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基于自适应模糊滑模控制的分数阶混沌系统的投影同步*

陈旭　李明　郑永爱†

(扬州大学 信息工程学院, 扬州　225127)

摘要　基于模糊控制理论和滑模控制理论以及自适应控制理论,研究了一类含有外部扰动的不确定分数阶混沌系统的混合投影同步问题.提出了一种自适应模糊滑模控制的分数阶混沌系统投影同步方法.模糊逻辑系统用来逼近未知的非线性函数和外部扰动,并且对逼近误差采用了自适应控制,同时构造了一种具有较强鲁棒性的分数阶积分滑模面.应用分数阶 Barbalat 引理设计了自适应模糊滑模控制器和参数自适应律.最后数值仿真结果验证了所提控制方法的有效性.

关键词　分数阶混沌系统,　模糊控制,　投影同步,　滑模控制

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引言

尽管分数阶微积分的发展历时三百余年,但由于理论相对复杂并且缺乏实际的应用背景,致使分数阶微积分理论发展十分缓慢[1].随着对分数阶微积分相关理论研究的不断深入,研究者普遍认为与传统的整数阶微积分相比,分数阶微积分可以更好地描述一些实际系统,促进了分数阶微积分在不同领域的广泛应用,完善了人们对客观世界的认识.自 1990 年,Pecora 和 Carroll[2] 对两个不同初始条件的相同混沌系统实现同步以来,就掀起了混沌同步问题的研究热潮.目前已有多种混沌同步方式被提出,例如:完全同步[3],广义同步[4],相同步[5],反同步[6],滞后同步[7] 等多种同步方式.1999 年 Mainieri 和 Rehacek[8] 在研究部分线性系统时,观察到驱动系统和响应系统之间的对应变量能够按照一定的比例因子进行演化的现象,提出了投影同步.随后被运用到了混沌加密领域,并取得了良好的效果.同时实现混沌系统同步的多种控制方法也相继提出,如线性反馈控制法[9],主动控制法[10],自适应控制法[11],滑模变结构控制法[12],模糊控制法[13] 等.滑模控制因具有良好的动态性能和较强的抗干扰能力和鲁棒性,得到广泛的关注.模糊控制具有不需要建立被控对象的数学模型和容错的特点,而被大量运用.

近十几年来,混沌系统的投影同步已成为非线性领域研究的热点之一.文献[14]提出了在驱动-响应系统参数发生失配的情况下,利用脉冲控制技术,实现驱动-响应系统的修正函数投影拟同步.文献[15]在模型不确定系统中采用模糊控制器和参数自适应律,实现了非对称不确定混沌系统的广义投影同步.文献[16]通过利用一种新的分数阶滑模面,设计了主动滑模控制器,实现了异结构分数阶超混沌系统投影同步,但是并未考虑系统存在外部扰动.文献[17]根据整数 Lyapunov 稳定性理论,在响应系统不确定的情况下,采用自适应模糊滑模控制策略,实现混沌系统的广义投影同步,其中模糊系统用来逼近未知的系统以及外部扰动项,该方法并未对逼近误差进行处理.

本文基于分数阶 Barbalat 引理,针对含有外部扰动和模型不确定的分数阶混沌系统投影同步问题,设计了自适应模糊滑模控制器,通过模糊逻辑系统来逼近未知的非线性项以及外部扰动项,并且对逼近误差采用了自适应控制,同时设计了具有较强

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† 通讯作者 E-mail:zhengyongai@163.com

鲁棒性的分数阶滑模面以增强系统的抗干扰能力.通过理论分析和数值仿真验证了该方法的有效性.

1 预备知识

常用的分数阶微积分的定义有三种,分别是Grunwald-Letnikov定义,Riemann-Liouville(R-L)定义和Caputo定义.本文选用Caputo定义[1],因为Caputo定义的系统初值与整数阶系统的初值一样,具有较好的物理意义,并且在工程实践当中应用较为广泛.

Caputo分数阶积分定义:

$$
{}_0I_t^{\alpha}f(t)={}_0D_t^{-\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-\tau)^{\alpha-1}f(\tau)d\tau \tag{1}
$$

其中$\Gamma(\cdot)$为欧拉Gamma函数:

$$
\Gamma(z)=\int_0^{\infty}t^{z-1}e^{-t}dt \tag{2}
$$

且有$\Gamma(z+1)=z\Gamma(z)$.Caputo分数阶微分定义为:

$$
{}_0D_t^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t\frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}}d\tau \tag{3}
$$

其中$n-1\leqslant\alpha\leqslant n$,$f(t)$为当$t>0$时在$[0,t]$内有$n+1$阶连续有界可导函数.

引理1[18] 考虑n维分数阶自治系统:

$$
D^{\alpha}\boldsymbol{x}=A\boldsymbol{x} \tag{4}
$$

其中$\alpha\in(0,1)$,$\boldsymbol{x}\in R^n$,$\boldsymbol{A}\in R^{n\times n}$,$D^{\alpha}\boldsymbol{x}=[D^{\alpha}x_1,D^{\alpha}x_2,\cdots,D^{\alpha}x_n]^T$.若:

1)当且仅当对矩阵$\boldsymbol{A}$的任意特征值$|\arg(eig(\boldsymbol{A}))|>\frac{\alpha\pi}{2}$恒成立,系统是渐近稳定的;

2)当且仅当对矩阵$\boldsymbol{A}$的任意特征值$|\arg(eig(\boldsymbol{A}))|\geqslant\frac{\alpha\pi}{2}$恒成立,系统是稳定的.

