

1:1 内共振情况下轻质材料层合板 动力学的奇异性分析*

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摘要 利用奇异性理论研究 1:1 内共振情况下的点阵夹芯板的非线性动力学分叉, 基于平均方程, 计算出有两个调谐参数和一个面内激励的限制切空间; 对于含有两个状态变量和三个参数的一般非线性动力学方程的奇异性理论进行了推广; 利用推广的奇异性理论得到 1:1 内共振情况下分叉方程余维 4 的普适开折, 画出了转迁集和分叉图; 当分叉、滞后和双极限点产生时, 两个调谐参数和面内激励之间的关系被确定, 数值结果表明, 在不同的分叉区域解的个数不同.

关键词 分叉方程, 奇异性理论, 普适开折, 转迁集

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引言

随着高新技术的发展, 人们已不再满足于材料单纯的轻质化, 而是寻找兼有轻质化和其他某种或几种优良性能相结合的先进材料以适应不同的需求, 作为一种生物材料中普遍存在的典型结构, 多孔介质在自然界动植物体中发挥着不可替代的生理功能. 在这些构型中, 3D-Kagome 结构拥有更加稀少的杆件布局 and 更小的相对密度. 3D-Kagome 结构是由两个正四面体对顶连接而成的网架结构, 将这种核心连接在面板上就形成了 3D-Kagome 点阵夹芯板^[1-3]. 通过实验和数值模拟得到结论, 相对于其他构型的点阵夹芯板, 在同一相对密度情况下, 3D-Kagome 点阵夹芯板具有更高的强度和抗屈曲性能^[4-6]. 此外, 3D-Kagome 点阵夹芯板在功能性方面的表现也比较突出, 例如具有优异的驱动和致动性能, 能够在受到很小的内部抵抗力的情况下获得很大范围内的整体变形. 点阵材料在航空航天等领域有着广泛的应用, 可以减轻飞行器的重量, 同时保证结构的强度和刚度满足要求.

自从上世纪 70 年代, Golubitsky、Schaeffer 和 Stewart 等将奇异性理论和群论方法引入分叉问题

的研究后, 分叉理论得到了越来越多的关注, 从而也推动了奇异性理论的发展. 1981 年, Golubitsky 和 Langford^[7] 研究了退化 Hopf 分叉问题的分类和开折. 1982 年, Golubitsky^[8] 和 Martinet^[9] 等讨论了在强等价下光滑映射芽的开折, 给出了各种形式的通有开折定理. 1985 年, Golubitsky 和 Schaeffer^[10] 得到了余维数不大于 3 的分叉问题的分类, 其中状态变量只有一个且具有 Z_2 对称性. 1986 年, Keyfitz^[11] 给出了余维数不大于 7 的分叉问题的分类, 其中状态变量只有一个但不具对称性, 给出了各种形式的通有开折定理, 但是这些研究都是针对单状态变量、单分叉参数的分叉系统而言. 随后人们将奇异性理论推广到了多状态变量的分叉研究中, 1986 年, Golubitsky^[12] 研究了单参数两状态变量余维数不超过 2 的分叉问题的分类, 其中状态变量关于二面体群 4D 对称, 1988 年, Melbourne^[13] 得到了单参数三个状态变量余维数不超过 1 的分叉问题的分类.

在开始的研究中, 研究工作没有考虑分叉参数的对称性. 1986 年, Gaffney^[14] 将幂单代数群和幂零 Lie 代数应用于多参数分叉问题中, 给出了 D (Γ)-等价. 1996 年, Furter 和 Sitta^[15] 等考虑了分叉

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参数的对称性,研究了余维数不大于1的分叉问题的分类.2003年,高守平和李养成等^[16]讨论了状态变量和分叉参数具有不同对称性的余维数不超过1的分叉问题的分类,其中状态变量关于二面体群 D_4 对称,分叉参数关于 S_1 对称.2006年,郭瑞芝^[17]给出了状态变量和分叉参数具不同对称性的岔问题的分类及识别条件,其中状态变量关于二面体群 D_3 对称,分叉参数关于 $O(2)$ 对称.2007年,崔登兰和李养成等^[18]也研究了含有两组状态变量且参数具有对称性的等变分叉问题.

随着研究的进一步深入,开始了多分叉参数的研究工作.1993年,Lavassani等^[19]通过奇异性理论研究了等变多参数分叉,给出了多参数分叉问题的有限确定定理和正规型,并讨论了多参数分叉的稳定性问题.2000年,胡凡努和李养成^[20]将状态变量分为两组,一组状态变量可以独立变化,而另一组状态变量则依赖于前一组状态变量,研究了该类分叉问题的通有开折.2003年,高守平和李养成^[21]研究了多参数等变分歧问题及其开折,给出了通有的开折定理.但是人们通常将分歧(分叉)问题中的状态变量看作是“平等”的,并不加以区分.2005年,郭瑞芝和李养成^[22]研究了含有两组状态变量的多参数等变分歧问题在左右等价群下的开折,得出了通有开折的充要条件.2010年,秦朝红和陈予恕等^[23-26]研究了含有两个状态变量和两个分叉参数的分叉系统的奇异性理论,并给出了含有两个分叉参数系统的转迁集的计算方法.

本文以点阵夹芯板为切入点,计算1:1内共振情况下点阵夹芯板的非线性动力学方程,推广含有两个状态变量和三个及三个以上参数的一般非线性动力学分叉方程的奇异性理论,利用推广的奇异性理论计算复合层合板的非线性动力学分叉方程的普适开折,同时计算普适开折的转迁集,并进一步讨论若干重要参数对稳定性的影响.

1 点阵夹芯板非线性动力学分叉方程的普适开折

1.1 点阵夹芯板的非线性动力学方程

考虑横向激励与面内激励联合作用下四边简支点阵夹芯板,在板的中面上建立坐标系 oxy ,板在 x 和 y 方向的长度分别为 a 和 b ,厚度为 h ,设夹芯

板中面上任一点在 x 、 y 和 z 方向的位移分别为 u 、 v 和 w ,板承受沿 z 方向的横向激励 $f=F(x,y)\cos\Omega_1 t$ 与沿 y 方向作用于 $x=0$ 和 $x=a$ 的面内激励 $p=p_0+p_1\cos\Omega_2 t$ 联合作用,这里, Ω_1 和 Ω_2 分别为横向激励与面内激励的频率.点阵夹芯板由点阵夹芯层与覆盖在上面的两层蒙皮组成,蒙皮采用各向同性材料,其厚度为 h_f ,芯层厚度为 h_c .其动力学方程为:

$$\ddot{w}_1 + \mu_1 \dot{w}_1 + \beta_{11} w_1 + \beta_{16} (p_0 - p_1 \cos(\Omega_2 t)) w_1 + \beta_{12} w_1 w_2^2 + \beta_{13} w_1^2 w_2 + \beta_{14} w_1^3 + \beta_{15} w_2^3 = \beta_{17} F_1 \cos(\Omega_1 t) \quad (1a)$$

$$\ddot{w}_2 + \mu_2 \dot{w}_2 + \beta_{21} w_2 + \beta_{26} (p_0 - p_1 \cos(\Omega_2 t)) w_2 + \beta_{22} w_1^2 w_2 + \beta_{23} w_1 w_2^2 + \beta_{24} w_2^3 + \beta_{25} w_1^3 = \beta_{27} F_2 \cos(\Omega_1 t) \quad (1b)$$

其中, w_1 为第一阶模态的振幅, w_2 为第二阶模态的振幅, μ_1 和 μ_2 表示对应于两阶模态的阻尼, F_1 和 F_2 表示对应于两阶模态的横向激励幅值, $\beta_{ij}(i=1,2;j=1\cdots 7)$ 表示材料参数.方程(1)的详细推导过程及其所有参数表达式参见文献[27].

1.2 在1:1内共振情况下点阵夹芯板动力学方程摄动分析

利用多尺度法进行研究,将方程(1)中的阻尼项、参数激励项、热激励项和非线性项添加小扰动项 ε ,考虑点阵夹芯板的主参数共振-1:1内共振的情况,共振关系如下:

$$2\omega_1 = \Omega_1 - \varepsilon\sigma_1, \quad 2\omega_2 = \Omega_2 - \varepsilon\sigma_2, \quad \Omega_1 = \Omega_2 \quad (2)$$

式中 ω_1 和 ω_2 为相应线性系统的第一阶和第二阶固有频率, σ_1 和 σ_2 为系统的调谐参数,为了方便处理,令 $\Omega_1 = 1$.

