frac{k_1}{m_1}r(t)-\frac{(m_1+m_2)}{mL}\frac{c_1}{m_1}\dot{r}(t)\right\}\tag{5}$$

式中 ω_n 为简支梁的固有频率:

$$\omega_n=\frac{i^2\pi^2}{L^2}\sqrt{\frac{EI}{m}}$$

$$\tag{6}$$

式(5)可以简化为^[18]:

$$\ddot{q}_i(t)+\omega_n^2q_i(t)=2\sin\left(\frac{i\pi vt}{L}\right)\left(\frac{-(m_1+m_2)g}{mL}\right)$$

$$\tag{7}$$

根据两端简支的等截面梁边界条件可得到简

支梁的广义坐标:

$$q_i(t)=\frac{A_i}{1-B_i^2}\left[\sin\left(\frac{i\pi vt}{L}\right)-B_i\sin(\omega_nt)\right]$$

(8)

式中:

$$A_i=\frac{-2(m_1+m_2)gL^3}{\pi^4EI}$$
$$B_i=\frac{i\pi v}{L\omega_n}$$

则简支梁动挠度的解析解为:

$$w(x,t)=\sum_{i=1}^N\frac{A_i}{1-B_i}\left\{\sin\left(\frac{i\pi x}{L}\right)\left[\sin\left(\frac{i\pi vt}{L}\right)-B_i\sin(\omega_nt)\right]\right\}$$

(9)

本文以湖南省某跨度为 50m 的预应力混凝土简支箱梁桥为例,桥梁抗弯刚度为 $EI=5.28\times10^{11}\text{Nm}^2$,单位长度质量为 $m=32840\text{kg/m}$.车辆参数选择重型车辆^[19]: $m_1=190\text{kg}$, $m_2=10109\text{kg}$, $k_1=2060000\text{N/m}$, $c_1=900\text{Ns/m}$, $k_2=75000\text{N/m}$, $c_2=30000\text{Ns/m}$.

分别采用解析和数值方法得到了桥梁跨中动挠度的时程曲线(如图 2 所示),表 1 列出了不同车速下跨中最大动挠度及相对偏差.

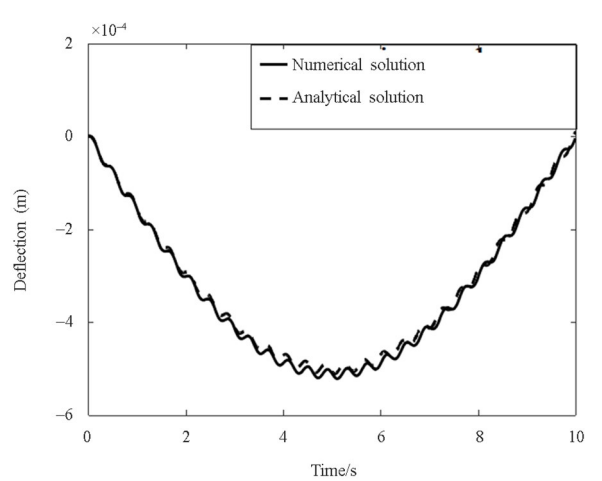


图 2 简支梁桥跨中动挠度时程曲线
Fig.2 Time-history curve of dynamic deflection in the middle span of a simply supported beam bridge

由图 2 和表 1 可知,不同车速下跨中动挠度数值解与解析解变化趋势相同,得到的跨中动挠度最大值也基本一致,相对偏差小于 3%,两种方法的分析结果吻合良好.

