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1∶2内共振条件下蜂窝夹芯板的两倍周期运动研究*

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摘要 主要利用推广的四维次谐 Melnikov 方法研究一类面内载荷与横向载荷联合作用下四边简支矩形蜂窝夹芯板的周期运动.首先,通过引入周期变换和相应的 Poincaré 映射,获得一个四维次谐 Melnikov 向量函数,通过对该向量函数简单零点的研究,得到一类四维非线性非自治系统周期运动的存在性判定定理.然后,利用推广的四维次谐 Melnikov 方法研究了 1∶2内共振情况下蜂窝夹芯板的周期运动,获得了系统存在两倍周期运动的参数域.最后,对系统进行数值模拟,验证了理论分析的正确性.

关键词 次谐 Melnikov 方法, 周期运动, 蜂窝夹芯板

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引言

蜂窝夹芯板被广泛应用于航空航天等领域,由于航空和航天飞行器严格的运行要求和严酷的运行环境,对于蜂窝夹芯板的非线性动力学研究显得尤为重要.Burlayenko 和 Sadowski[1] 研究了蒙皮与芯层的剥离对泡沫和蜂窝夹芯板自由振动特性的影响.黄丽娟等[2] 利用双协调自由界面模态综合法研究了周期局域共振蜂窝夹层板弯曲振动的频响特性.Liu 等[3] 提出了一种分析正方形蜂窝夹芯板弯曲、屈曲和振动特性的半解析方法.Zhang 等[4] 利用广义 Melnikov 方法研究了四边简支矩形蜂窝夹芯板的全局分叉和多脉冲混沌动力学.Sekine 等[5] 研究了蜂窝夹芯板的振动特性,文中将蜂窝芯层考虑为具有剪切变形的厚板,复合材料蒙皮考虑为薄板.Yu 和 Cleghorn[6] 研究了简支矩形蜂窝夹芯板的自由振动,分别运用了经典薄板理论、Reissner-Mindlin 理论和 Reddy 三阶剪切变形理论建立了蜂窝夹芯板的力学模型.Li 和 Zhu[7] 运用改进的 Reddy 三阶剪切变形理论研究了蜂窝夹芯板的自由振动,对 Reddy 三阶剪切变形理论引入了剪切修正因子.Alijani 等[8] 实验研究了蜂窝和泡沫夹芯板的非线性振动特性,在定幅激励情况下,通过缓慢的上下扫频过程获得了夹芯板的非线性频率响应曲线.

Melnikov 方法在研究非线性系统的动力学特性中起到了很重要的作用,郭翔鹰等[9] 利用高维 Melnikov 方法研究了复合材料层合板的混沌运动. Chicone[10] 结合 Lyapunov-Schmidt 退化方法和次谐 Melnikov 理论研究了高维非线性系统的周期解. Boinn[11] 将谐波平衡法与次谐 Melnikov 理论相结合研究了耦合 Van der Pol 振子次谐轨道的分叉. Yagasaki[12] 利用次谐 Melnikov 理论研究了一类退化共振情况下非线性系统的周期解.Sun 等人[13] 利用次谐 Melnikov 理论,研究了压电复合材料层合板的周期解.

本文首先利用周期变换和 Poincaré 映射推广四维非线性非自治次谐 Melnikov 理论,然后利用该理论研究蜂窝夹芯板的周期运动,理论上给出系统存在 2 倍周期运动的参数域,最后利用数值模拟验证理论的正确性.

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1　四维次谐 Melnikov 方法

本节中,研究如下一类四维非线性非自治系统的周期运动:

$$\dot{x}=\boldsymbol{JDH}_1(x)+\varepsilon g(x,y,\omega t;u)$$
$$\dot{y}=\boldsymbol{JDH}_2(y)+\varepsilon f(x,y,\omega t;u) \tag{1}$$

其中:

$$x=(x_1,x_2)\in R^2,y=(y_1,y_2)\in R^2,$$
$$\boldsymbol{DH}_1(x)=\left(\frac{\partial H_1(x_1,x_2)}{\partial x_1},\frac{\partial H_1(x_1,x_2)}{\partial x_2}\right)^T,$$
$$\boldsymbol{DH}_2(y)=\left(\frac{\partial H_2(y_1,y_2)}{\partial y_1},\frac{\partial H_2(y_1,y_2)}{\partial y_2}\right)^T,$$

$0<\varepsilon\ll 1,u\in R$ 为参数, $\boldsymbol{J}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$. 并且,$H_j$: $R^2\to R$ 足够光滑,$g(x,y,\varphi;u)$ 和 $f(x,y,\varphi;u)$ 是关于 φ 周期为 2π 的函数.