设方程(1)的一致渐近解为:

$$w(x,t,\varepsilon) = w_0(x,T_0,T_1) + \varepsilon w_1(x,T_0,T_1) \quad (3)$$

其中 $T_0 = t, T_1 = \varepsilon t$.

则有微分算子:

$$\frac{d}{dt} = \frac{\partial}{\partial T_0} + \frac{\partial}{\partial T_1} \frac{\partial T_1}{\partial t} + \cdots = D_0 + \varepsilon D_1 + \cdots \quad (4a)$$

$$\frac{d^2}{dt^2} = (D_0 + \varepsilon D_1 + \cdots)^2 = D_0^2 + 2\varepsilon D_0 D_1 + \cdots \quad (4b)$$

其中 $D_0 = \frac{\partial}{\partial T_0}, D_1 = \frac{\partial}{\partial T_1}$.

将式(3)和(4)带入方程(1)中,比较方程两边摄动参数 ε 同阶次的系数,得到如下方程:

ε^0 阶:

$$D_0^2 x_{10} + \frac{1}{4} x_{10} = 0 \quad (5a)$$

$$D_0^2 x_{20} + \frac{1}{4} x_{20} = 0 \quad (5b)$$

ε^1 阶:

$$D_0^2 x_{11} + \frac{1}{4} x_{11} = -2D_0 D_1 x_{10} - \mu_1 D_0 x_{10} + \frac{1}{2} \sigma_1 x_{10} + \beta_{16} x_{10} p_1 \cos t - \beta_{13} x_{10}^2 x_{20} - \beta_{14} x_{10}^3 - \beta_{15} x_{20}^3 - \beta_{12} x_{10} x_{20}^2 + \beta_{17} F_1 \cos t \quad (6a)$$

$$D_0^2 x_{21} + \frac{1}{4} x_{21} = -2D_0 D_1 x_{20} - \mu_2 D_0 x_{20} + \frac{1}{2} \sigma_2 x_{20} + \beta_{26} x_{20} p_1 \cos t - \beta_{23} x_{10} x_{20}^2 - \beta_{24} x_{20}^3 - \beta_{25} x_{10}^3 - \beta_{22} x_{10}^2 x_{20} + \beta_{27} F_2 \cos t \quad (6b)$$

方程(5a)和(5b)的解可以写成如下复数形式:

$$x_{10} = A_1(T_1) e^{\frac{i}{2}T_0} + \bar{A}_1(T_1) e^{-\frac{i}{2}T_0} \\ x_{20} = A_2(T_1) e^{\frac{i}{2}T_0} + \bar{A}_2(T_1) e^{-\frac{i}{2}T_0} \quad (7)$$

式中 \bar{A}_1 和 \bar{A}_2 分别是 A_1 和 A_2 的复共轭。

将方程(7)代入方程(6a)和方程(6b)中得到:

$$D_0^2 x_{11} + \frac{1}{4} x_{11} = (iD_1 A_1 - \frac{1}{2} i\mu_1 A_1 + \frac{1}{2} \sigma_1 A_1 + \frac{1}{2} \beta_{16} p_1 \bar{A}_1 - \beta_{13} A_1^2 \bar{A}_2 - 3\beta_{14} A_1^2 \bar{A}_1 - 2\beta_{13} A_1 \bar{A}_1 A_2 - 3\beta_{15} A_2^2 \bar{A}_2 - 2\beta_{12} A_1 A_2 \bar{A}_2 - \beta_{12} \bar{A}_1 A_2^2) e^{\frac{i}{2}T_0} + cc + NST \quad (8a)$$

$$D_0^2 x_{21} + \frac{1}{4} x_{21} = (-iD_1 A_2 - \frac{1}{2} i\mu_2 A_2 + \frac{1}{2} \sigma_2 A_2 + \frac{1}{2} \beta_{26} p_1 \bar{A}_2 - \beta_{23} \bar{A}_1 A_2^2 - 3\beta_{24} A_2^2 \bar{A}_2 - 2\beta_{23} A_1 \bar{A}_1 A_2 - 3\beta_{25} A_1^2 \bar{A}_1 - 2\beta_{22} A_1 A_2 \bar{A}_1 - \beta_{22} \bar{A}_2 A_1^2) e^{\frac{i}{2}T_0} + cc + NST \quad (8b)$$

其中 cc 和 NST 分别表示方程(8)右端函数的复数部分和长期项。

A_1 和 A_2 可以表示为下列形式:

$$A_2 = \frac{1}{2} a_2 e^{i\varphi_2} \quad (9)$$

消除方程(8)中可以产生长期项的部分,将方程(9)代入方程(8),将实部与虚部分离,得到极坐标形式的四维平均方程为:

$$a_1 = -\frac{\mu_1 a_1}{2} - \frac{\beta_{16} p_1 a_1}{2} \sin(2\varphi_1) - \frac{\beta_{13} a_1^2 a_2}{4} \sin(\varphi_1 - \varphi_2) - \left(\frac{\beta_{13} a_1^2 a_2}{2} + \frac{3\beta_{15} a_2^3}{4} + \frac{\beta_{12} a_1 a_2^2}{2} \cos(\varphi_2 - \varphi_1) \right) \sin(\varphi_2 - \varphi_1) \quad (10a)$$

$$a_1 \dot{\varphi}_1 = -\frac{\mu_1 a_1}{2} - \frac{\beta_{16} p_1 a_1}{2} \cos(2\varphi_1) + \frac{3\beta_{14} a_1^3}{4} + \frac{\beta_{12} a_1 a_2^2}{4} + \frac{\beta_{13} a_1^2 a_2}{4} \cos(\varphi_1 - \varphi_2) +$$

$$\left(\frac{\beta_{13} a_1^2 a_2}{2} + \frac{3\beta_{15} a_2^3}{4} + \frac{\beta_{12} a_1 a_2^2}{2} \cos(\varphi_2 - \varphi_1) \right) \cos(\varphi_2 - \varphi_1) \quad (10b)$$

$$\dot{a}_2 = -\frac{\mu_2 a_2}{2} - \frac{\beta_{26} p_1 a_2}{2} \sin(2\varphi_2) - \left(\frac{\beta_{23} a_1 a_2^2}{4} + \frac{3\beta_{25} a_1^3}{4} + \frac{\beta_{22} a_1^2 a_2}{2} \cos(\varphi_1 - \varphi_2) \right) \sin(\varphi_1 - \varphi_2) \quad (10c)$$

$$a_2 \dot{\varphi}_2 = -\frac{\mu_2 a_2}{2} - \frac{\beta_{26} p_1 a_2}{2} \cos(2\varphi_2) + \frac{3\beta_{24} a_2^3}{4} + \frac{\beta_{22} a_1^2 a_2}{4} + \frac{\beta_{23} a_1 a_2^2}{4} \cos(\varphi_2 - \varphi_1) + \left(\frac{\beta_{23} a_1 a_2^2}{2} + \frac{3\beta_{25} a_1^3}{4} + \frac{\beta_{22} a_1^2 a_2}{2} \cos(\varphi_1 - \varphi_2) \right) \cos(\varphi_1 - \varphi_2) \quad (10d)$$