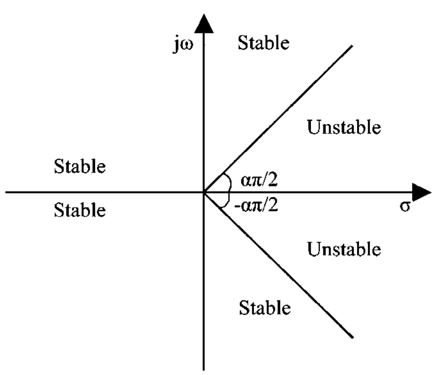

图1 分数阶系统稳定区域

Fig.1 Stability diagram of fractional-order system

分数阶系统的稳定区域如图1所示.由图1可知,如果分数阶系统的系数矩阵$\boldsymbol{A}$的所有特征值的实部都不大于零,则系统是渐近稳定的.

引理2[19] (分数阶Barbalat引理)若$V(t)$在$C^1(R^+)$上是一致连续有界函数,同时$\lim I^{\alpha}V(t)\to L$(L是有界常数),如果$V(t)$是一个正函数,则当$t\to\infty$时$V(t)\to0$.

引理3[20] 令$\boldsymbol{x}(t)\in R^n$为连续可微函数,则对于任意$t\geqslant0$有:

$$
2^{-1}D^{\alpha}\boldsymbol{x}^T(t)\boldsymbol{x}(t)\leqslant\boldsymbol{x}^T(t)D^{\alpha}\boldsymbol{x}(t) \tag{5}
$$

其中$\alpha\in(0,1)$.

引理4[21] (分数阶单调性原理)若$D_t^{\alpha}\boldsymbol{x}(t)\leqslant0$,则$\boldsymbol{x}(t)$在$[0,+\infty)$上单调减少;若$D_t^{\alpha}\boldsymbol{x}(t)\geqslant0$,则$\boldsymbol{x}(t)$在$[0,+\infty)$上单调增加.

引理5 若$\|\cdot\|_1$和$\|\cdot\|$分别为R^n上的1-范数和2-范数,则存在正常数c和c'使得$c\|\boldsymbol{x}\|\leqslant\|\boldsymbol{x}\|_1\leqslant c'\|\boldsymbol{x}\|$,$\forall x\in R^n$.

定理1 (万能逼近公式)设$f(x)$是紧集Ω上定义的连续函数,则对$\forall$常数$\varepsilon>0$,都存在合适的模糊逻辑系统逼近函数$\hat{f}(x)$,使得$\sup_{\Omega}|f(x)-\hat{f}(x)|\leqslant\varepsilon$.

2 同步控制器的设计及稳定性分析

2.1 问题描述

设分数阶驱动混沌系统和响应混沌系统分别为:

$$
D_t^{\alpha}\boldsymbol{x}(t)=f(\boldsymbol{x}(t)) \tag{6}
$$

$$
D_t^{\alpha}\boldsymbol{y}(t)=g(\boldsymbol{y}(t))+\boldsymbol{u}(t)+\boldsymbol{d}(t) \tag{7}
$$

其中$\boldsymbol{x}(t)=[x_1(t),\cdots,x_n(t)]^T\in R^n$,$\boldsymbol{y}(t)=[y_1(t),\cdots,y_n(t)]^T\in R^n$分别是驱动系统和响应系统的可测状态向量,$f,g:R^n\to R^n$为未知的非线性函数.$\boldsymbol{d}(t)=[d_1(t),d_2(t),\cdots,d_n(t)]^T\in R^{n\times1}$是未知的外部扰动,$\boldsymbol{u}(t)=(u_1,u_2,\cdots,u_n)^T\in R^{n\times1}$是待设计的响应系统控制器.

假设1 外部扰动$d(t)$是一个有界连续函数,即存在未知常数ρ,使得对所有t满足如下不等式:

$$
|d_i|<\rho<+\infty\ (\forall t>0) \tag{8}
$$

定义系统的投影同步误差:

$\boldsymbol{e}=\boldsymbol{y}-\boldsymbol{\lambda}\boldsymbol{x}$,$\boldsymbol{e}=(e_1,e_2,\cdots e_n)^T\in R^{n\times1}$,

$\boldsymbol{\lambda}=diag(\lambda_1,\lambda_2,\cdots,\lambda_n)^T\in R^{n\times n}$,$\|\cdot\|$表示向量2-范数,则在任意初始值$\boldsymbol{x}(0)$,$\boldsymbol{y}(0)$下,若存在:

$$\lim_{t\to\infty}\|\boldsymbol{e}(t)\|=\lim_{t\to\infty}\|\boldsymbol{y}(t)-\boldsymbol{\lambda x}(t)\|=0 \tag{9}$$

则称驱动系统(6)与响应系统(7)实现混合投影同步.

根据系统(6)和(7)误差系统可以表示为：

$$D^{\alpha}\boldsymbol{e}(t)=g(\boldsymbol{y}(t))+\boldsymbol{u}(t)+\boldsymbol{d}(t)-\boldsymbol{\lambda f}(x(t)) \tag{10}$$

2.2　设计自适应模糊滑模控制器

设计积分滑模面：

$$s_i(t)=e_i(t)+\eta_i D^{-\alpha}e_i(t)$$

$$D^{\alpha}s_i=D^{\alpha}e_i+\eta_i e_i \tag{11}$$

$$D^{\alpha}s_i=g_i(\boldsymbol{y}(t))-\lambda_i f_i(\boldsymbol{x}(t))+u_i+d_i+\eta_i e_i \tag{12}$$

当系统发生滑模运动时，$s_i(t)=0$，即 $D^{\alpha}e_i(t)=-\eta_i e_i(t)$，$\eta_i>0$ 根据引理 1，此时系统是渐近稳定的，从而 $\boldsymbol{e}\to 0$，即驱动系统与响应系统将实现混合投影同步.