对未扰动系统作如下假设:

A1. 每个方程都有一族周期轨道,其表达式分别为: $L_1=\{x^{h_1}\mid H_1(x_1,x_2)=h_1\}$ 和 $L_2=\{y^{h_2}\mid H_2(y_1,y_2)=h_2\}$,其中 $h_j\in K$,K 为开区间, $j=1,2$.

A2. $x^{h_1}(t)$ 和 $y^{h_2}(t)$ 关于 h 是 C^r 的,其周期记为 T_1 和 T_2.

A3. 存在互素的正整数 m_j 和 n_j,$j=1,2$,满足 $\frac{T_1(h_1)}{T}=\frac{m_1}{n_1},\frac{T_2(h_2)}{T}=\frac{m_2}{n_2}$,其中 $T=\frac{2\pi}{\omega}$.

假设 m_0 为 m_1 和 m_2 的最小公倍数,我们研究由未扰动系统的一族闭轨经过小参数扰动后在其附近产生周期为 m_0T 的周期振动问题.

对系统(1)引入如下周期变换:

$$x=G(\theta_1,h_1)=q\left(\frac{T_1(h_1)}{2\pi}\theta_1,h_1\right)$$
$$y=P(\theta_2,h_2)=p\left(\frac{T_2(h_2)}{2\pi}\theta_2,h_2\right) \tag{2}$$

将方程(2)带入系统(1)可得到如下极坐标形式的四维非线性非自治系统:

$$\dot{h}_1=\varepsilon F_1(h_1,h_2,\theta_1,\theta_2,\omega t)$$
$$\dot{\theta}_1=\Omega_1(h_1)+\varepsilon Q_1(h_1,h_2,\theta_1,\theta_2,\omega t)$$
$$\dot{h}_2=\varepsilon F_2(h_1,h_2,\theta_1,\theta_2,\omega t)$$
$$\dot{\theta}_2=\Omega_2(h_2)+\varepsilon Q_2(h_1,h_2,\theta_1,\theta_2,\omega t) \tag{3}$$

其中:

$$Q_1=\Omega_1\frac{g\wedge G_{h_1}}{\boldsymbol{JDH}_1(G)\wedge G_{h_1}},\ \Omega_1=\frac{2\pi}{T_1},$$
$$Q_2=\Omega_2\frac{f\wedge P_{h_2}}{\boldsymbol{JDH}_2(P)\wedge P_{h_2}},\ \Omega_2=\frac{2\pi}{T_2},$$
$$F_1=\boldsymbol{DH}_1(G)g,\ F_2=\boldsymbol{DH}_2(P)f \tag{4}$$

令 $h=(h_1,h_2)$,$\theta=(\theta_1,\theta_2)$,在相空间 $R^2\times T^2\times S^1$ 中定义如下形式的横截面:

$$\sum=\{(h,\theta,\varphi)\in R^2\times T^2\times S^1\mid\varphi=0\} \tag{5}$$

其中 $T^2=S^1\times S^1$ 为二维环面.

对系统(3)定义如下 Poincaré 映射:

$$P_\varepsilon:(h(0),\theta(0))\to(h(T),\theta(T)) \tag{6}$$

其中:

$(h(t),\theta(t),\omega t)=(h_1(t),h_2(t),\theta_1(t),\theta_2(t),\omega t)$ 为系统(3)的解.

因此 m_0 次复合映射 $P_\varepsilon{}^{m_0}$为:

$$P_\varepsilon{}^{m_0}:(h(0),\theta(0))\to(h(m_0T),\theta(m_0T)) \tag{7}$$

其中 m_0 为 m_1 和 m_2 的最小公倍数.系统(3)的周期解的存在性等价于 $P_\varepsilon{}^{m_0}$的不动点的存在性.经计算可得:

$$P_\varepsilon\, m_0:(h(0),\theta(0))\to(h(m_0T),\theta(m_0T))$$
$$\begin{pmatrix}h_{01} & \theta_{01}\\ h_{02} & \theta_{02}\end{pmatrix}\to\begin{pmatrix}h_{01}+\varepsilon M_1 & \theta_{01}+\Omega_1(h_{01})m_0T+\varepsilon M_2\\ h_{02}+\varepsilon M_3 & \theta_{02}+\Omega_2(h_{02})m_0T+\varepsilon M_4\end{pmatrix}+O(\varepsilon^2) \tag{8}$$