让方程(10)的左边都等于零,同时在方程(10a)和方程(10b)中消除 $\varphi_2 - \varphi_1$,在方程(10c)和方程(10d)中消除 $\varphi_1 - \varphi_2$,得到:

$$\left[-\frac{1}{2} \mu_1 a_1 - \frac{1}{8} \gamma_{15} a_2^2 a_1 \sin 2(\varphi_2 - \varphi_1) - \frac{1}{2} \gamma_{18} f_T \sin \varphi_1 + \frac{1}{8} \gamma_{14} a_1^2 a_2 \sin(\varphi_2 - \varphi_1) \right]^2 + \left[\frac{1}{2} \sigma_1 a_1 + \frac{3}{8} \gamma_{13} a_1^3 + \frac{1}{4} \gamma_{15} a_1 a_2^2 + \frac{1}{8} \gamma_{15} a_1 a_2^2 \cos 2(\varphi_2 - \varphi_1) - \frac{1}{2} \gamma_{18} f_T \cos \varphi_1 + \frac{1}{8} \gamma_{14} a_1^2 a_2 \cos(\varphi_2 - \varphi_1) \right]^2 = \left(\frac{1}{2} \gamma_{12} a_2 + \frac{1}{4} \gamma_{14} a_1^2 a_2 + \frac{3}{8} \gamma_{16} a_2^3 \right)^2 \quad (11a)$$

$$\left[-\frac{1}{2} \mu_2 a_2 - \frac{1}{8} \gamma_{25} a_2^2 a_2 \sin 2(\varphi_1 - \varphi_2) - \frac{1}{2} \gamma_{28} f_T \sin \varphi_2 + \frac{1}{8} \gamma_{24} a_1 a_2^2 \sin(\varphi_1 - \varphi_2) \right]^2 + \left[\frac{1}{2} \sigma_2 a_2 + \frac{3}{8} \gamma_{23} a_2^3 + \frac{1}{4} \gamma_{25} a_1^2 a_2 + \frac{1}{8} \gamma_{25} a_1^2 a_2 \cos 2(\varphi_1 - \varphi_2) - \frac{1}{2} \gamma_{28} f_T \cos \varphi_2 + \frac{1}{8} \gamma_{24} a_1 a_2^2 \cos(\varphi_1 - \varphi_2) \right]^2 = \left(\frac{1}{2} \gamma_{22} a_1 + \frac{1}{4} \gamma_{24} a_1 a_2^2 + \frac{3}{8} \gamma_{26} a_1^3 \right)^2 \quad (11b)$$

展开方程(11),得到:

$$k_{11} a_1^6 + k_{12} a_2^6 + k_{13} a_1^5 a_2 + k_{14} a_1 a_2^5 + k_{15} a_1^4 a_2^2 + k_{16} a_1^2 a_2^4 + k_{17} a_1^3 a_2^3 + k_{18} \mu_1 a_1^3 a_2 + k_{19} \sigma_1 a_1^3 a_2 + k_{110} p_1 a_1^3 a_2 + k_{111} p_1 a_1^4 + k_{112} \sigma_1 a_1^4 + k_{113} \sigma_1 a_1^2 a_2^2 + k_{114} p_1 a_1^2 a_2^2 + k_{115} p_1^2 a_1^2 +$$

$$k_{116}\mu_1 p_1 a_1^2 + k_{117} p_1 \sigma_1 a_1^2 + \sigma_1^2 a_1^2 + \mu_1^2 a_1^2 = 0 \quad (12a)$$

$$\begin{aligned} & k_{21} a_2^6 + k_{22} a_1^6 + k_{23} a_1 a_2^5 + k_{24} a_1^5 a_2 + k_{25} a_1^2 a_2^4 + k_{26} a_1^4 a_2^2 + \\ & k_{27} a_1^3 a_2^3 + k_{28} \mu_2 a_1 a_2^3 + k_{29} \sigma_2 a_1 a_2^3 + \\ & k_{210} p_1 a_1 a_2^3 + k_{211} p_1 a_2^4 + k_{212} \sigma_2 a_2^4 + \\ & k_{213} \sigma_2 a_1^2 a_2^2 + k_{214} p_1 a_1^2 a_2^2 + k_{215} p_1^2 a_2^2 + \\ & k_{216} \mu_2 p_1 a_2^2 + k_{217} p_1 \sigma_2 a_2^2 + \sigma_2^2 a_2^2 + \mu_2^2 a_2^2 = 0 \quad (12b) \end{aligned}$$

式中:

$$k_{11} = \frac{9\beta_{14}^2}{4}, k_{12} = -\frac{9\beta_{15}^2}{4}, k_{13} = \frac{3\beta_{13}\beta_{14}}{2} \cos(\varphi_1 - \varphi_2),$$

$$k_{14} = -3\beta_{12}\beta_{15} \cos(\varphi_2 - \varphi_1), k_{15} = \frac{3\beta_{12}\beta_{14}}{2},$$

$$k_{16} = \beta_{12}^2 \sin^2(\varphi_2 - \varphi_1) - \frac{3\beta_{13}\beta_{15}}{2},$$

$$k_{17} = -\frac{\beta_{12}\beta_{13}}{2} \cos(\varphi_1 - \varphi_2), k_{18} = \beta_{13} \sin(\varphi_1 - \varphi_2),$$

$$k_{19} = -\beta_{13} \cos(\varphi_1 - \varphi_2), k_{110} = -\beta_{13}\beta_{16} \cos(3\varphi_1 - \varphi_2),$$

$$k_{111} = -3\beta_{14}\beta_{16} \cos(2\varphi_1), k_{112} = -3\beta_{14},$$

$$k_{113} = -\beta_{12}, k_{114} = -\beta_{12}\beta_{16} \cos(2\varphi_1), k_{115} = \beta_{16}^2,$$

$$k_{116} = 2\beta_{16} \sin(2\varphi_1), k_{117} = 2\beta_{16} \cos(2\varphi_1),$$

$$k_{21} = \frac{9\beta_{24}^2}{4}, k_{22} = -\frac{9\beta_{25}^2}{4}, k_{23} = \frac{3\beta_{23}\beta_{24}}{2} \cos(\varphi_2 - \varphi_1),$$

$$k_{24} = -3\beta_{22}\beta_{25} \cos(\varphi_1 - \varphi_2), k_{25} = \frac{3\beta_{22}\beta_{24}}{2},$$

$$k_{26} = \beta_{22}^2 \sin^2(\varphi_1 - \varphi_2) - \frac{3\beta_{23}\beta_{25}}{2},$$

$$k_{27} = -\frac{\beta_{22}\beta_{23}}{2} \cos(\varphi_2 - \varphi_1), k_{28} = \beta_{23} \sin(\varphi_2 - \varphi_1),$$

$$k_{29} = -\beta_{23} \cos(\varphi_2 - \varphi_1), k_{210} = -\beta_{23}\beta_{26} \cos(3\varphi_2 - \varphi_1),$$

$$k_{211} = -3\beta_{24}\beta_{26} \cos(2\varphi_2), k_{212} = -3\beta_{24},$$

$$k_{213} = -\beta_{22}, k_{214} = -\beta_{22}\beta_{26} \cos(2\varphi_2), k_{215} = \beta_{26}^2,$$

$$k_{216} = 2\beta_{26} \sin(2\varphi_2), k_{217} = 2\beta_{26} \cos(2\varphi_2) \quad (12c)$$

根据 $\varphi_1 = 0$ 和 $\varphi_2 = 0$, φ_1 和 φ_2 是 $[-\pi, \pi]$ 中的任意固定值.

1.3 在 1:1 内共振情况下点阵夹芯板动力学方程分叉分析

令:

$$g = (g_1, g_2) \in \vec{E}_{z, \lambda} \quad (13)$$

式中:

$$\begin{aligned} g_1 = & k_{11} a_1^6 + k_{12} a_2^6 + k_{13} a_1^5 a_2 + k_{14} a_1 a_2^5 + k_{15} a_1^4 a_2^2 + \\ & k_{16} a_1^2 a_2^4 + k_{17} a_1^3 a_2^3 + k_{18} \mu_1 a_1^3 a_2 + k_{19} \sigma_1 a_1^3 a_2 + \end{aligned}$$

$$k_{110} p_1 a_1^3 a_2 + k_{111} p_1 a_1^4 + k_{112} \sigma_1 a_1^4 +$$

$$k_{113} \sigma_1 a_1^2 a_2^2 + k_{114} p_1 a_1^2 a_2^2 + k_{115} p_1^2 a_1^2 +$$

$$k_{116} \mu_1 p_1 a_1^2 + k_{117} p_1 \sigma_1 a_1^2 + \sigma_1^2 a_1^2 + \mu_1^2 a_1^2 \quad (14a)$$

$$g_2 = k_{21} a_2^6 + k_{22} a_1^6 + k_{23} a_1 a_2^5 + k_{24} a_1^5 a_2 + k_{25} a_1^2 a_2^4 +$$

$$k_{26} a_1^4 a_2^2 + k_{27} a_1^3 a_2^3 + k_{28} \mu_2 a_1 a_2^3 + k_{29} \sigma_2 a_1 a_2^3 +$$

$$k_{210} p_1 a_1 a_2^3 + k_{211} p_1 a_2^4 + k_{212} \sigma_2 a_2^4 +$$

$$k_{213} \sigma_2 a_1^2 a_2^2 + k_{214} p_1 a_1^2 a_2^2 + k_{215} p_1^2 a_2^2 +$$

$$k_{216} \mu_2 p_1 a_2^2 + k_{217} p_1 \sigma_2 a_2^2 + \sigma_2^2 a_2^2 + \mu_2^2 a_2^2 \quad (14b)$$

$$z = (a_1, a_2), \lambda = (\sigma_1, \sigma_2, p_1) \quad (14c)$$

1.3.1 限制切空间

定理 1.1 芽 $g(z, \lambda)$ 的限制切空间 $\mathbf{RT}(g, 1)$

能够表示为:

$$\mathbf{RT}(g, 1) = \mathbf{M}^2 + \mathbf{M} \langle \sigma_1, \sigma_2, p_1 \rangle \quad (15)$$