因而等价控制器设计为：

$$u_i=-g_i(\boldsymbol{y}(t))+\lambda_i f_i(\boldsymbol{x}(t))-d_i-\eta_i e_i \tag{13}$$

由于驱动系统和响应系统以及外部扰动未知，所以等价控制器(13)的设计无法实现. 为了能够控制和辨识驱动系统和响应系统以及外部扰动，由定理 1 知通过利用模糊逻辑系统 $\hat{g}_i(\boldsymbol{y}(t))$ $\hat{f}_i(\boldsymbol{x}(t))$ $\hat{h}_i(t)$ 对未知的非线性函数 $g_i(\boldsymbol{y}(t))$，$f_i(\boldsymbol{x}(t))$ 和外部扰动 $d_i(t)$ 进行逼近.

模糊规则定义为：

$R^{(l)}$：if z_1 is A_1^l, and…, and

z_n is A_n^l, then $\hat{f}_i$ is $B_{1i}{}^l$, and $\hat{g}_i$ is $B_{2i}{}^l$

if s_i is S^l, then $\hat{h}_i$ is H^l

$(i=1,2,\cdots,n;l=1,2,\cdots,N)$

模糊系统可描述为：

$$\begin{cases}\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i(t))=\boldsymbol{\theta}_i^T(t)\boldsymbol{\varphi}(\boldsymbol{z}(t))\\ \hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i(t))=\boldsymbol{\beta}_i^T(t)\boldsymbol{\varphi}(\boldsymbol{z}(t))\\ \hat{h}_i(\boldsymbol{s}(t),\boldsymbol{\gamma}_i(t))=\boldsymbol{\gamma}_i^T(t)\boldsymbol{\phi}(s_i(t))\end{cases} \tag{14}$$

其中 $\boldsymbol{\varphi}(\boldsymbol{z}(t))=(\boldsymbol{\varphi}_1(\boldsymbol{z}(t)),\boldsymbol{\varphi}_2(\boldsymbol{z}(t)),\cdots,\boldsymbol{\varphi}_N(\boldsymbol{z}(t)))^T$ 和 $\boldsymbol{\phi}(\boldsymbol{s}(t))=(\phi_1(s_i(t)),\phi_2(s_i(t)),\cdots,\phi_n(s_i(t)))^T$ 是模糊基函数，$\boldsymbol{\theta}_i(t)$，$\boldsymbol{\beta}_i(t)$，$\boldsymbol{\gamma}_i(t)$ 是模糊系统的可调参数向量.

$$\boldsymbol{\varphi}_l(z(t))=\frac{\prod_{j=1}^{n}\mu_{A_j^l}(z_j)}{\sum_{l=1}^{N}\prod_{j=1}^{n}\mu_{A_j^l}(z_j)},$$

$$\phi_l(s_i(t))=\frac{\mu_{S^l}(s_i)}{\sum_{l=1}^{N}\mu_{S^l}(s_i)},$$

$\mu_{A_j^l}(z_j)$ 和 $\mu_{S^l}(s_i)$ 分别是 z_j 和 δ 隶属度函数.

令模糊系统的最优估计参数向量为 $\boldsymbol{\theta}_i^*$，$\boldsymbol{\beta}_i^*$，$\boldsymbol{\gamma}_i^*$，通常可假设 $\boldsymbol{\theta}_i^*$，$\boldsymbol{\beta}_i^*$，$\boldsymbol{\gamma}_i^*$ 是常数向量.

$$\begin{cases}\boldsymbol{\theta}_i^*=\arg\min\limits_{\theta_i\in\Omega_g}\left[\left|\sup\limits_{y(t)\in\Omega_y}|\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i)-g_i(\boldsymbol{y}(t))\right|\right]\\ \boldsymbol{\beta}_i^*=\arg\min\limits_{\beta_i\in\Omega_f}\left[\left|\sup\limits_{x(t)\in\Omega_x}|\hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i)-f_i(\boldsymbol{x}(t))\right|\right]\\ \boldsymbol{\gamma}_i^*=\arg\min\limits_{\gamma_i\in\Omega_\gamma}\left[\left|\sup\limits_{s(t)\in\Omega_s}|\hat{h}_i(s_i(t),\boldsymbol{\gamma}_i)-(\rho+\delta)\mathrm{sgn}(s_i)\right|\right]\end{cases} \tag{15}$$

$\Omega_g,\Omega_f,\Omega_\gamma,\Omega_x,\Omega_y$ 和 Ω_s 分别是 $\boldsymbol{\theta}_i,\boldsymbol{\beta}_i,\boldsymbol{\gamma}_i,\boldsymbol{x}(t)$，$\boldsymbol{y}(t)$ 和 $\boldsymbol{s}(t)$ 的集合，$\hat{h}_i(s_i(t),\boldsymbol{\gamma}_i^*)=(\rho+\delta)\mathrm{sgn}(s_i)$，$\delta$ 是一个正数，$\mathrm{sgn}(\cdot)$ 是符号函数，$\rho+\delta-|d_i|=k_i(k_i>0)$ $i=1,2,\cdots,n$.