表 1 解析解与数值解所得桥梁跨中最大动挠度及相对偏差
Table 1 Maximum dynamic deflection and relative deviation of bridge mid-span obtained from analytical and numerical solutions

Speed(m/s)	5	10	15	20
Analytical solution(mm)	-0.5107	-0.5182	-0.5256	-0.5345
Numerical solution(mm)	-0.5214	-0.5290	-0.5383	-0.5477
Relative deviation(%)	2.05	2.04	2.36	2.41

3 车-桥耦合系统动力响应的影响因素分析

3.1 车速

采用上述数值方法,分别计算了车速为 5m/s、15m/s、25m/s 时,简支梁跨中动挠度、动弯矩、动剪力,见图 3~图 5.结果表明,不同车速时,跨中动挠度、动弯矩、动剪力的变化趋势一致,随着车速的增加,以上动力响应的最大值逐渐增大.车辆在桥梁上行驶时,桥梁各个截面的动挠度、动弯矩和动剪力会在某一位置和某个瞬间达到最大值,且在车速不同时,最大值也不同.通过数值分析可以得到,不同车速下,桥梁所有截面的动挠度、动弯矩、动剪力全局最大值以及最大值所在位置.其中,桥梁动挠度、动弯矩、动剪力全局最大值定义为:

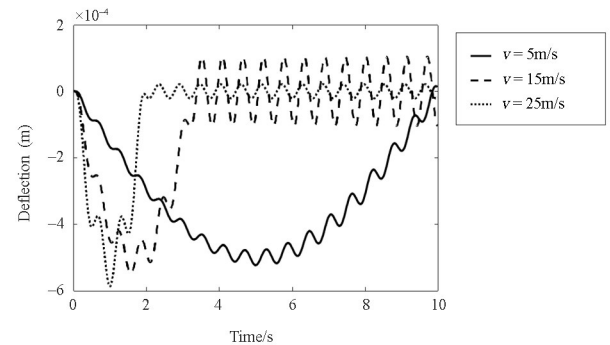


图 3 车速对简支梁桥跨中动挠度的影响
Fig.3 Mid-span dynamic deflection of bridges at different speeds

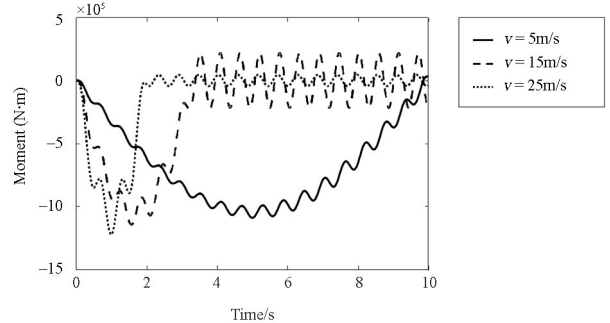


图 4 车速对简支梁桥跨中动弯矩的影响
Fig.4 Mid-span bending moment at different speeds

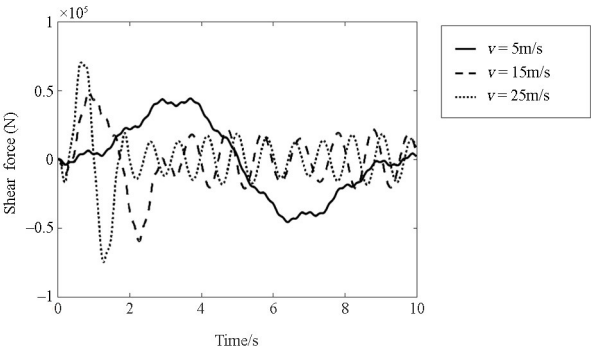


图 5 车速对简支梁桥跨中动剪力的影响
Fig.5 Mid-span shear force at different speeds

$$w_{\max} = \max \left\{ w(x, t), 0 \leq x \leq l, 0 \leq t \leq \frac{l}{v} \right\} \quad (10)$$

$$M_{b\max} = \max \left\{ M_b(x, t), 0 \leq x \leq l, 0 \leq t \leq \frac{l}{v} \right\} \quad (11)$$

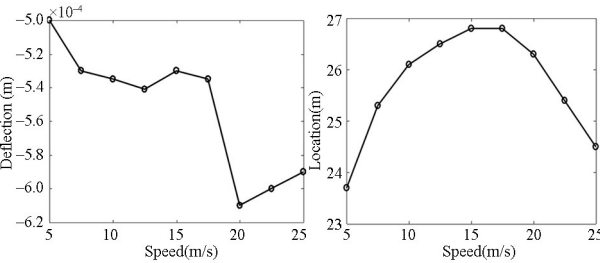
$$Q_{b\max} = \max \left\{ Q_b(x, t), 0 \leq x \leq l, 0 \leq t \leq \frac{l}{v} \right\} \quad (12)$$