证明: 根据命题 1.4 (文献 [10] 中, 第二册 169 页), $\mathbf{RT}(g, 1)$ 的生成元有 14 个:

$$\begin{aligned} & (g_1, 0), (g_2, 0), (0, g_1), (0, g_2), a_1(g_{1, a_1}, g_{2, a_1}), \\ & a_2(g_{1, a_1}, g_{2, a_1}), \sigma_1(g_{1, a_1}, g_{2, a_1}), \sigma_2(g_{1, a_1}, g_{2, a_1}), \\ & p_1(g_{1, a_1}, g_{2, a_1}), a_1(g_{1, a_2}, g_{2, a_2}), a_2(g_{1, a_2}, g_{2, a_2}), \\ & \sigma_1(g_{1, a_2}, g_{2, a_2}), \sigma_2(g_{1, a_2}, g_{2, a_2}), p_1(g_{1, a_2}, g_{2, a_2}) \quad (16) \end{aligned}$$

式中:

$$\begin{aligned} g_{1, a_1} = & 6k_{11} a_1^5 + 5k_{13} a_1^4 a_2 + k_{14} a_2^5 + 4k_{15} a_1^3 a_2^2 + \\ & 2k_{16} a_1 a_2^4 + 3k_{17} a_1^2 a_2^3 + 3k_{18} \mu_1 a_1^2 a_2 + \\ & 3k_{19} \sigma_1 a_1^2 a_2 + 3k_{110} p_1 a_1^2 a_2 + 4k_{111} p_1 a_1^3 + \\ & 4k_{112} \sigma_1 a_1^3 + 2k_{113} \sigma_1 a_1 a_2^2 + 2k_{114} p_1 a_1 a_2^2 + \\ & 2k_{115} p_1^2 a_1 + 2k_{116} \mu_1 p_1 a_1 + 2k_{117} p_1 \sigma_1 a_1 + \\ & 2\sigma_1^2 a_1 + 2\mu_1^2 a_1 \quad (17a) \end{aligned}$$

$$\begin{aligned} g_{2, a_1} = & 6k_{22} a_1^5 + k_{23} a_2^5 + 5k_{24} a_1^4 a_2 + 2k_{25} a_1 a_2^4 + \\ & 4k_{26} a_1^3 a_2^2 + 3k_{27} a_1^2 a_2^3 + k_{28} \mu_2 a_2^3 + k_{29} \sigma_2 a_2^3 + \\ & k_{210} p_1 a_2^3 + 2k_{213} \sigma_2 a_1 a_2^2 + 2k_{214} p_1 a_1 a_2^2 \quad (17b) \end{aligned}$$

$$\begin{aligned} g_{1, a_2} = & 6k_{12} a_2^5 + k_{13} a_1^5 + 5k_{14} a_1 a_2^4 + 2k_{15} a_1^4 a_2 + \\ & 4k_{16} a_1^2 a_2^3 + 3k_{17} a_1^3 a_2^2 + k_{18} \mu_1 a_1^3 + k_{19} \sigma_1 a_1^3 + \\ & k_{110} p_1 a_1^3 + 2k_{113} \sigma_1 a_1^2 a_2 + 2k_{114} p_1 a_1^2 a_2 \quad (17c) \end{aligned}$$

$$\begin{aligned} g_{2, a_2} = & 6k_{21} a_2^5 + 5k_{23} a_1 a_2^4 + k_{24} a_1^5 + 4k_{25} a_1^2 a_2^3 + \\ & 2k_{26} a_1^4 a_2 + 3k_{27} a_1^3 a_2^2 + 3k_{28} \mu_2 a_1 a_2^2 + \\ & 3k_{29} \sigma_2 a_1 a_2^2 + 3k_{210} p_1 a_1 a_2^2 + 4k_{211} p_1 a_2^3 + \\ & 4k_{212} \sigma_2 a_2^3 + 2k_{213} \sigma_2 a_1^2 a_2 + 2k_{214} p_1 a_1^2 a_2 + \\ & 2k_{215} p_1^2 a_2 + 2k_{216} \mu_2 p_1 a_2 + 2k_{217} p_1 \sigma_2 a_2 + \\ & 2\sigma_2^2 a_2 + 2\mu_2^2 a_2 \quad (17d) \end{aligned}$$

$\mathbf{M}^2 + \mathbf{M} \langle \sigma_1, \sigma_2, p_1 \rangle$ 的生成元有 18 个:

$$\begin{aligned}
& (a_1^2, 0), (a_1 a_2, 0), (a_2^2, 0), (\sigma_1 a_1, 0), \\
& (\sigma_2 a_1, 0), (p_1 a_1, 0), (\sigma_1 a_2, 0), (\sigma_2 a_2, 0), \\
& (p_1 a_2, 0), (0, a_1^2), (0, a_1 a_2), (0, a_2^2), \\
& (0, \sigma_1 a_1), (0, \sigma_2 a_1), (0, p_1 a_1), (0, \sigma_1 a_2), \\
& (0, \sigma_2 a_2), (0, p_1 a_2) \quad (18)
\end{aligned}$$

下面证明, 当 $(a_1, a_2, \sigma_1, \sigma_2, p_1) = (0, 0, 0, 0, 0)$ 时, (16) 和 (18) 之间存在一个可逆矩阵 A , 使得两者可以互相表示:

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = A \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (19)$$

式中:

$$\begin{aligned}
u_1 = & ((g_1, 0) \quad (g_2, 0) \quad (0, g_1) \quad (0, g_2) \\
& a_1(g_{1,a_1}, g_{2,a_1}) \quad a_2(g_{1,a_1}, g_{2,a_1}) \quad \sigma_1(g_{1,a_1}, g_{2,a_1}))_{1 \times 7}^T \quad (20a)
\end{aligned}$$

$$\begin{aligned}
u_2 = & (\sigma_2(g_{1,a_1}, g_{2,a_1}) \quad p_1(g_{1,a_1}, g_{2,a_1}) \quad a_1(g_{1,a_2}, g_{2,a_2}) \\
& a_2(g_{1,a_2}, g_{2,a_2}) \quad \sigma_1(g_{1,a_2}, g_{2,a_2}) \quad \sigma_2(g_{1,a_2}, g_{2,a_2}) \\
& p_1(g_{1,a_2}, g_{2,a_2}))_{1 \times 7}^T \quad (20b)
\end{aligned}$$

$$\begin{aligned}
v_1 = & ((a_1^2, 0) \quad (a_1 a_2, 0) \quad (a_2^2, 0) \quad (\sigma_1 a_1, 0) \\
& (\sigma_2 a_1, 0) \quad (p_1 a_1, 0) \quad (\sigma_1 a_2, 0) \quad (\sigma_2 a_2, 0) \\
& (p_1 a_2, 0))_{1 \times 9}^T \quad (20c)
\end{aligned}$$

$$\begin{aligned}
v_2 = & ((0, a_1^2) \quad (0, a_1 a_2) \quad (0, a_2^2) \quad (0, \sigma_1 a_1) \\
& (0, \sigma_2 a_1) \quad (0, p_1 a_1) \quad (0, \sigma_1 a_2) \quad (0, \sigma_2 a_2) \\
& (0, p_1 a_2))_{1 \times 9}^T \quad (20d)
\end{aligned}$$

$$A_1 = \begin{pmatrix} C_1 & C_2 & C_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_4 & C_5 & C_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_7 & C_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_7 & C_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_7 & 0 & 0 & C_2 & 0 & 0 \end{pmatrix}_{7 \times 9} \quad (20e)$$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_1 & C_2 & C_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_4 & C_5 & C_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6C_4 & C_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6C_4 & C_8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6C_4 & 0 & 0 & C_8 & 0 & 0 \end{pmatrix}_{7 \times 9} \quad (20f)$$

$$A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & C_7 & 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_7 & 0 & 0 & C_2 \\ C_9 & 6C_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_9 & 6C_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_9 & 0 & 0 & 6C_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_9 & 0 & 0 & 6C_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_9 & 0 & 0 & 6C_3 \end{pmatrix}_{7 \times 9} \quad (20g)$$

$$A_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 6C_4 & 0 & 0 & C_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6C_4 & 0 & 0 & C_8 \\ C_5 & C_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_5 & C_{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_5 & 0 & 0 & C_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_5 & 0 & 0 & C_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_5 & 0 & 0 & C_{10} \end{pmatrix}_{7 \times 9} \quad (20h)$$