设模糊系统的参数误差和最优估计误差分别为：

$$\tilde{\boldsymbol{\theta}}_i=\boldsymbol{\theta}_i-\boldsymbol{\theta}_i^*,\ \tilde{\boldsymbol{\beta}}_i=\boldsymbol{\beta}_i-\boldsymbol{\beta}_i^*,\ \tilde{\boldsymbol{\gamma}}_i=\boldsymbol{\gamma}_i-\boldsymbol{\gamma}_i^* \tag{16}$$

$$\boldsymbol{\varepsilon}_i(\boldsymbol{y}(t))=g_i(\boldsymbol{y}(t))-\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i^*) \tag{17}$$

$$\tau_i(\boldsymbol{x}(t))=f_i(\boldsymbol{x}(t))-\hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i^*) \tag{18}$$

假设模糊系统的最优估计误差有界，即 $|\varepsilon_i(\boldsymbol{y}(t))|\leqslant\bar{\varepsilon}_i$，$|\tau_i(\boldsymbol{x}(t))|\leqslant\bar{\tau}_i$($\bar{\varepsilon}_i>0$，$\bar{\tau}_i>0$ 是未知常数)，$\hat{\bar{\varepsilon}}_i$，$\hat{\bar{\tau}}_i$ 是 $\bar{\varepsilon}_i$，$\bar{\tau}_i$ 的估计值，估计误差为 $\tilde{\bar{\varepsilon}}_i=\hat{\bar{\varepsilon}}_i-\bar{\varepsilon}_i$，$\tilde{\bar{\tau}}_i=\hat{\bar{\tau}}_i-\bar{\tau}_i$.

经过简单的推导，未知的非线性函数估计误差可以写为：

$$\begin{aligned}&\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i(t))-g_i(\boldsymbol{y}(t))\\&=\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i(t))-\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i{}^*)+\\&\quad\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i{}^*)-g_i(\boldsymbol{y}(t))\\&=\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i(t))-\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i{}^*)-\varepsilon_i(\boldsymbol{y}(t))\\&=\tilde{\boldsymbol{\theta}}_i^T(t)\boldsymbol{\varphi}_i(\boldsymbol{y}(t))-\varepsilon_i(\boldsymbol{y}(t))\end{aligned} \tag{19}$$

$$\begin{aligned}&\hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i(t))-f_i(\boldsymbol{x}(t))\\&=\hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i(t))-\hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i{}^*)+\\&\quad\hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i{}^*)-f_i(\boldsymbol{y}(t))\\&=\hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i(t))-\hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i{}^*)-\tau_i(\boldsymbol{x}(t))\\&=\tilde{\boldsymbol{\beta}}_i^T(t)\boldsymbol{\varphi}_i(\boldsymbol{x}(t))-\tau_i(\boldsymbol{x}(t))\end{aligned} \tag{20}$$

根据上面的讨论，控制器设计为：

$$\begin{aligned}u_i=&-\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i)+\lambda_i\hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i)-\\&\hat{h}_i(s_i(t),\boldsymbol{\gamma}_i)-\eta_i e_i-(\hat{\bar{\varepsilon}}_i+\lambda_i\hat{\bar{\tau}}_i)\mathrm{sgn}(s_i)\end{aligned} \tag{21}$$

自适应律：

$$\begin{cases}D^{\alpha}\boldsymbol{\theta}_i=r_is_i\boldsymbol{\varphi}_i(\boldsymbol{y}(t))\\D^{\alpha}\boldsymbol{\beta}_i=-\sigma_is_i\lambda_i\boldsymbol{\varphi}_i(\boldsymbol{x}(t))\\D^{\alpha}\boldsymbol{\gamma}_i=\rho_is_i\boldsymbol{\phi}(s_i)\\D^{\alpha}\tilde{\bar{\varepsilon}}_i=k_i|s_i|\\D^{\alpha}\tilde{\bar{\tau}}_i=\lambda_i\omega_i|s_i|\end{cases}\tag{22}$$

其中 $r_i,\sigma_i,\rho_i,k_i,\omega_i>0$ 为设计参数,因为常数的 α 阶 Caputo 导数为 0,所以有

$$D^{\alpha}\tilde{\boldsymbol{\theta}}_i=D^{\alpha}\boldsymbol{\theta}_i,$$

$$D^{\alpha}\tilde{\boldsymbol{\beta}}_i=D^{\alpha}\boldsymbol{\beta}_i,D^{\alpha}\tilde{\boldsymbol{\gamma}}_i=D^{\alpha}\boldsymbol{\gamma}_i\quad i=1,2,\cdots,n.$$

$$\begin{aligned}D^{\alpha}s_i&=g_i(\boldsymbol{y}(t))-\hat{g}_i(\boldsymbol{y}(t),\boldsymbol{\theta}_i)+\lambda_i(\hat{f}_i(\boldsymbol{x}(t),\boldsymbol{\beta}_i)-\\&\quad\lambda_if_i(\boldsymbol{x}(t))+\hat{h}_i(s_i(t),\boldsymbol{\gamma}_i^*)-\\&\quad\hat{h}_i(\boldsymbol{s}(t),\boldsymbol{\gamma}(t))-\hat{h}_i(s_i(t),\boldsymbol{\gamma}_i^*)+\\&\quad d_i-(\hat{\bar{\varepsilon}}_i+\lambda_i\hat{\bar{\tau}}_i)\operatorname{sgn}(s_i)\\&=-\tilde{\boldsymbol{\theta}}_i^T(t)\boldsymbol{\varphi}_i(\boldsymbol{y}(t))+\varepsilon_i(\boldsymbol{y}(t))-\\&\quad\hat{h}_i(s_i(t),\boldsymbol{\gamma}_i^*)+d_i-\tilde{\boldsymbol{\gamma}}_i^T\boldsymbol{\phi}(s_i)+\\&\quad\lambda_i(\tilde{\boldsymbol{\beta}}_i^T(t)\boldsymbol{\varphi}_i(\boldsymbol{x}(t))-\tau_i(\boldsymbol{x}(t)))-\\&\quad(\hat{\bar{\varepsilon}}_i+\lambda_i\hat{\bar{\tau}}_i)\operatorname{sgn}(s_i)\end{aligned}\tag{23}$$

定理 2 对于误差系统(10),设计如式(21)所表示的控制器和式(22)所表示的自适应律,则误差系统(10)的运动轨迹稳定到滑模面上,即实现驱动系统(6)和响应系统(7)的混合投影同步.