其中,由材料力学知识可知,梁的弯矩、剪力与挠度的关系为:

$$M_b(x, t) = -EI \frac{\partial^2 w(x, t)}{\partial x^2} \quad (13)$$

$$Q_b(x, t) = -EI \frac{\partial^3 w(x, t)}{\partial x^3} \quad (14)$$

图 6~图 8 为车速对桥梁动挠度、动弯矩、动剪力全局最大值及所在位置的影响情况。

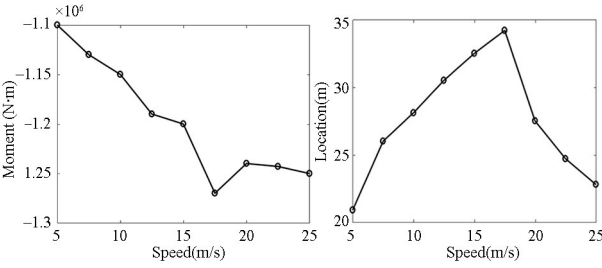


(a) 动挠度全局最大值 (b) 动挠度全局最大值发生的位置
(a) The global maximum deflection (b) Location of the global maximum deflection

图 6 不同车速下桥梁动挠度全局最大值及发生的位置
Fig.6 Global maximum deflection and location of the bridge's dynamic deflection at different speeds

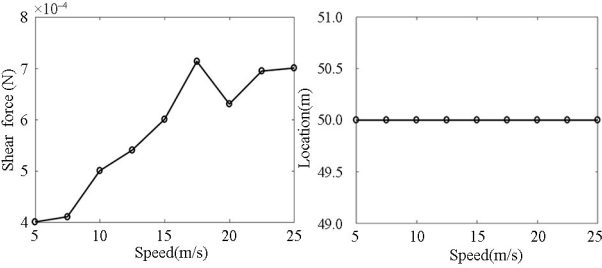
由图 6~图 8 可知,当车辆以 20m/s 行驶时,桥梁动挠度全局最大值最大,出现在跨中附近,距离上桥端为 26m 左右;当车辆以 17.5m/s 行驶时,桥梁动弯矩全局最大值最大,出现的位置偏离跨中位置大约 10m 左右;当车辆以 17.5m/s 行驶时,桥

梁动剪力全局最大值最大,最大剪力始终出现在车辆下桥的位置.因此,车速不同,引起的桥梁动力响应的全局最大值不同,除了动剪力响应,其他响应发生的位置也有所不同。



(a) 弯矩全局最大值 (b) 弯矩全局最大值位置
(a) The global maximum bending moment (b) Location of the global maximum bending moment

图 7 不同车速下桥梁弯矩全局最大值及发生的位置
Fig.7 Global maximum value and location of bridge bending moment at different speeds



(a) 剪力全局最大值 (b) 剪力全局最大值位置
(a) The global maximum shear force (b) Location of the global maximum shear force

图 8 不同车速下桥梁剪力全局最大值及发生的位置
Fig.8 Global maximum value and location of bridge shear force at different speeds

3.2 车距

当两辆车同时上桥时,车辆参数与单车相同,前后车之间的车距分别为 10m、20m、30m 时,简支梁跨中动挠度、动弯矩、动剪力的时程曲线,如图 9~图 10 所示.结果表明,随着车距的增加,桥梁跨中动挠度、动弯矩、动剪力逐渐减小。

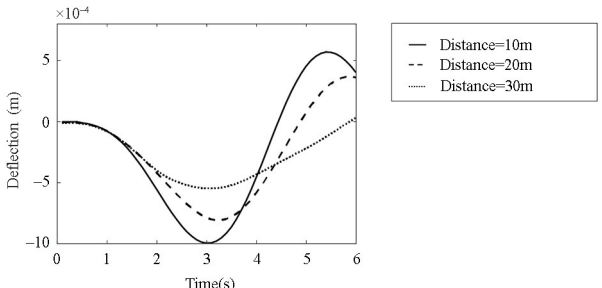


图 9 不同车距下桥梁跨中动挠度
Fig.9 Mid-span dynamic deflection of bridge under different vehicle distances