式中:

$$\begin{aligned}
C_1 = & k_{11} a_1^4 + k_{13} a_1^3 a_2 + k_{15} a_1^2 a_2^2 + k_{16} a_2^4 + k_{17} a_1 a_2^3 + \\
& k_{18} \mu_1 a_1 a_2 + k_{19} \sigma_1 a_1 a_2 + k_{110} p_1 a_1 a_2 + k_{111} p_1 a_1^2 + \\
& k_{112} \sigma_1 a_1^2 + k_{113} \sigma_1 a_2^2 + k_{114} p_1 a_2^2 + k_{115} p_1^2 + \\
& k_{116} \mu_1 p_1 + k_{117} p_1 \sigma_1 + \sigma_1^2 + \mu_1^2 \quad (21a)
\end{aligned}$$

$$C_2 = k_{14} a_2^4 \quad (21b)$$

$$C_3 = k_{12} a_2^4 \quad (21c)$$

$$C_4 = k_{22} a_1^4 \quad (21d)$$

$$C_5 = k_{24} a_1^4 \quad (21e)$$

$$\begin{aligned}
C_6 = & k_{21} a_2^4 + k_{23} a_1 a_2^3 + k_{25} a_1^2 a_2^2 + k_{26} a_1^4 + k_{27} a_1^3 a_2 + \\
& k_{28} \mu_2 a_1 a_2 + k_{29} \sigma_2 a_1 a_2 + k_{210} p_1 a_1 a_2 + \\
& k_{211} p_1 a_2^2 + k_{212} \sigma_2 a_2^2 + k_{213} \sigma_2 a_1^2 + k_{214} p_1 a_1^2 + \\
& k_{215} p_1^2 + k_{216} \mu_2 p_1 + k_{217} p_1 \sigma_2 + \sigma_2^2 + \mu_2^2 \quad (21f)
\end{aligned}$$

$$\begin{aligned}
C_7 = & 6k_{11} a_1^4 + 5k_{13} a_1^3 a_2 + 4k_{15} a_1^2 a_2^2 + 2k_{16} a_2^4 + \\
& 3k_{17} a_1 a_2^3 + 3k_{18} \mu_1 a_1 a_2 + 3k_{19} \sigma_1 a_1 a_2 + \\
& 3k_{110} p_1 a_1 a_2 + 4k_{111} p_1 a_1^2 + 4k_{112} \sigma_1 a_1^2 + \\
& 2k_{113} \sigma_1 a_2^2 + 2k_{114} p_1 a_2^2 + 2k_{115} p_1^2 + 2k_{116} \mu_1 p_1 + \\
& 2k_{117} p_1 \sigma_1 + 2\sigma_1^2 + 2\mu_1^2 \quad (21g)
\end{aligned}$$

$$\begin{aligned}
C_8 = & k_{23} a_2^4 + 5k_{24} a_1^4 + 2k_{25} a_1 a_2^3 + 4k_{26} a_1^3 a_2 + \\
& 3k_{27} a_1^2 a_2^2 + k_{28} \mu_2 a_2^2 + k_{29} \sigma_2 a_2^2 + k_{210} p_1 a_2^2 + \\
& 2k_{213} \sigma_2 a_1 a_2 + 2k_{214} p_1 a_1 a_2 \quad (21h)
\end{aligned}$$

$$\begin{aligned}
C_9 = & k_{13} a_1^4 + 5k_{14} a_2^4 + 2k_{15} a_1^3 a_2 + 4k_{16} a_1 a_2^3 + \\
& 3k_{17} a_1^2 a_2^2 + k_{18} \mu_1 a_1^2 + k_{19} \sigma_1 a_1^2 + k_{110} p_1 a_1^2 +
\end{aligned}$$

$$2k_{113}\sigma_1 a_1 a_2 + 2k_{114}p_1 a_1 a_2 \quad (21i)$$

$$C_{10} = 6k_{21}a_2^4 + 5k_{23}a_1 a_2^3 + 4k_{25}a_1^2 a_2^2 + 2k_{26}a_1^4 + 3k_{27}a_1^3 a_2 + 3k_{28}\mu_2 a_1 a_2 + 3k_{29}\sigma_2 a_1 a_2 + 3k_{210}p_1 a_1 a_2 + 4k_{211}p_1 a_2^2 + 4k_{212}\sigma_2 a_2^2 + 2k_{213}\sigma_2 a_1^2 + 2k_{214}p_1 a_1^2 + 2k_{215}p_1^2 + 2k_{216}\mu_2 p_1 + 2k_{217}p_1 \sigma_2 + 2\sigma_2^2 + 2\mu_2^2 \quad (21j)$$

因为 $\mu_1 > 0, \mu_2 > 0$, 当 $a_1 = a_2 = \sigma_1 = \sigma_2 = p_1 = 0$ 时,

$$C_1 = \mu_1^2 \neq 0, C_2 = 0, C_3 = 0, C_4 = 0, C_5 = 0, C_6 = \mu_2^2 \neq 0, C_7 = 2\mu_1^2 \neq 0, C_8 = 0, C_9 = 0, C_{10} = 2\mu_2^2 \neq 0 \quad (22)$$

这样矩阵 A 等于:

$$\begin{pmatrix} A'_1 & A'_2 \\ A'_3 & A'_4 \end{pmatrix} \quad (23)$$

式中:

$$A_1 = \begin{pmatrix} \mu_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_2^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\mu_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\mu_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu_1^2 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{7 \times 9} \quad (24a)$$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_1^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_2^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{7 \times 9} \quad (24b)$$

$$A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 2\mu_1^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{7 \times 9} \quad (24c)$$

$$A_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\mu_2^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\mu_2^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\mu_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\mu_2^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\mu_2^2 \end{pmatrix}_{7 \times 9} \quad (24d)$$

在 14×18 的矩阵 A 中, 容易发现第 1 行和第 5 行线性相关, 第 4 行和第 11 行线性相关, 第 7、8、9、13、14、15 列是空的, 剔除第 5、11 行和第 7、8、9、13、14、15 列, 剩下一个 12×12 的矩阵, 很容易证明 12×12 的矩阵的行列式的值是非零的。

证明完成。

1.3.2 简单识别

定理 1.2 令:

$$g(z, \lambda) = h(z, \lambda) + q(z, \lambda) \quad (25)$$

式中:

$$h = (\mu_1^2 a_1^2, \mu_2^2 a_2^2) \quad (26a)$$

$$q = (k_{11}a_1^6 + k_{12}a_2^6 + k_{13}a_1^5 a_2 + k_{14}a_1 a_2^5 + k_{15}a_1^4 a_2^2 + k_{16}a_1^2 a_2^4 + k_{17}a_1^3 a_2^3 + k_{18}\mu_1 a_1^3 a_2 + k_{19}\sigma_1 a_1^3 a_2 + k_{110}p_1 a_1^3 a_2 + k_{111}p_1 a_1^4 + k_{112}\sigma_1 a_1^4 + k_{113}\sigma_1 a_1^2 a_2^2 + k_{114}p_1 a_1^2 a_2^2 + k_{115}p_1^2 a_1^2 + k_{116}\mu_1 p_1 a_1^2 + k_{117}p_1 \sigma_1 a_1^2 + \sigma_1^2 a_1^2, k_{21}a_2^6 + k_{22}a_1^6 + k_{23}a_1 a_2^5 + k_{24}a_1^5 a_2 + k_{25}a_1^2 a_2^4 + k_{26}a_1^4 a_2^2 + k_{27}a_1^3 a_2^3 + k_{28}\mu_2 a_1 a_2^3 + k_{29}\sigma_2 a_1 a_2^3 + k_{210}p_1 a_1 a_2^3 + k_{211}p_1 a_2^4 + k_{212}\sigma_2 a_2^4 + k_{213}\sigma_2 a_1^2 a_2^2 + k_{214}p_1 a_1^2 a_2^2 + k_{215}p_1^2 a_2^2 + k_{216}\mu_2 p_1 a_2^2 + k_{217}p_1 \sigma_2 a_2^2 + \sigma_2^2 a_2^2) \quad (26b)$$