证明: 构造 Lyapunov 函数如下:

$$\begin{aligned}V(t)&=\sum_{i=1}^{n}V_i(t)\\&=\sum_{i=1}^{n}\frac{1}{2}s_i^2+\sum_{i=1}^{n}\frac{1}{2r_i}\tilde{\boldsymbol{\theta}}_i^T\tilde{\boldsymbol{\theta}}_i+\sum_{i=1}^{n}\frac{1}{2\sigma_i}\tilde{\boldsymbol{\beta}}_i^T\tilde{\boldsymbol{\beta}}_i+\\&\quad\sum_{i=1}^{n}\frac{1}{2\rho_i}\tilde{\boldsymbol{\gamma}}_i^T\tilde{\boldsymbol{\gamma}}_i+\sum_{i=1}^{n}\frac{1}{2k_i}\tilde{\bar{\varepsilon}}_i^T\tilde{\bar{\varepsilon}}_i+\sum_{i=1}^{n}\frac{1}{2\omega_i}\tilde{\bar{\tau}}_i^T\tilde{\bar{\tau}}_i\end{aligned}$$

根据引理 3,5 得:

$$\begin{aligned}D^{\alpha}V&\leqslant\sum_{i=1}^{n}s_iD^{\alpha}s_i+\sum_{i=1}^{n}\frac{1}{r_i}\tilde{\boldsymbol{\theta}}_i^TD^{\alpha}\tilde{\boldsymbol{\theta}}_i+\\&\quad\sum_{i=1}^{n}\frac{1}{\sigma_i}\tilde{\boldsymbol{\beta}}_i^TD^{\alpha}\tilde{\boldsymbol{\beta}}_i+\sum_{i=1}^{n}\frac{1}{\rho_i}\tilde{\boldsymbol{\gamma}}_i^TD^{\alpha}\tilde{\boldsymbol{\gamma}}_i+\\&\quad\sum_{i=1}^{n}\frac{1}{k_i}\tilde{\bar{\varepsilon}}_i^TD^{\alpha}\tilde{\bar{\varepsilon}}_i+\sum_{i=1}^{n}\frac{1}{\omega_i}\tilde{\bar{\tau}}_i^TD^{\alpha}\tilde{\bar{\tau}}_i\\&=\sum_{i=1}^{n}s_i\varepsilon_i(\boldsymbol{y}(t))-\sum_{i=1}^{n}\lambda_is_i\tau_i(\boldsymbol{x}(t))-\\&\quad\sum_{i=1}^{n}s_i\hat{h}_i(s_i(t),\boldsymbol{\gamma}_i^*)+\sum_{i=1}^{n}s_id_i-\\&\quad\sum_{i=1}^{n}|s_i|\hat{\bar{\varepsilon}}_i-\sum_{i=1}^{n}\lambda_i|s_i|\hat{\bar{\tau}}_i+\\&\quad\sum_{i=1}^{n}\tilde{\bar{\varepsilon}}_i|s_i|+\sum_{i=1}^{n}\lambda_i\tilde{\bar{\tau}}_i|s_i|\\&\leqslant\sum_{i=1}^{n}|s_i|\bar{\varepsilon}_i+\sum_{i=1}^{n}\lambda_i|s_i|\bar{\tau}_i-\\&\quad\sum_{i=1}^{n}|s_i|\hat{\bar{\varepsilon}}_i-\sum_{i=1}^{n}\lambda_i|s_i|\hat{\bar{\tau}}_i-\\&\quad\sum_{i=1}^{n}|s_i|(\rho+\delta)+\sum_{i=1}^{n}|s_i||d_i|+\\&\quad\sum_{i=1}^{n}\tilde{\bar{\varepsilon}}_i|s_i|+\sum_{i=1}^{n}\lambda_i\tilde{\bar{\tau}}_i|s_i|\\&=-\sum_{i=1}^{n}|s_i|(\rho+\delta)+\sum_{i=1}^{n}|s_i||d_i|\\&=-\sum_{i=1}^{n}|s_i|(\rho+\delta-|d_i|)\\&=-\sum_{i=1}^{n}k_i|s_i|\\&\leqslant-kc\|\boldsymbol{s}\|\leqslant0\end{aligned}\tag{24}$$

其中 $k=\min\{k_1,k_2,,,k_n\}$,由引理 4 可知 $V(t)\leqslant V(0)$,两边同时取 α 阶积分得 $\lim\limits_{t\to\infty}kcD_t^{-\alpha}\|\boldsymbol{s}\|\leqslant\lim\limits_{t\to\infty}(V(0)-V(t))\leqslant V(0)\leqslant\infty$,由引理 2 知当 $t\to\infty$ 时 $\|\boldsymbol{s}\|\to0$,由于系统在滑模面 $\boldsymbol{s}=0$ 上有 $\boldsymbol{e}\to0$,故可实现驱动系统与响应系统间的混合投影同步.

3 数值仿真

为了验证本文的控制器和自适应律的有效性,选取分数阶 Rössler 混沌系统和分数阶 Arneodo 混沌系统为例进行研究.在仿真中对变量 $z_i(i=1,2,3)$ 选取隶属度函数为 $\mu_{A_j^l}(z_j)=e^{\frac{-(z(j)-a(l))^2}{0.36}}$,对变量 $s_i(i=1,2,3)$ 选取隶属度函数 $\mu_{S_j^l}(s_j)=e^{\frac{-(s(j)-b(l))^2}{0.64}}$,模糊规则数 $N=5$,$a=[-1,-0.5,0,0.5,1]$,$b=[-2,-1,0,1,2]$.模糊系统可调向量的初值 $\boldsymbol{\theta}_1(0)$,$\boldsymbol{\theta}_2(0)$,$\boldsymbol{\theta}_3(0)\boldsymbol{\beta}_1(0)$,$\boldsymbol{\beta}_2(0)$,$\boldsymbol{\beta}_3(0)$,$\boldsymbol{\gamma}_1(0)$,$\boldsymbol{\gamma}_2(0)$,$\boldsymbol{\gamma}_3(0)$ 均为 5 维随机向量.

