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隔振对象重量变化对准零刚度隔振器隔振性能的影响 *

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摘要 本文主要研究隔振对象重量变化对一类准零刚度隔振器隔振性能的影响，并给出了新的研究结果．文中使用欧拉屈曲梁构建负刚度调节结构并设计了隔振系统的平衡位置可调机构．假设系统有轻微的过载和超载，推导了系统的动力学方程并进行求解，定义了非线性隔振系统的力传递率及位移传递率来评价系统的隔振性能．对线性隔振系统，超载会让隔振频率略微降低，共振放大峰略微增大．对于准零刚度隔振系统，力传递率和线性系统类似，但是对于位移传递率，过载会导致系统固有频率和共振放大峰均升高，反而不利于隔振．研究结果可以对此类隔振系统的使用场合以及对过载和轻载的选择有工程指导意义．

关键词 负刚度， 准零刚度， 非线性隔振器， 力传递率， 位移传递率

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引言

刚度和阻尼对被动隔振器来说是最基本的两个参数，对于由刚度和阻尼组成的单自由度隔振系统，刚度决定着固有频率，阻尼则控制着共振放大倍数．在工程应用时，刚度和阻尼都要选择的合适，刚度太大固有频率过大导致隔振效果降低，而刚度过小则承载能力缺失；阻尼太大会造成高频振动抑制效果变差，而阻尼太小则会引起较大的共振放大倍数．

理想的被动隔振器应在共振区提供大阻尼，高频区提供小阻尼；在静承载时刚度大，在振动时又能保持在小刚度．近十年来，很多学者在着手解决被动隔振中的矛盾，这其中，“准零刚度”隔振系统尤为热门．该隔振系统采取正负刚度并联的方式，使系统在平衡位置最低可以获得零刚度的特性，之所以称之为“准零刚度”，是因为其零刚度区间在理论上只存在一点，偏离平衡位置后刚度便会大于零．Liu[1]对不同的负刚度机构及此类隔振系统进行了详细的综述，陆泽琦[2]则对非线性被动隔振的最新进展进行了综述．Carrella[3,4]和 Kovacic[5-7]对三根弹簧组成的准零刚度系统进行了大量研究，其中也包括双层隔振系统．Platus[8]使用压杆的失稳来提供负刚度，对一类准零刚度隔振器进行了研究．刘兴天[9]使用欧拉屈曲梁构造负刚度调节器，探讨了其动力学特性；黄修长[10]则针对这个系统的刚度不完美项进行分析，讨论了缺陷对隔振效果的影响．Arrieta 等[11]、Shaw 等[12]以及国内的陆泽琦等[13]通过使用双稳态复合板构造负刚度机构也讨论了准零刚度隔振系统的复杂动力学特性，并进行了试验研究．彭志科等[14]则对一类含有非线性刚度和非线性阻尼的的隔振系统进行了传递率分析，在对非线性频响函数进行求解时，提出了基于 Volterra 级数的新方法．Sun 等人[15]提出了剪刀型的桁架结构来达到准零刚度．Zhou 和 Liu[16]、Wu 等[17]、Xu 等[18]以及 Robertson 等[19]通过采用永磁体或电磁体来获取负刚度使系统的动态刚度在平衡点处降至零．Wang 等[20]、Zhou 等[21]以及 Wang 等[22]针对凸轮-滚珠-弹簧机构构造负刚度机构，并研究了双层隔振及六自由度隔振系统在隔振性能上带来的优势．

然而，大多数文献的焦点都集中在如何构建负刚度机构以获取超低频隔振性能，而忽略了系统参数的影响．如上文提到的，准零刚度隔振系统的零刚度点理论上只存在于一点，考虑到系统中存在的摩擦等因素，要使得隔振对象平衡在零刚度点异常

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困难，而且隔振对象的重量往往无法调节.在进行工程应用时，必须考虑此类问题.Shaw[23]通过采用不对称的刚度结构来缓解重量带来的不利影响，而Abolfathi[24]则评价了系统参数的变化对力激励下准零刚度隔振系统的影响，同样，Wang[25]也讨论了参数不完美对系统隔振性能的影响.这些参数变化对于精密隔振尤为重要.

本文的落脚点定在隔振对象重量对准零刚度隔振系统的影响上，考虑隔振对象的重量相对于理想重量变化±10%对系统隔振性能的影响.通过采用力传递率和位移传递率的指标，来评估隔振性能的变化，发现了完全不同于线性系统的现象.为此类隔振系统的工程应用提供了有益的指导.

1 系统的动力学建模

在工程应用中，隔振器的许用负载是一个非常重要的设计参数，隔振器承载的重量必须在许用负载范围内.否则可能会导致隔振器受损或隔振性能恶化.定义隔振器性能最优时的载荷为额定载荷；当实际载荷大于或小于额定载荷时，隔振系统为过载和轻载.以超载情况为例，线性隔振器模型如图1所示，刚度为常数，隔振器因超载静变形变大.