可以得到 g 和 h 是强等价的。

证明: 根据定理 1.1, $\mathbf{M}^2 + \mathbf{M}\langle \sigma_1, \sigma_2, p_1 \rangle$ 的一个高阶项是:

$$\mathbf{M}^3 + \mathbf{M}^2 \langle \sigma_1, \sigma_2, p_1 \rangle \quad (27)$$

对于多项式 g , 我们观察发现:

$$\begin{aligned} & (k_{11}a_1^6, 0), (k_{12}a_2^6, 0), (k_{13}a_1^5 a_2, 0), \\ & (k_{14}a_1 a_2^5, 0), (k_{15}a_1^2 a_2^4, 0), (k_{16}a_1^4 a_2^2, 0), \\ & (k_{17}a_1^3 a_2^3, 0), (k_{18}\mu_1 a_1^3 a_2, 0), (0, k_{21}a_2^6), \\ & (0, k_{22}a_1^6), (0, k_{23}a_1 a_2^5), (0, k_{24}a_1^5 a_2), \\ & (0, k_{25}a_1^2 a_2^4), (0, k_{16}a_1^4 a_2^2), (0, k_{27}a_1^3 a_2^3) \text{ 和} \\ & (0, k_{28}\mu_2 a_1 a_2^3) \text{ 属于 } \mathbf{M}^3 \vec{\mathbf{E}}_{2, \lambda} \end{aligned} \quad (28a)$$

$$\begin{aligned}
& (k_{19}\sigma_1 a_1^3 a_2, 0), (k_{110}p_1 a_1^3 a_2, 0), (k_{111}p_1 a_1^4, 0), \\
& (k_{112}\sigma_1 a_1^4, 0), (k_{113}\sigma_1 a_1^2 a_2^2, 0), (k_{114}p_1 a_1^2 a_2^2, 0), \\
& (k_{115}p_1^2 a_1^2, 0), (k_{116}\mu_1 p_1 a_1^2, 0), (k_{117}p_1 \sigma_1 a_1^2, 0), \\
& (\sigma_1^2 a_1^2, 0), (0, k_{29}\sigma_2 a_1 a_2^3), (0, k_{210}p_1 a_1 a_2^3), \\
& (0, k_{211}p_1 a_2^4), (0, k_{212}\sigma_2 a_2^4), (0, k_{213}\sigma_2 a_1^2 a_2^2), \\
& (0, k_{214}p_1 a_1^2 a_2^2), (0, k_{215}p_1^2 a_2^2), (0, k_{216}\mu_2 p_1 a_2^2), \\
& (0, k_{217}p_1 \sigma_2 a_2^2) \text{ 和 } (\sigma_2^2 a_2^2, 0)
\end{aligned}$$

$$\text{属于 } \mathbf{M}^2 \langle \sigma_1, \sigma_2, p_1 \rangle \vec{\mathbf{E}}_{z,\lambda} \quad (28b)$$

因此, $q \in (\mathbf{M}^3 + \mathbf{M}^2 \langle \sigma_1, \sigma_2, p_1 \rangle) \vec{\mathbf{E}}_{z,\lambda}$

根据定理 4.1 (文献 [10] 中, 第二册 185 页),

可以得到 g 和 h 是强等价的.

证明完成.

在下面的讨论中, g 将被 h 取代.

引理 1.1 h 的非退化条件是:

$$16\mu_1^4 \mu_2^4 \neq 0 \quad (29)$$

满足非退化条件的 h 等价于:

$$(a_1^2, a_2^2) \quad (30)$$

证明: h 是关于 a_1, a_2 的二次齐次多项式, 根据

方程(2.7) (文献 [10] 中, 第一册 402 页),

$$Q(h) = \begin{vmatrix} 2\mu_1^2 a_1 & 0 \\ 0 & 2\mu_2^2 a_2 \end{vmatrix} = 4\mu_1^2 \mu_2^2 a_1 a_2 \quad (31)$$

根据方程(2.8) (文献 [10] 中, 第一册 402 页),

$$D = b^2 - 4ac = 16\mu_1^4 \mu_2^4 \quad (32)$$

因为 $\mu_1 > 0, \mu_2 > 0$,

$$16\mu_1^4 \mu_2^4 \neq 0 \quad (33)$$

h 的正规形式能够表示为:

$$(a_1^2, a_2^2) \quad (34)$$

证明完成.

观察表达式(34), h 的普适开折需要表达式

(34) 补足 h 的线性项和常数项.

定理 1.3 令:

$H(a_1, a_2, \sigma_1, \sigma_2, p_1) = (h_1(a_1, a_2, \sigma_1, \sigma_2, p_1), h_2(a_1, a_2, \sigma_1, \sigma_2, p_1))$ 是分叉问题 h 的一个 4 参数开折, 如果满足引理 1.1, 则 H 是 h 的普适开折, 当且仅当:

$$\det(Q) =$$

$$\det \begin{pmatrix} 0 & h_{1,a_1 a_1} & h_{1,a_1 a_2} & 0 & h_{2,a_1 a_1} & h_{2,a_1 a_2} \\ 0 & h_{1,a_2 a_1} & h_{1,a_2 a_2} & 0 & h_{2,a_2 a_1} & h_{2,a_2 a_2} \\ h_{1,\sigma_1} & h_{1,\sigma_1 a_1} & h_{1,\sigma_1 a_2} & h_{2,\sigma_1} & h_{2,\sigma_1 a_1} & h_{2,\sigma_1 a_2} \\ h_{1,\sigma_2} & h_{1,\sigma_2 a_1} & h_{1,\sigma_2 a_2} & h_{2,\sigma_2} & h_{2,\sigma_2 a_1} & h_{2,\sigma_2 a_2} \\ h_{1,p_1} & h_{1,p_1 a_1} & h_{1,p_1 a_2} & h_{2,p_1} & h_{2,p_1 a_1} & h_{2,p_1 a_2} \\ h_{1,\gamma} & h_{1,\gamma a_1} & h_{1,\gamma a_2} & h_{2,\gamma} & h_{2,\gamma a_1} & h_{2,\gamma a_2} \end{pmatrix} \neq 0 \quad (35)$$

式中 γ 是辅助参数, 这里:

$$(a_1, a_2, \sigma_1, \sigma_2, p_1, \gamma) = (0, 0, 0, 0, 0, 0)$$

证明: 矩阵 Q 能够被表示为:

$$Q = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)^T \quad (36)$$

式中:

$$\alpha_1 = (0, h_{1,a_1 a_1}, h_{1,a_1 a_2}, 0, h_{2,a_1 a_1}, h_{2,a_1 a_2}) \quad (37a)$$

$$\alpha_2 = (0, h_{1,a_2 a_1}, h_{1,a_2 a_2}, 0, h_{2,a_2 a_1}, h_{2,a_2 a_2}) \quad (37b)$$

$$\alpha_3 = (h_{1,\sigma_1}, h_{1,\sigma_1 a_1}, h_{1,\sigma_1 a_2}, h_{2,\sigma_1}, h_{2,\sigma_1 a_1}, h_{2,\sigma_1 a_2}) \quad (37c)$$

$$\alpha_4 = (h_{1,\sigma_2}, h_{1,\sigma_2 a_1}, h_{1,\sigma_2 a_2}, h_{2,\sigma_2}, h_{2,\sigma_2 a_1}, h_{2,\sigma_2 a_2}) \quad (37d)$$

$$\alpha_5 = (h_{1,p_1}, h_{1,p_1 a_1}, h_{1,p_1 a_2}, h_{2,p_1}, h_{2,p_1 a_1}, h_{2,p_1 a_2}) \quad (37e)$$

$$\alpha_6 = (h_{1,\gamma}, h_{1,\gamma a_1}, h_{1,\gamma a_2}, h_{2,\gamma}, h_{2,\gamma a_1}, h_{2,\gamma a_2}) \quad (37f)$$