例 1 考虑分数阶 Rössler 混沌系统作为驱动系统:

$$\begin{cases}D^{\alpha}x_1=-x_2-x_3\\D^{\alpha}x_2=x_1+ax_2\\D^{\alpha}x_3=bx_1-(c-x_1)x_3\end{cases}\tag{25}$$

设计如下响应系统:

$$\begin{cases}D^{\alpha}y_1=-y_2-y_3+0.3\sin(3t)+u_1\\D^{\alpha}y_2=y_1+ay_2+0.3\cos(4t)+u_2\\D^{\alpha}y_3=by_1-(c-y_1)y_3-0.3\sin(3t)+u_3\end{cases}\tag{26}$$

令 $\alpha=0.95, a=0.34, b=0.4, c=4.5$.

控制器取为:

$$\begin{cases} u_1=-\hat{g}_1(\boldsymbol{y}(t),\boldsymbol{\theta}_1)+\lambda_1\hat{f}_1(\boldsymbol{x}(t),\boldsymbol{\beta}_1)- \\ \qquad \hat{h}_1(\boldsymbol{s}(t),\boldsymbol{\gamma}_1(t))-\eta_1 e_1-(\hat{\bar{\varepsilon}}_1+\lambda_1\hat{\bar{\tau}}_1)\operatorname{sgn}(s_1) \\ u_2=-\hat{g}_2(\boldsymbol{y}(t),\boldsymbol{\theta}_2)+\lambda_2\hat{f}_2(\boldsymbol{x}(t),\boldsymbol{\beta}_2)- \\ \qquad \hat{h}_2(\boldsymbol{s}(t),\boldsymbol{\gamma}_2(t))-\eta_2 e_2-(\hat{\bar{\varepsilon}}_2+\lambda_2\hat{\bar{\tau}}_2)\operatorname{sgn}(s_2) \\ u_3=-\hat{g}_3(\boldsymbol{y}(t),\boldsymbol{\theta}_3)+\lambda_3\hat{f}_3(\boldsymbol{x}(t),\boldsymbol{\beta}_3)- \\ \qquad \hat{h}_3(\boldsymbol{s}(t),\boldsymbol{\gamma}_3(t))-\eta_3 e_3-(\hat{\bar{\varepsilon}}_3+\lambda_3\hat{\bar{\tau}}_3)\operatorname{sgn}(s_3) \end{cases} \tag{27}$$

参数自适应律为:

$$\begin{cases} D^\alpha\boldsymbol{\theta}_1=r_1 s_1\boldsymbol{\varphi}_1(\boldsymbol{y}(t)) \\ D^\alpha\boldsymbol{\theta}_2=r_2 s_2\boldsymbol{\varphi}_2(\boldsymbol{y}(t)) \\ D^\alpha\boldsymbol{\theta}_3=r_3 s_3\boldsymbol{\varphi}_3(\boldsymbol{y}(t)) \end{cases} \quad \begin{cases} D^\alpha\boldsymbol{\beta}_1=-\sigma_1 s_1\lambda_1\boldsymbol{\varphi}_1(\boldsymbol{x}(t)) \\ D^\alpha\boldsymbol{\beta}_2=-\sigma_2 s_2\lambda_2\boldsymbol{\varphi}_2(\boldsymbol{x}(t)) \\ D^\alpha\boldsymbol{\beta}_3=-\sigma_3 s_3\lambda_3\boldsymbol{\varphi}_3(\boldsymbol{x}(t)) \end{cases}$$

$$\begin{cases} D^\alpha\boldsymbol{\gamma}_1=\rho_1 s_1\boldsymbol{\phi}(s_1) \\ D^\alpha\boldsymbol{\gamma}_2=\rho_2 s_2\boldsymbol{\phi}(s_2) \\ D^\alpha\boldsymbol{\gamma}_3=\rho_3 s_3\boldsymbol{\phi}(s_3) \end{cases} \quad \begin{cases} D^\alpha\ \tilde{\bar{\varepsilon}}_1=k_1\ |s_1| \\ D^\alpha\ \tilde{\bar{\varepsilon}}_2=k_2\ |s_2| \\ D^\alpha\ \tilde{\bar{\varepsilon}}_3=k_3\ |s_3| \end{cases}$$

$$\begin{cases} D^\alpha\ \tilde{\bar{\tau}}_1=\lambda_1\omega_1\ |s_1| \\ D^\alpha\ \tilde{\bar{\tau}}_2=\lambda_2\omega_2\ |s_2| \\ D^\alpha\ \tilde{\bar{\tau}}_3=\lambda_3\omega_3\ |s_3| \end{cases} \tag{28}$$

设定驱动系统的初始值 $\boldsymbol{x}(0)=[2,1,3]^T$,取受控响应系统的初始值为 $\boldsymbol{y}(0)=[2,2,3]^T$,$(r_1,r_2,r_3)=(100,200,350)$,$\sigma_1=\sigma_2=\sigma_3=0.02$,$(\rho_1,\rho_2,\rho_3)=(200,300,200)$,模糊系统的逼近误差的估计初值为 $\hat{\bar{\varepsilon}}_1=\hat{\bar{\varepsilon}}_3=\hat{\bar{\tau}}_1=\hat{\bar{\tau}}_3=1.8$,$\hat{\bar{\varepsilon}}_2=\hat{\bar{\tau}}_2=1.2$,$k_1=k_2=k_3=0.0009$,$\omega_1=\omega_2=\omega_3=0.0009$,$\eta_1=\eta_2=\eta_3=60$,比例因子选为 $\lambda_1=-2$,$\lambda_2=-1.8$,$\lambda_3=1$,时间步长 $h=0.01$.其数值仿真结果如图 2 和图 3 所示.