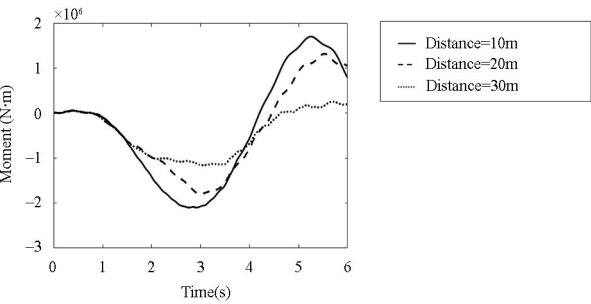


图 10 不同车距下桥梁跨中动弯矩
Fig.10 Mid-span bending moment of bridge under different vehicle distances

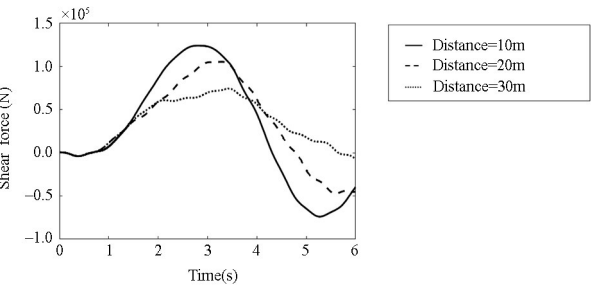
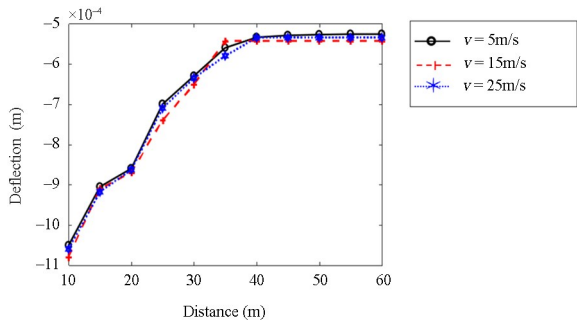
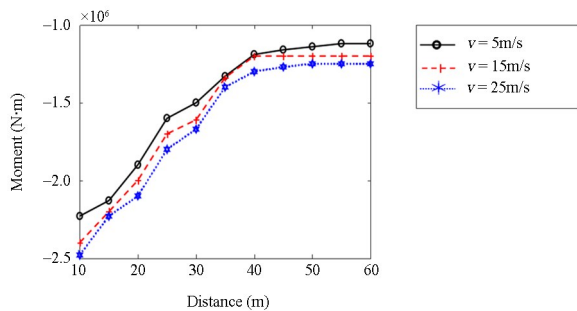


图 11 不同车距下桥梁跨中动剪力
Fig.11 Mid-span shear force of bridge under different vehicle distances

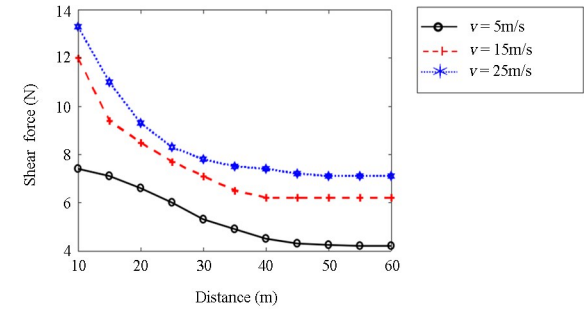
图 12 为两车同时上桥时,分别以车速 5m/s、15m/s、25m/s,车距以 5m 为增量步长,从 10m 增加到 60m 通过桥梁,桥梁跨中动挠度、动弯矩、动剪力全局最大值变化规律。



(a) 动挠度
(a) Deflection



(b) 动弯矩
(b) Bending moment



(c) 动剪力
(c) Shear force

图 12 不同车距下桥梁动挠度、弯矩、剪力全局最大值
Fig.12 Global maximum values of dynamic deflection, bending moment and shear force of bridges under different vehicle distances