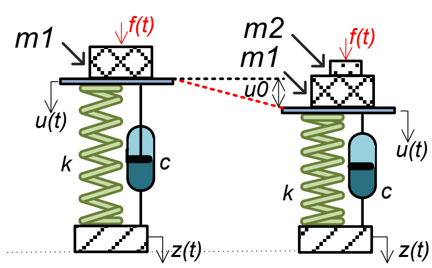

图1 线性隔振系统的过载示意图

Fig.1 Model of a linear isolator and that with overload

本文采用欧拉屈曲梁的方式获取负刚度进而构建准零刚度系统，隔振系统模型如图2所示.当欧拉屈曲梁的两个端点在一条水平线时，对应准零刚度系统，此时如果减小或增加隔振对象重量，则平衡点会上移或者下移.通常，无论是轻载或过载，我们仍希望隔振对象平衡在零刚度点.因此，本文引入了一个垂向弹簧调节装置，轻载时，弹簧支撑块向下移动，过载时弹簧支撑块向上移动，以此来保证隔振对象平衡在零刚度点.

考虑轻载和过载情况，系统的动力学方程由公

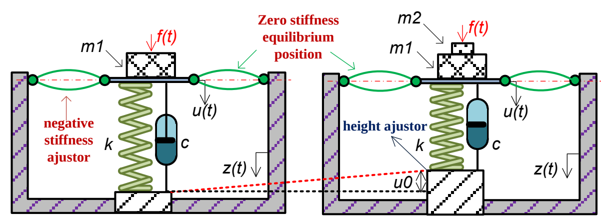

图2 准零刚度隔振系统的过载示意图

Fig.2 Model of a quasi-zero stiffness isolator and that with overload

式(1)和公式(2)表示，分别对应于力激励和基础激励工况.

$$(m_1+m_2\ddot{u})(t)+c\dot{u}(t)+kL(k_3\tilde{u}^3)=f(t) \tag{1}$$

$$(m_1+m_2)\ddot{\delta}(t)+c\dot{\delta}(t)+kL(k_3\tilde{\delta}^3)=-\left(m_1+m_2\right)\ddot{z}(t) \tag{2}$$

其中 k_3 为立方刚度系数，其表达形式为：

$$k_3=\left(\left(\frac{1}{2\gamma^2}+\frac{2}{\gamma\left(\left(\pi\tilde{q}_0\right)^2-4\gamma+12\right)}\right)+\frac{\pi\tilde{q}_0}{\gamma\left(\left(\left(\pi\tilde{q}_0\right)^2-4\gamma+4\right)^{\frac{3}{2}}-\pi\tilde{q}_0\left(\left(\pi\tilde{q}_0\right)^2-4\gamma+4\right)\right)}\right) \tag{3}$$

公式(3)中的定义见参考文献[9].令 $\tilde{u}=\dfrac{u}{L}$，$\tilde{\delta}=\dfrac{\delta}{L}$，$\tilde{F}_e=\dfrac{F_e}{kL}$，$\tilde{Z}_0=\dfrac{Z_0}{L}$，$\omega_n=\sqrt{k/m_1}$，$\zeta=\dfrac{c}{2m_1\omega_n}$，$\omega_n t=\tau$，$\Omega=\omega/\omega_n$，$\mu=m_2/m_1$，可以将公式(1)和公式(2)写成如下统一格式：

$$(1+\mu)\Delta''+2\zeta\Delta'+k_3\Delta^3=\rho a_0\cos(\Omega\tau+\theta) \tag{4}$$

使用谐波平衡法求解方程(4)，其稳态主响应的解可设为：

$$\Delta=A\cos\left(\Omega\tau+\psi\right) \tag{5}$$

将公式(5)代入到公式(4)中，经过一系列推导，可得：

$$\frac{9}{16}k_3^2A_1^6-\frac{3}{2}k_3\left(1+\mu\right)\Omega^2A_1^4+\left(\left(1+\mu\right)^2\Omega^4+4\zeta^2\Omega^2\right)A_1^2-\rho^2a_0^2=0 \tag{6}$$