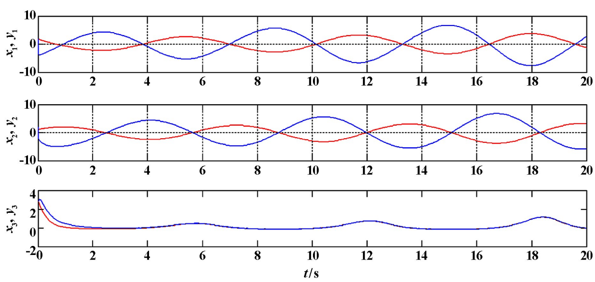

图 2　驱动系统与响应系统的状态响应曲线

Fig.2　State response curves of drive system and response system

图 2 为驱动系统和响应系统的状态响应曲线,图 3 为系统同步误差随时间变化的状态响应轨迹,从仿真结果可知,对含有扰动的不确定分数阶混沌系统在所设计的控制器和自适应律的作用下,e_1,e_2,e_3 随着时间的变化很快趋于零,即驱动系统和响应系统很快实现混合投影同步,并且曲线变化平滑具有较好的控制效果和鲁棒性.

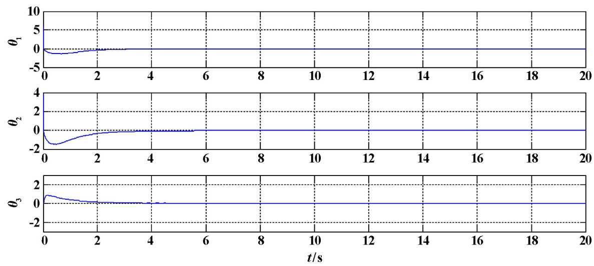

图 3　同步误差与时间状态轨迹

Fig.3　State trajectories of the synchronization error and time

例 2　考虑分数阶 Arneodo 混沌系统作为驱动系统:

$$\begin{cases} D^\alpha x_1=x_2 \\ D^\alpha x_2=x_3 \\ D^\alpha x_3=-a_1x_1-b_1x_2-c_1x_3-d_1x_1^3 \end{cases} \tag{29}$$

分数阶 Rössler 混沌系统作为响应系统:

$$\begin{cases} D^\alpha y_1=-y_2-y_3+0.5\sin(5t)+u_1 \\ D^\alpha y_2=y_1+a_2y_2+0.2+0.3\cos(4t)+u_2 \\ D^\alpha y_3=b_2y_1-(c_2-y_1)y_3-0.6\sin(3t)+u_3 \end{cases} \tag{30}$$

令 $\alpha=0.95$,$a_1=-5.5$,$b_1=-3.5$,$c_1=1$,$d_1=1$,$a_2=0.34$,$b_2=0.4$,$c_2=4.5$.

控制器取为:

$$\begin{cases} u_1=-\hat{g}_1(\boldsymbol{y}(t),\boldsymbol{\theta}_1)+\lambda_1\hat{f}_1(\boldsymbol{x}(t),\boldsymbol{\beta}_1)- \\ \qquad \hat{h}_1(\boldsymbol{s}(t),\boldsymbol{\gamma}_1(t))-\eta_1 e_1-(\hat{\bar{\varepsilon}}_1+\lambda_1\hat{\bar{\tau}}_1)\operatorname{sgn}(s_1) \\ u_2=-\hat{g}_2(\boldsymbol{y}(t),\boldsymbol{\theta}_2)+\lambda_2\hat{f}_2(\boldsymbol{x}(t),\boldsymbol{\beta}_2)- \\ \qquad \hat{h}_2(\boldsymbol{s}(t),\boldsymbol{\gamma}_2(t))-\eta_2 e_2-(\hat{\bar{\varepsilon}}_2+\lambda_2\hat{\bar{\tau}}_2)\operatorname{sgn}(s_2) \\ u_3=-\hat{g}_3(\boldsymbol{y}(t),\boldsymbol{\theta}_3)+\lambda_3\hat{f}_3(\boldsymbol{x}(t),\boldsymbol{\beta}_3)- \\ \qquad \hat{h}_3(\boldsymbol{s}(t),\boldsymbol{\gamma}_3(t))-\eta_3 e_3-(\hat{\bar{\varepsilon}}_3+\lambda_3\hat{\bar{\tau}}_3)\operatorname{sgn}(s_3) \end{cases} \tag{31}$$