由图 12 可以看出,不同车速下车距越大,桥梁动挠度、动弯矩、动剪力全局最大值越小;当达到临界车距时,三个最大值不再减小且与单车同车速下情况相同,说明达到临界车距后,两车对动力响应的全局最大值叠加影响消失。

3.3 桥梁模态截断阶数

进行数值计算时,分别截取简支梁第 1 阶模态、前 6 阶模态、前 7 阶模态、前 8 阶模态,计算桥梁的动力响应,如图 13~图 15 所示,为车速 15m/s,

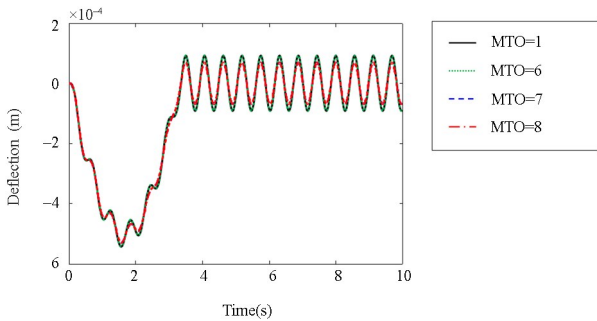


图 13 模态截断阶数对桥梁跨中动挠度的影响
Fig.13 Mid-span dynamic deflection of bridge under different bridge modal truncation orders(MTO)

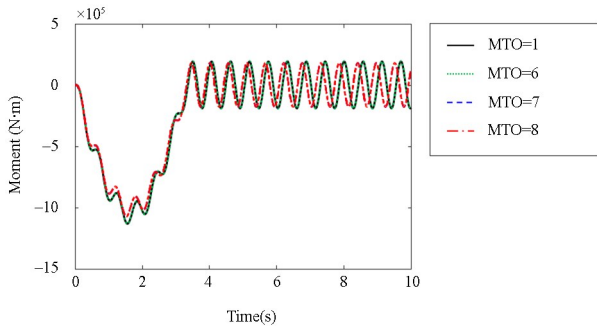


图 14 模态截断阶数对桥梁跨中动弯矩的影响
Fig.14 Mid-span bending moment of bridge under different bridge modal truncation orders(MTO)

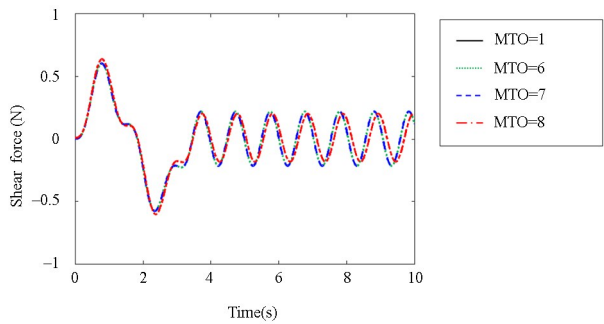


图 15 模态截断阶数对桥梁跨中动剪力的影响
Fig.15 Mid-span shear force of bridge under different bridge modal truncation orders(MTO)

取不同模态截断阶数时,简支梁跨中动挠度、动弯矩、动剪力的时程曲线.结果表明,数值计算时,简支梁的模态截断阶数对跨中动挠度影响较小,若取低阶模态计算,跨中弯矩最大值计算结果偏大,跨中剪力最大值计算结果偏小.

图 16~图 18 为车速为 15m/s 时,模态截断阶数对桥梁动挠度、动弯矩、动剪力全局最大值的影响.