不同激励条件下的准零刚度隔振系统的响应曲线由公式(7)决定.分力激励和基础激励的条件可以分别求解其响应，分别为：

$$\Omega_{1,2}^f=\frac{\sqrt{\left(\frac{3}{4}\left(1+\mu\right)k_3A^2-2\zeta^2\right)\pm\frac{1}{A}\sqrt{\left(4\zeta^4A^2-3k_3\left(1+\mu\right)\zeta^2A^4\right)+\left(1+\mu\right)^2F_e^2}}}{\left(1+\mu\right)} \tag{7}$$

$$\Omega_{1,2}^{d}=\frac{1}{(1+\mu)}\sqrt{\frac{\left(\frac{3}{4}k_3(1+\mu)A^4-2\zeta^2A^2\right)\pm A^2\sqrt{\frac{9}{16}(1+\mu)^2Z_0^2k_3^2A^2+4\zeta^4-3(1+\mu)k_3\zeta^2A^2}}{(A^2-Z_0^2)}} \tag{8}$$

力激励下的响应由公式(8)表示,当两条曲线相等时,这时候对应着响应的最大值:

$$A_{0peak}^{f}=\sqrt{\frac{2\zeta^3+\sqrt{4\zeta^6+3(1+\mu)^3F_e^2k_3}}{3k_3(1+\mu)\zeta}} \tag{9}$$

取得最大响应值时对应的频率为

$$\Omega_{peak}^{f}=\frac{\sqrt{\frac{\sqrt{4\zeta^6+3(1+\mu)^3F_e^2k_3}-6\zeta^3}{4\zeta}}}{(1+\mu)} \tag{10}$$

同样可以求得基础激励下系统的最大响应以及对应的频率

$$A_{peak}^{d}=\frac{2\zeta^2}{\sqrt{\left(3(1+\mu)k_3\zeta^2-\frac{9}{16}(1+\mu)^2Z_0^2k_3^2\right)}} \tag{11}$$

$$\Omega_{peak}^{d}=\frac{1}{(1+\mu)}\sqrt{\frac{\left(\frac{3}{4}k_3(1+\mu)A^4-2\zeta^2A^2\right)}{(A^2-Z_0^2)}} \tag{12}$$

为比较方便,定义等效线性系统为去除负刚度调节机构之后的隔振系统,对等效线性隔振系统的要求是在相同载重下,其静变形和准零刚度隔振系统保持一致.对应的响应为:

$$\Delta_0=\frac{\rho a_0}{\sqrt{\left(1-(1+\mu)\Omega^2\right)^2+4\zeta^2\Omega^2}} \tag{13}$$

2　重量变化对传递率的影响

对隔振器性能进行评价的最重要指标之一就是传递率,线性系统传递率再次毋庸赘述,对于准零刚度隔振系统,各传递率定义为:

准零刚度隔振系统的力传递率可以写成:

$$T_{nf}=\sqrt{\frac{(k_3A^3)^2+4\zeta^2\Omega^2A^2}{F_e}} \tag{14}$$

其中 A 为准零刚度隔振系统的稳态响应幅值.

准零刚度隔振系统的相对位移传递率为:

$$T_{nrd}=A/Z_0 \tag{15}$$

准零刚度隔振系统的绝对位移传递率为:

$$T_{nad}=U_0/Z_0 \tag{16}$$

隔振系统重量对准零刚度隔振系统的影响见图3到图5所示.为对比方便,图中分别画出了对应等效线性系统的传递率曲线.考虑工程实际应用,讨论了过载和轻载的情况.力激励幅值定为 $F_e=5\times10^{-4}$,位移激励幅值定为 $U_0=2\times10^{-3}$,系统阻尼比定为 $\zeta=0.01$.考虑到隔振器的选用都有一定的承载范围,因此本文仅考虑±10%以内的重量偏离.对于线性系统,重量增加后,其固有频率降低但是共振放大峰加大,对于准零刚度隔振系统,从图3中可以看出,其趋势和线性系统类似,峰值点频率降低,最大响应增大.通过图中也可以观察到,实际上±10%的重量变化对力传递率的影响是有限的.

对于位移传递率,如图4和图5所示,情况和力传递率相比发生很大变化,线性系统的表现不变,但是对准零刚度隔振系统,过载会导致共振峰增大而且共振频率也增大,这不利于对基础位移振动的隔离.因此,在采用准零刚度隔振器进行振动隔离时,为获得更好的隔振性能,可以规定额定载荷的区间在轻载和额定负载之间,尽量避免过载,可以实现更好的隔振性能.此外,轻载也可以让非线性系统中的跳变现象消失,这对隔振也是有益的.