其中:

$$M_1=\int_0^{m_0T}F_1(h_{01},\theta_{01}+\Omega_1t,h_{02},\theta_{02}+\Omega_2t,\omega t)dt$$
$$M_2=\int_0^{m_0T}Q_1(h_{01},\theta_{01}+\Omega_1t,h_{02},\theta_{02}+\Omega_2t,\omega t)dt+\frac{d\Omega_1}{dh_1}\int_0^{m_0T}\int_0^t F_1dsdt$$
$$M_3=\int_0^{m_0T}F_2(h_{01},\theta_{01}+\Omega_1t,h_{02},\theta_{02}+\Omega_2t,\omega t)dt$$
$$M_4=\int_0^{m_0T}Q_2(h_{01},\theta_{01}+\Omega_1t,h_{02},\theta_{02}+\Omega_2t,\omega t)dt+\frac{d\Omega_2}{dh_2}\int m_0T_0\int_0^t F_2dsdt \tag{9}$$

我们可得到如下四维非线性系统周期存在性判定定理[14]:

定理　假设未扰动系统的周期轨道的初值 $(h_{01}^*,\theta_{01}^*,h_{02}^*,\theta_{02}^*)$满足如下条件:

$$\frac{T_1(h_{01}^*)}{T}=\frac{m_1}{n_1},\frac{T_2(h_{02}^*)}{T}=\frac{m_2}{n_2} \tag{10}$$

并且下列两组条件之一成立,即:

(1)对于非退化共振情况 $\frac{d\Omega_i}{dh_{0i}}(h_{0i}^*)\neq 0(i=1,2)$,

$$M_i(h_{01}^*,\theta_{01}^*,h_{02}^*,\theta_{02}^*)=0(i=1,3)$$

$$\left[\frac{\partial(M_1,M_3)}{\partial(\theta_{01},\theta_{02})}\right]_{(h_{01}^*,\theta_{01}^*,h_{02}^*,\theta_{02}^*)}\neq 0 \tag{11}$$

(2)对于退化共振情况 $\frac{d\Omega_i}{dh_{0i}}(h_{0i}^*)=0,(i=1,2)$,

$$M_i(h_{01}^*,\theta_{01}^*,h_{02}^*,\theta_{02}^*)=0(i=1,\cdots,4)$$

$$\left[\frac{\partial(M_1,M_2,M_3,M_4)}{\partial(h_{01},\theta_{01},h_{02},\theta_{02})}\right]_{(h_{01}^*,\theta_{01}^*,h_{02}^*,\theta_{02}^*)}\neq 0 \tag{12}$$

则对于 $0<\varepsilon\ll 1$,映射 $P_\varepsilon\ m_0$ 在 $(h_{01}^*,\theta_{01}^*,h_{02}^*,\theta_{02}^*)$ 点附近存在一个不动点.因此系统(1)在 $\left(x\left(t+\frac{\theta_{01}^*}{\Omega_1(h_{01}^*)}\right),y\left(t+\frac{\theta_{02}^*}{\Omega_2(h_{02}^*)}\right)\right)$ 附近存在一个周期轨道,周期为 m_0T.

2 蜂窝夹芯板的周期运动

本节采用上述理论研究蜂窝夹芯板的两倍周期运动.蜂窝夹芯板受到 x 方向的面内均布载荷与横向均布载荷联合作用.夹芯板的长、宽、高分别为 a、b 和 H,直角坐标 Oxy 位于夹芯板的中性面内,z 轴向下,设板内任一点沿 x、y 和 z 方向的位移分别为 u、v 和 w,沿 x 方向作用的面内载荷为 $p=p_0-p_1\cos\Omega_2 t$,横向载荷为 $f=F(x,y)\cos\Omega_1 t$.蜂窝夹芯板分为三层,上下蒙皮是相同的各向同性材料,蒙皮层厚度为 h_f,芯层为正六角形蜂窝构型,蜂窝芯轴向为坐标 z 方向,蜂窝芯厚度为 h_c.