将 h 代入方程(37), 得到:

$$\alpha_1 = (0, 2, 0, 0, 0, 0), \alpha_2 = (0, 0, 0, 0, 0, 2),$$

$$\alpha_3 = (0, 0, 0, 0, 0, 0), \alpha_4 = (0, 0, 0, 0, 0, 0),$$

$$\alpha_5 = (0, 0, 0, 0, 0, 0), \alpha_6 = (0, 0, 0, 0, 0, 0) \quad (38)$$

在方程(38), 仅有 2 个向量 α_1 和 α_2 是线性无关的, 因此存在补足 h 的 4 个线性无关的向量:

$$(0, \lambda' a_1), (\lambda'' a_2, 0), (\lambda, \lambda), (-\gamma, \gamma) \quad (39)$$

式中 λ, λ' 和 λ'' 的取值是 σ_1, σ_2 和 p_1 三个分叉参数中的任意一个, 且互相之间取值不同.

这样, 我们得到:

$$H = (a_1^2 + \lambda'' a_2 + \lambda - \gamma, a_2^2 + \lambda' a_1 + \lambda + \gamma) \quad (40)$$

把方程(40)代入方程(35), 得到:

$$\det(Q) \neq 0. \quad (41)$$

反过来, 在方程(38)中, 仅有向量 α_1 和 α_2 是线性无关, 因为 $\det(Q) \neq 0$, 补足 h 的线性无关的向量需要 4 个:

$$(0, \lambda' a_1), (\lambda'' a_2, 0), (\lambda, \lambda), (-\gamma, \gamma) \quad (42)$$

这样, 得到:

$$H=(a_1^2+\varepsilon_2\lambda''a_2+\varepsilon_1\lambda-\varepsilon_4\gamma,a_2^2+\varepsilon_3\lambda'a_1+\varepsilon_1\lambda+\varepsilon_4\gamma)$$

证明完成.

1.3.3 多项式空间的维数

定理 1.4 多项式空间能够简化为:

$$\mathbf{R}\{(dh)_{z,\lambda}(Y_1), \dots, (dh)_{z,\lambda}(Y_m), h_\lambda, \lambda h_\lambda, \lambda^2 h_\lambda, \dots\} \\ =\mathbf{R}\{(a_1, 0), (0, a_2)\} \quad (43)$$

证明: 推导方程(43), 建立下面的矩阵关系:

$$\begin{pmatrix} u'_1 \\ u'_2 \end{pmatrix} = B \begin{pmatrix} v'_1 \\ v'_2 \end{pmatrix} = \begin{pmatrix} B_1 & B_2 \\ B_3 & B_4 \end{pmatrix} \begin{pmatrix} v'_1 \\ v'_2 \end{pmatrix} \quad (44)$$

式中:

$$v'_1 = \begin{pmatrix} (h_{1,a_1}, 0) & (h_{1,a_2}, 0) & (h_{1,\sigma_1}, 0) \\ (h_{1,\sigma_2}, 0) & (h_{1,p_1}, 0) \end{pmatrix}_{1 \times 5}^T \quad (45a)$$

$$v'_2 = \begin{pmatrix} (0, h_{2,a_1}) & (0, h_{2,a_2}) & (0, h_{2,\sigma_1}) \\ (0, h_{2,\sigma_2}) & (0, h_{2,p_1}) \end{pmatrix}_{1 \times 5}^T \quad (45b)$$

$$v'_1 = \begin{pmatrix} (a_1, 0) & (a_2, 0) & (\sigma_1, 0) & (\sigma_2, 0) \\ (p_1, 0) \end{pmatrix}_{1 \times 5}^T \quad (45c)$$

$$v'_2 = \begin{pmatrix} (0, a_1) & (0, a_2) & (0, \sigma_1) & (0, \sigma_2) \\ (0, p_1) \end{pmatrix}_{1 \times 5}^T \quad (45d)$$

$$B_1 = \begin{pmatrix} D_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (45e)$$

$$B_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (45f)$$

$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (45g)$$

$$B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & D_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (45h)$$

显然:

$$D_1 = 2, D_2 = 2 \quad (46)$$

我们发现 $(u'_1 \quad u'_2)^T$ 能够被 $(a_1, 0)$ 和 $(0, a_2)$

两个向量表示, 因此,

$$\mathbf{R}\{(dh)_{z,\lambda}(Y_1), \dots, (dh)_{z,\lambda}(Y_m), h_\lambda, \lambda h_\lambda, \lambda^2 h_\lambda, \dots\} \\ =\mathbf{R}\{(a_1, 0), (0, a_2)\} \quad (47)$$

证明完成.

1.3.4 奇异性理论推广

根据定理 1.1, 有下面的关系:

$$h \in \mathbf{M}^2 \subset \mathbf{M}^2 + \mathbf{M}\langle \sigma_1, \sigma_2, p_1 \rangle \quad (48)$$

得到:

$$(a_1^2, 0), (0, a_2^2) \text{ 属于 } \mathbf{M}^2 \vec{\mathbf{E}}_{z,\lambda} \quad (49)$$

明显, 多项式 h 不包含属于 $\mathbf{M}\langle \sigma_1, \sigma_2, p_1 \rangle \vec{\mathbf{E}}_{z,\lambda}$ 的单项式.

因此, 对奇异性理论作如下的推广:

定义 1.1 设 $\mathbf{T}(h, \mathbf{1}) \subset \vec{\mathbf{E}}_{z,\lambda}$ 有有限余维, 如果 $h \in \langle h \rangle \subset \mathbf{T}(h, \mathbf{1})$, 则 h 在 $\mathbf{T}(h, \mathbf{1})$ 中的补空间的一组基所需参数个数最小时, 叫做 h 在 $\mathbf{T}(h, \mathbf{1})$ 的余维数, 记作 n ; $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补空间的一组基所需参数个数最小时, 叫做 $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 的余维数, 记作 m .

引理 1.2 h 在 $\vec{\mathbf{E}}_{z,\lambda}$ 的余维数, 等于 $n+m$.

证明: 根据定义 1.1, $\mathbf{T}(h, \mathbf{1}) \subset \vec{\mathbf{E}}_{z,\lambda}$ 有有限余维, 存在 n 个线性无关的向量补足 $\langle h \rangle$, 使得:

$$\langle h \rangle \oplus \mathbf{p}(n) = \mathbf{T}(h, \mathbf{1}) \quad (50)$$

式中 $\mathbf{p}(n)$ 是一个有限维子空间.

同理, 存在 m 个线性无关的向量补足 $\mathbf{T}(h, \mathbf{1})$, 使得

$$\mathbf{T}(h, \mathbf{1}) \oplus \mathbf{p}'(m) = \vec{\mathbf{E}}_{z,\lambda} \quad (51)$$

式中 $\mathbf{p}'(m)$ 是一个有限维子空间.

把方程 (50) 代入方程 (51), 得到

$$\langle h \rangle \oplus \mathbf{p}(n) \oplus \mathbf{p}'(m) = \vec{\mathbf{E}}_{z,\lambda} \quad (52)$$

因此, $\langle h \rangle$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 的余维数, 等于 $n+m$, 即 h 在 $\vec{\mathbf{E}}_{z,\lambda}$ 的余维数, 等于 $n+m$.

证明完成.

推论 1.1 $n=6-2-m$.

证明: 根据定理 1.3, 存在 6 个线性无关的向量, 但是在方程 (38) 中, 仅有两个线性无关的向量 α_1 和 α_2 . 根据引理 1.1, 可知 $2+m+n=6$, 即: $n=6-2-m$.

证明完成.

引理 1.3 让辅助参数 $\gamma \in R$ 作用在单项式 $\lambda^{l_1} x^{l_2}$ 上, 如果 $l_1 \neq 0$, 则 $|\gamma|$ 能够被剔除, $\text{sgn}(\gamma) \lambda^{l_1} x^{l_2}$ 将被获得.