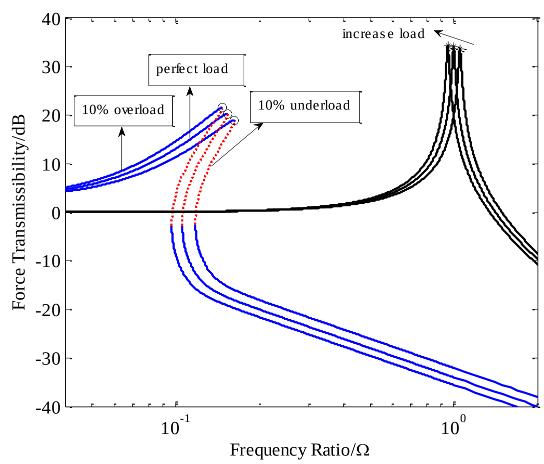

图3　重量变化对隔振系统力传递率影响
('o' 准零刚度共振峰,'*'线性刚度共振峰)

Fig.3　Effects of load imperfection on the force transmissibility of the QZS and the equivalent linear isolator

3　结论

本文主要对隔振对象重量的变化对准零刚度隔振器隔振性能的影响进行了研究.假设系统有轻微的过载和超载,推导了系统的动力学方程并进行

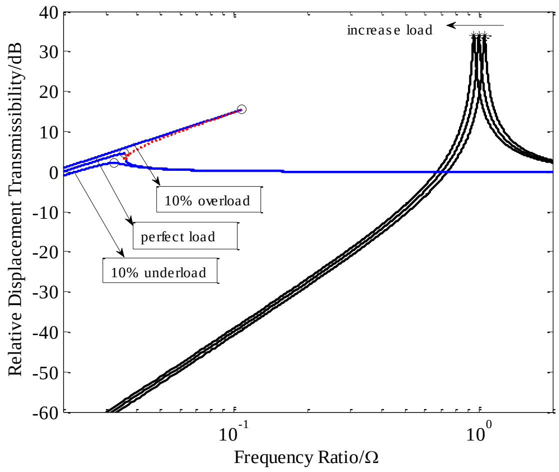

图 4 重量变化对隔振系统相对位移传递率影响
(‘o’ 准零刚度共振峰,‘*’线性刚度共振峰)

Fig.4 Effects of load imperfection on the relative displacement transmissibility of the QZS and the equivalent linear isolator

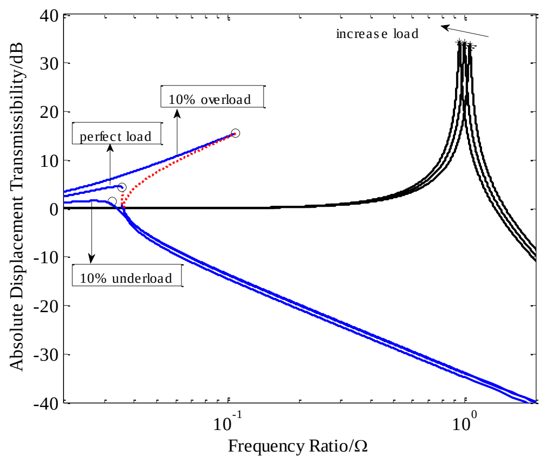

图 5 重量变化对隔振系统绝对位移传递率影响
(‘o’ 准零刚度共振峰,‘*’线性刚度共振峰)

Fig.5 Effects of load imperfection on the absolute displacement transmissibility of the QZS and the equivalent linear isolator

了求解,定义了力传递率以及位移传递率来评价隔振系统的隔振性能.研究结果表明:

1)对于力传递率,隔振对象重量变化对准零刚度隔振系统的影响和线性系统类似,隔振对象重量增大导致共振峰变大但固有频率降低;

2)对于位移传递率,隔振对象重量增大反而会导致准零刚度隔振系统的共振峰和固有频率均增大,这和线性系统显著不同;

3)准零刚度隔振系统用于隔离基础振动,轻载还可以让跳变线性消失,这也对隔振有利.

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EFFECT OF LOAD IMPERFECTION ON VIBRATION ISOLAITON PERFORMANCE OF A QUASI-ZERO-STIFFNESS ISOLATOR *

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Abstract A quasi-zero-stiffness (QZS) isolator was configured by combining an Euler buckled beam negative stiffness corrector and a linear isolator. Making the load balanced at the zero-stiffness point, the QZS isolator can offer ultra-low frequency vibration isolation performance. However, overload or underload usually happens in practices. This paper inrestigated the effects of load imperfection on the performance of the QZS isolator by using force and displacement transmissibility, which was also compared with the linear isolator. Assuming slight overload or underload and adjusting the actual load to the zero-stiffness position for the QZS isolator, the dynamic response of the vibration isolation system was obtained by using the harmonic balance method. For the linear system, the resonant frequency decreases and the peak transmissibility increases under overload. In the case of force transmissibility, the QZS system is similar to the linear system. However, the displacement transmissibility is opposite. Increasing load makes both the peak and resonance frequency of the absolute and relative displacement transmissibility shift to higher values. The results presented here can be a useful guideline when design such a kind of vibration isolator.

Key words negative stiffness, quasi-zero stiffness, nonlinear isolator, force transmissibility, displacement transmissibility

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