利用 Hamilton 原理和 Galerkin 方法,我们得到如下两自由度非线性非自治动力学方程[15]:

$$\ddot{w}_1+c_1\dot{w}_1-(\alpha_1+\alpha_7p_0)w_1+\alpha_7p_1\cos\Omega_2tw_1-\alpha_2w_1w_2{}^2-\alpha_3w_1{}^2w_2-\alpha_4w_1{}^3-\alpha_5w_2{}^3=\alpha_6F_1\cos\Omega_1t \tag{13a}$$

$$\ddot{w}_2+c_2\dot{w}_2-(\beta_1+\beta_7p_0)w_2+\beta_7p_1\cos\Omega_2tw_2-\beta_2w_1{}^2w_2-\beta_3w_1w_2{}^2-\beta_4w_2{}^3-\beta_5w_1{}^3=\beta_6F_2\cos\Omega_1t \tag{13b}$$

对方程(13)引入如下变换:

$$x_1=w_1,x_2=\dot{w}_1,x_3=w_2,x_4=\dot{w}_2 \tag{14}$$

方程(13)可转化为如下形式:

$$\begin{aligned}
&\dot{x}_1=x_2\\
&\dot{x}_2=-\omega_1^2x_1+\varepsilon(\alpha_2x_1x_3^2+\alpha_3x_1^2x_3+\alpha_4x_1^3+\alpha_5x_3^3-c_1x_2-f_1x_1\cos\Omega_2t+F_1\cos\Omega_1t)\\
&\dot{x}_3=x_4\\
&\dot{x}_4=-\omega_2^2x_3+\varepsilon(\beta_2x_1^2x_3+\beta_3x_1x_3^2+\beta_4x_3^3+\beta_5x_1^3-c_2x_4-f_2x_3\cos\Omega_2t+F_2\cos\Omega_1t)
\end{aligned} \tag{15}$$

其中:

$$-\omega_1^2\rightarrow\alpha_1+\alpha_7p_0,\varepsilon f_1\rightarrow\alpha_7P_1,\varepsilon F_1\rightarrow\alpha_6F_1,-\omega_2^2\rightarrow\beta_1+\beta_7p_0,\varepsilon f_2\rightarrow\beta_7P_1,\varepsilon F_2\rightarrow\beta_6F_2 \tag{16}$$

考虑如下共振关系:

$$\Omega_1=\Omega_2=2\omega_1,\omega_1=\omega,\omega_2=2\omega \tag{17}$$

因此,我们可得:

$$\frac{m_1}{n_1}=2,\frac{m_2}{n_2}=1,m_0=2,T=\frac{2\pi}{\Omega_1}=\frac{\pi}{\omega} \tag{18}$$

利用改进的四维次谐 Melnikov 方法,可得蜂窝夹芯板在 1:2内共振情况下的次谐 Melnikov 函数为:

$$M_1=-\frac{\pi f_1h_{01}}{\omega}\sin2\theta_{01}-2\pi c_1h_{01} \tag{19}$$

$$M_2=-\frac{\pi f_1}{2\omega^2}\cos2\theta_{01}-\frac{3\pi\alpha_4h_{01}}{2\omega^3} \tag{20}$$

$$M_3=2\pi F_2\cos2\theta_{02}\sqrt{\frac{h_{02}}{\omega}}-4\pi c_2h_{02} \tag{21}$$

$$M_4=-\frac{\pi\beta_2h_{01}}{2\omega^3\sqrt{\omega}}\cos2\theta_{02}-\frac{3\pi\beta_4h_{02}}{8\omega^3\sqrt{\omega}}\cos2\theta_{02}-\frac{\pi c_2}{\omega\sqrt{\omega}}\sin2\theta_{02} \tag{22}$$

若 $\sin2\theta_{01}^*=\frac{-2c_1\omega}{f_1},\cos2\theta_{02}^*=\frac{2c_2\omega}{F_2}\sqrt{\frac{h_{02}^*}{\omega}}$ 成立,则我们可得:

$$M_1(h_0^*,\theta_0^*)=M_3(h_0^*,\theta_0^*)=0 \tag{23}$$

另一方面,我们有:

$$\det N(h_0,\theta_0)\big|_{(h_0^*,\theta_0^*)}=\begin{vmatrix}0&a_{12}&0&0\\a_{21}&a_{22}&0&0\\0&0&a_{33}&a_{34}\\a_{41}&0&a_{43}&a_{44}\end{vmatrix} \tag{24}$$