参数自适应律为:

$$\begin{cases} D^\alpha\boldsymbol{\theta}_1=r_1 s_1\boldsymbol{\varphi}_1(\boldsymbol{y}(t)) \\ D^\alpha\boldsymbol{\theta}_2=r_2 s_2\boldsymbol{\varphi}_2(\boldsymbol{y}(t)) \\ D^\alpha\boldsymbol{\theta}_3=r_3 s_3\boldsymbol{\varphi}_3(\boldsymbol{y}(t)) \end{cases} \quad \begin{cases} D^\alpha\boldsymbol{\beta}_1=-\sigma_1 s_1\lambda_1\boldsymbol{\varphi}_1(\boldsymbol{x}(t)) \\ D^\alpha\boldsymbol{\beta}_2=-\sigma_2 s_2\lambda_2\boldsymbol{\varphi}_2(\boldsymbol{x}(t)) \\ D^\alpha\boldsymbol{\beta}_3=-\sigma_3 s_3\lambda_3\boldsymbol{\varphi}_3(\boldsymbol{x}(t)) \end{cases}$$

$$\begin{cases} D^\alpha\boldsymbol{\gamma}_1=\rho_1 s_1\boldsymbol{\phi}(s_1) \\ D^\alpha\boldsymbol{\gamma}_2=\rho_2 s_2\boldsymbol{\phi}(s_2) \\ D^\alpha\boldsymbol{\gamma}_3=\rho_3 s_3\boldsymbol{\phi}(s_3) \end{cases} \quad \begin{cases} D^\alpha\ \tilde{\bar{\varepsilon}}_1=k_1\ |s_1| \\ D^\alpha\ \tilde{\bar{\varepsilon}}_2=k_2\ |s_2| \\ D^\alpha\ \tilde{\bar{\varepsilon}}_3=k_3\ |s_3| \end{cases}$$

$$\begin{cases} D^{\alpha}\,\tilde{\tilde{\tau}}_1=\lambda_1\omega_1\,|\,s_1\,| \\ D^{\alpha}\,\tilde{\tilde{\tau}}_2=\lambda_2\omega_2\,|\,s_2\,| \\ D^{\alpha}\,\tilde{\tilde{\tau}}_3=\lambda_3\omega_3\,|\,s_3\,| \end{cases} \tag{32}$$

设定驱动系统的初始值 $\boldsymbol{x}(0)=[0.7,0.3,1.3]^T$,取受控响应系统的初始值为 $\boldsymbol{y}(0)=[0.8,0.4,1.3]^T$,$(r_1,r_2,r_3)=(600,600,600)$,$(\rho_1,\rho_2,\rho_3)=(800,800,800)$,$\sigma_1=\sigma_2=\sigma_3=0.001$ 模糊系统的逼近误差的估计初值为 $\hat{\varepsilon}_2=\hat{\tau}_2=1.2$,$\hat{\varepsilon}_1=\hat{\varepsilon}_3=\hat{\tau}_1=\hat{\tau}_3=1.8$,$k_1=k_2=k_3=0.001$,$\omega_1=\omega_2=\omega_3=0.001$,$\eta_1=\eta_2=\eta_3=60$,比例因子选为 $\lambda_1=-1$,$\lambda_2=1.5$,$\lambda_3=2$,时间步长 $h=0.01$.其数值仿真结果如图 4 和图 5 所示.

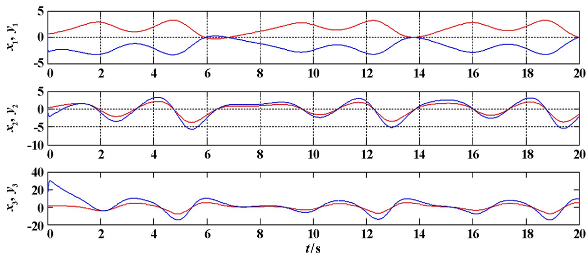

图 4 驱动系统与响应系统的状态响应曲线

Fig.4 State response curves of drive system and response system

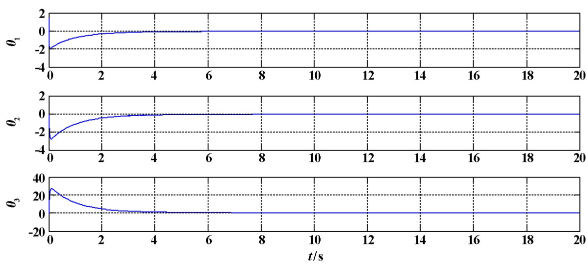

图 5 同步误差与时间状态轨迹

Fig.5 State trajectories of the synchronization error and time

从仿真图 4 和图 5 可知,对含有扰动的异结构分数阶混沌系统和在所设计的控制器和自适应律的作用下,e_1,e_2,e_3 随着时间的变化很快趋于零,即驱动系统和响应系统很快实现混合投影同步,并且曲线变化平滑具有较好的控制效果和鲁棒性.

4 结论

基于分数阶 Barbalat 理论,采用了自适应模糊滑模控制策略,实现了含有扰动项的不确定混沌系统的混合投影同步.本文的方法不需要知道被控对象的精确模型,有较强的抗干扰能力.最后,以分数阶 Rössler 混沌系统和分数阶 Arneodo 混沌系统为例进行了数值仿真,仿真结果验证了该控制方法的有效性.

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PROJECTIVE SYNCHRONIZATION OF FRACTIONAL ORDER CHAOTIC SYSTEM BASED ON ADAPTIVE FUZZY SLIDING MODE CONTROL *

Chen Xu Li Ming Zheng Yongai†

(*College of Information Engineering*, *Yangzhou University*, *Yangzhou* 225127, *China*)

Abstract Based on the fuzzy control theory, the sliding mode control theory and the adaptive control theory, a mixed projective synchronization problem for a class of uncertain fractional chaotic systems with external perturbations is investigated. An adaptive fuzzy sliding mode control is proposed for the projective synchronization method of fractional order chaotic systems. The fuzzy logic system is used to approximate the unknown nonlinear function and external perturbation, the adaptive control is adopted for the approximation error, and a fractional integral sliding mode with strong robustness is constructed. Moreover, adaptive fuzzy sliding mode controller and parameter adaptive law are designed based on the fractional order Barbalat lemma. Finally, the validity of the proposed method is verified by the numerical simulation results.

Key words fractional-order chaotic system, fuzzy control, projective synchronization, sliding mode control

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† Corresponding author E-mail:zhengyongai@163.com