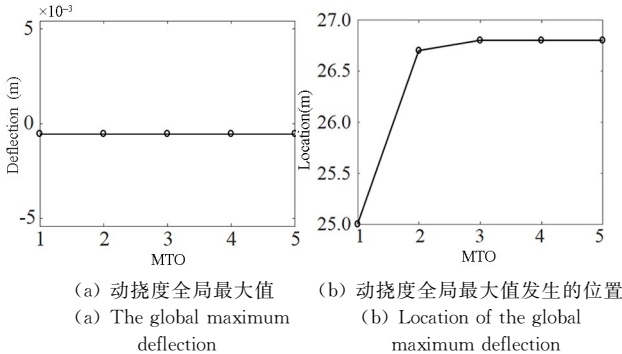


图 16 不同桥梁模态截断阶数下桥梁动挠度全局最大值及位置
Fig.16 Global maximum deflection and location of deflection of bridge under different bridge modal truncation orders(MTO)

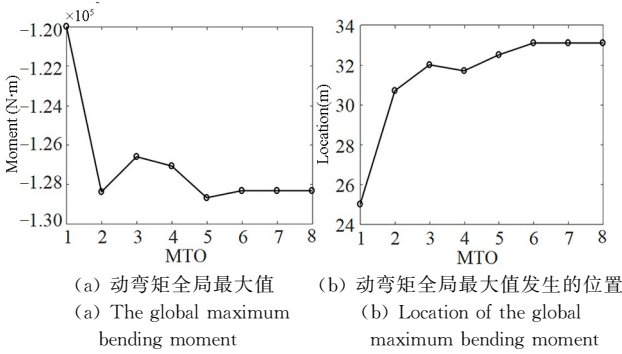


图 17 不同桥梁模态截断阶数下桥梁动弯矩全局最大值及位置
Fig.17 Global maximum value and location of bridge bending moment under different bridge modal truncation orders(MTO)

由图 16~图 18 可知,计算桥梁动挠度全局最大值时,桥梁模态截断阶数影响较小,随着截断阶数增

大,动挠度全局最大值发生的位置稳定在 26.8m 处,为了提高桥梁动挠度计算精度,桥梁模态截断阶数应大于 3 阶;计算桥梁动弯矩全局最大值时,桥梁模态截断阶数影响较大,若取低阶模态计算,计算结果偏小,动弯矩全局最大值发生的位置稳定在 33.1m 处,为提高精度,模态截断阶数应大于 6 阶;计算桥梁动剪力全局最大值时,桥梁模态截断阶数影响较大,若取低阶模态计算,计算结果偏小,为提高精度,模态截断阶数应大于 7 阶,模态截断阶数对动剪力最大值发生的位置没有影响,始终在车辆下桥的梁端.

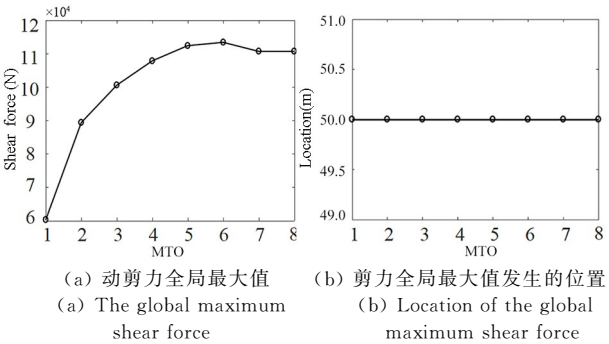


图 18 不同桥梁模态截断阶数下桥梁动剪力全局最大值及位置
Fig.18 Global maximum value and location of bridge shear force under different bridge modal truncation orders(MTO)

4 结论

本文建立了考虑桥面不平顺的车-简支梁桥耦合系统模型,计算了车速、车距、桥梁模态截断阶数对桥梁动挠度、动弯矩、动剪力全局最大值及最大值发生的位置的影响.结果表明:

- (1) 车速对简支梁动挠度、动弯矩、动剪力全局最大值以及最大值发生的位置影响较大,三个最大值对应的特定车速不同.
- (2) 车距对简支梁动挠度、动弯矩、动剪力全局最大值影响较大.随着车距增大,两车对动力响应最大值的叠加效果逐渐削弱,当达到某个车距时,叠加效果消失,与单车过桥的情况相同.
- (3) 简支梁模态截断阶数对计算桥梁动挠度影响较小,对桥梁动弯矩和动剪力影响较大.因此,计算简支梁桥动挠度、动弯矩和动剪力时,其模态截断阶数宜分别大于 3 阶、6 阶和 7 阶.

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