其中 $a_{12}=\frac{-2\pi f_1h_{01}^*}{\omega}\cos2\theta_{01}^*$

$$a_{21}=-\frac{3\pi\alpha_4}{2\omega^3},\ a_{22}=\frac{\pi f_1\sin2\theta_{01}^*}{\omega^2}$$

$$a_{33}=-4\pi c_2+\frac{\pi h_{01}^*h_{02}^*F_2\cos2\theta_{02}^*}{\sqrt{\omega h_{02}^*}}$$

$$a_{34}=-4\pi F_2\sin2\theta_{02}^*\sqrt{\frac{h_{02}^*}{\omega}}$$

$$a_{41}=-\frac{\pi\beta_2}{2\omega^3\sqrt{\omega}}\cos2\theta_{02}^*$$

$$a_{43}=-\frac{3\pi\beta_4}{8\omega^3\sqrt{\omega}}\cos2\theta_{02}^*$$

$$a_{44}=\frac{2\pi\sin2\theta_{02}^*(4\pi\beta_2h_{01}^*+3\pi\beta_4h_{02}^*)}{8\omega^3\sqrt{\omega}} \tag{25}$$

经计算,当 $a_{21}\neq0a_{12}\neq0$,$a_{33}a_{44}\neq a_{34}a_{43}$ 成立时,$\det N(h_0,\theta_0)|_{(h_0^*,\theta_0^*)}\neq0$. 由定理得蜂窝板在 $\left(q\left(t+\frac{\theta_{01}^*}{\Omega_1},h_{01}^*\right),p\left(t+\frac{\theta_{02}^*}{\Omega_2},h_{02}^*\right)\right)$ 附近存在周期为 $2T$ 的周期轨道.

3　数值模拟

我们应用四阶 Runge-Kutta 法对方程(15)进行数值模拟来验证横向与面内载荷联合作用下蜂窝夹芯板存在2倍周期轨道,如图1和图2所示.

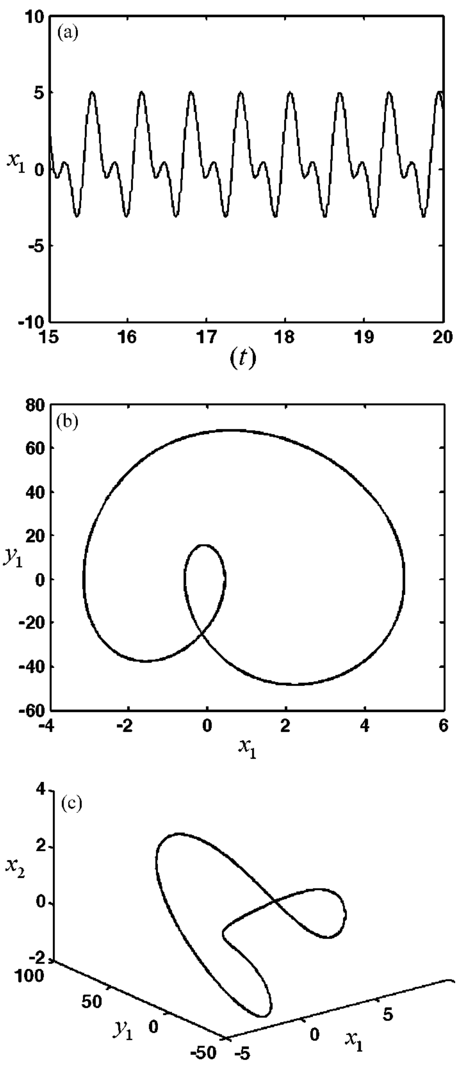

图 1　参数 P_1 下蜂窝夹芯板的两倍周期运动

Fig.1　Period-1 motion of the plate with P_1

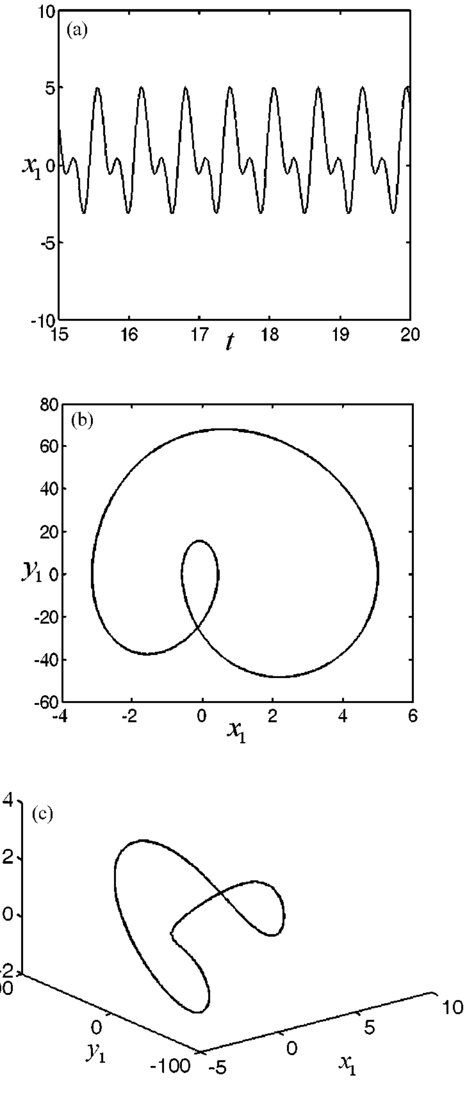

图 2　参数 P_2 下蜂窝夹芯板的两倍周期运动

Fig.2　Period-2 motion of the plate with P_2

图(a)在平面(t,x_1)上的波形图,图(b)表示在平面(x_1,y_1)上的二维相图,图(c)表示在空间(x_1,y_1,x_2)上的三维相图.数值模拟所选参数均满足理论得到的条件.

图 1 表明蜂窝夹芯板存在两倍周期运动,参数为:

$F_1=F_2=600$, $f_1=100$, $f_2=50$, $c_1=3$, $c_2=0.9$, $\omega_1=10$, $\omega_2=20$, $\Omega_1=\Omega_2=20$, $\alpha_2=-2.32$, $\alpha_3=1.7$, $\alpha_4=-2.3$, $\alpha_5=1.336$, $\beta_2=-20.26$, $\beta_3=15.7$, $\beta_4=-30.28$, $\beta_5=12.5$, $x_{10}=x_{20}=0$, $y_{10}=0.01$, $y_{20}=0.1$,此组参数记为 P_1.

图 2 表明蜂窝板也存在两倍周期运动,其参数为:

$F_1 = F_2 = 600, f_1 = 100, f_2 = 80, c_1 = 6,$
$c_2 = 2.1, \omega_1 = 10, \omega_2 = 20, \Omega_1 = \Omega_2 = 20,$
$\alpha_2 = -5, \alpha_3 = 2, \alpha_4 = -3, \alpha_5 = 2.5,$
$\beta_2 = -10.26, \beta_3 = 5.7, \beta_4 = -30, \beta_5 = 22.5,$
$x_{10} = x_{20} = 0, y_{10} = 0.01, y_{20} = 0.1$,此组参数记为 P_2.

5 结论

本文首先推广了四维次谐 Melnikov 方法,我们引入周期变换将系统化为极坐标形式的非线性非自治系统,然后对该系统建立相应的 Poincaré 映射,通过对映射不动点的研究得到一个四维次谐 Melnikov 向量函数,通过对该向量函数零点的研究,我们得到一类四维非线性非自治系统周期运动的存在性判定定理.然后,利用改进的方法研究了蜂窝夹芯板在 1:2内共振情况下的周期运动.我们利用定理计算得到次谐 Melnikov 向量函数,同时获得系统存在两倍周期运动的参数条件.最后,利用数值方法得到蜂窝夹芯板的二维、三维相图和波形图,验证了理论分析的正确性,研究结果丰富了蜂窝夹芯板的非线性动力学研究.

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PERIOD-2 MOTION OF A HONEYCOMB SANDWICH PLATE UNDER 1∶2 INTERNAL RESONANCE *

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Abstract The periodic motion of a simply-supported rectangular honeycomb sandwich plate under the combination of the parametric and transverse excitations is investigated by the improved four-dimensional subharmonic Melnikov method. Based on periodic transformations and Poincaré map, a four-dimensional subharmonic Melnikov function is obtained. Then, a main theorem, which can be used to determine the existence of the periodic orbits for the four-dimensional non-autonomous nonlinear dynamical system, is presented. The periodic motion of the honeycomb sandwich plate under 1∶2 internal resonance is then studied by the improved method. The certain parameter regions where the period-two motion may occur are obtained. Numerical simulations are also carried out to verify the analytical predictions.

Key words subharmonic Melnikov method, periodic motion, honeycomb sandwich plate

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