证明: 对于 $\gamma \lambda^{l_1} x^{l_2}$, 如果 $l_1 \neq 0$, $|\gamma|$ 能够嵌入在 λ^{l_1} 中, 得到 $\text{sgn}(\gamma) \lambda^{l_1}$, 这是因为 γ 和 λ 都是参数, 有共同的辅助特性.

证明完成.

在后面的讨论中, 为了简化符号, $\text{sgn}(\gamma)$ 被记作 ε .

1.3.5 普适开折

定理 1.5 h 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的余维数是 4, 方程 (34) 的普适开折是:

$$G_1 = (a_1^2 + 2\varepsilon_{12}\sigma_1 a_2 + \varepsilon_{11}p_1 - \gamma, a_2^2 + 2\varepsilon_{13}\sigma_2 a_1 + \varepsilon_{11}p_1 + \gamma) \quad (53a)$$

$$G_2 = (a_1^2 + 2\varepsilon_{22}\sigma_2 a_2 + \varepsilon_{21}p_1 - \gamma, a_2^2 + 2\varepsilon_{23}\sigma_1 a_1 + \varepsilon_{21}p_1 + \gamma) \quad (53b)$$

$$G_3 = (a_1^2 + 2\varepsilon_{32}\sigma_2 a_2 + \varepsilon_{31}\sigma_1 - \gamma, a_2^2 + 2\varepsilon_{33}p_1 a_1 + \varepsilon_{31}\sigma_1 + \gamma) \quad (53c)$$

$$G_4 = (a_1^2 + 2\varepsilon_{42}p_1 a_2 + \varepsilon_{41}\sigma_1 - \gamma, a_2^2 + 2\varepsilon_{43}\sigma_2 a_1 + \varepsilon_{41}\sigma_1 + \gamma) \quad (53d)$$

$$G_5 = (a_1^2 + 2\varepsilon_{52}\sigma_1 a_2 + \varepsilon_{51}\sigma_2 - \gamma, a_2^2 + 2\varepsilon_{53}p_1 a_1 + \varepsilon_{51}\sigma_2 + \gamma) \quad (53e)$$

$$G_6 = (a_1^2 + 2\varepsilon_{62}p_1 a_2 + \varepsilon_{61}\sigma_2 - \gamma, a_2^2 + 2\varepsilon_{63}\sigma_1 a_1 + \varepsilon_{61}\sigma_2 + \gamma) \quad (53f)$$

式中 γ 是辅助参数, $\varepsilon_{ij} = +1, 0, -1; i = 1, \dots, 6; j = 1, 2, 3$.

证明: 根据方程 (2.7) (文献 [10] 中, 第二册 211 页), 可知:

$$\mathbf{T}(h, \mathbf{1}) = (\mathbf{M}^2 + \mathbf{M}\langle \sigma_1, \sigma_2, p_1 \rangle) \vec{\mathbf{E}}_{z,\lambda} + \mathbf{R}\{(a_1, 0), (0, a_2)\} \quad (54)$$

$\text{ItrT}(h, \mathbf{1})$ 是包含在 $\mathbf{T}(h, \mathbf{1})$ 中的极大理想:

$$\text{ItrT}(h, \mathbf{1}) = (\mathbf{M}^2 + \mathbf{M}\langle \sigma_1, \sigma_2, p_1 \rangle) \vec{\mathbf{E}}_{z,\lambda} \quad (55)$$

可知:

$$\begin{aligned} [\text{ItrT}(h, \mathbf{1})]^\perp = & \mathbf{R}\{(a_1, 0), (a_2, 0), (\sigma_1, 0), \\ & (\sigma_2, 0), (p_1, 0), (-1, 0), \\ & (0, a_1), (0, a_2), (0, \sigma_1), \\ & (0, \sigma_2), (0, p_1), (0, 1)\} \end{aligned} \quad (56)$$

维数是 12.

显然, $\mathbf{R}\{(a_1, 0), (0, a_2)\}$ 的维数是 2, 因此存在 $[\text{ItrT}(g, \mathbf{1})]^\perp$ 的子空间的一组基补足 $\mathbf{R}\{(a_1, 0), (0, a_2)\}$, 即:

$$\begin{aligned} & (a_2, 0), (0, a_1), (\sigma_1, 0), (0, \sigma_1), (\sigma_2, 0), \\ & (0, \sigma_2), (p_1, 0), (0, p_1), (-1, 0), (0, 1) \end{aligned} \quad (57)$$

根据定理 1.3, 简化方程 (57),

$$\begin{aligned} & (a_2, 0), (0, a_1), (\sigma_1, \sigma_1), (\sigma_2, \sigma_2), \\ & (p_1, p_1), (-1, 1) \end{aligned} \quad (58)$$

$\mathbf{M}\langle \sigma_1, \sigma_2, p_1 \rangle \vec{\mathbf{E}}_{z,\lambda}$ 的生成元有 12 个:

$$\begin{aligned} & (\sigma_1 a_1, 0), (\sigma_2 a_1, 0), (p_1 a_1, 0), (\sigma_1 a_2, 0), \\ & (\sigma_2 a_2, 0), (p_1 a_2, 0), (0, \sigma_1 a_1), (0, \sigma_2 a_1), \\ & (0, p_1 a_1), (0, \sigma_1 a_2), (0, \sigma_2 a_2), (0, p_1 a_2) \end{aligned} \quad (59)$$

在方程 (58) 中, 对于向量 $(a_2, 0)$ 和 $(0, a_1)$, 存在两个辅助参数 β_1 和 β_2 , 使得 $(\beta_1 a_2, 0)$ 和 $(0, \beta_2 a_1)$ 发生. 根据定理 1.3, $(\beta_1 a_2, 0)$ 和 $(\sigma_1 a_2, 0)$, $(\sigma_2 a_2, 0)$, $(p_1 a_2, 0)$ 三个向量中的任意一个线性相关; $(0, \beta_2 a_1)$ 和 $(0, \sigma_1 a_1)$, $(0, \sigma_2 a_1)$, $(0, p_1 a_1)$ 三个向量中的任意一个线性相关. 因此, $(a_2, 0)$ 和 $(0, a_1)$ 将被剔除.

在 h 的普适开折中, 存在 4 个辅助参数 $\alpha_1, \alpha_2, \alpha_3$ 和 γ , 使得 $(\alpha_1 p_1, \alpha_1 p_1)$, $(\alpha_2 \sigma_1, \alpha_2 \sigma_1)$, $(\alpha_3 \sigma_2, \alpha_3 \sigma_2)$ 和 $(-\gamma, \gamma)$ 发生, 根据引理 1.3, $(\varepsilon_1 p_1, \varepsilon_1 p_1)$, $(\varepsilon_2 \sigma_1, \varepsilon_2 \sigma_1)$, $(\varepsilon_3 \sigma_2, \varepsilon_3 \sigma_2)$ 和 $(-\gamma, \gamma)$ 发生. 因为 p_1, σ_1, σ_2 和 γ 都是参数, 根据定理 1.3, 把四个向量 $(\varepsilon_1 p_1, \varepsilon_1 p_1)$, $(\varepsilon_2 \sigma_1, \varepsilon_2 \sigma_1)$, $(\varepsilon_3 \sigma_2, \varepsilon_3 \sigma_2)$ 和 $(-\gamma, \gamma)$ 代入方程 (37) 中, 仅有两个向量是线性无关的, 根据定义 1.1, $m = 2$.

根据定理 1.3, 在矩阵 \mathbf{Q} 中, 可知向量 $(-\gamma, \gamma)$ 必然发生.

因此, $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补基是 $(\varepsilon_1 p_1, \varepsilon_1 p_1)$ 和 $(-\gamma, \gamma)$, 或者 $(\varepsilon_2 \sigma_1, \varepsilon_2 \sigma_1)$ 和 $(-\gamma, \gamma)$, 或者 $(\varepsilon_3 \sigma_2, \varepsilon_3 \sigma_2)$ 和 $(-\gamma, \gamma)$.

为了简化符号, $(\varepsilon_2 \sigma_1, \varepsilon_2 \sigma_1)$ 记作 $(\varepsilon_1 \sigma_1, \varepsilon_1 \sigma_1)$, $(\varepsilon_3 \sigma_2, \varepsilon_3 \sigma_2)$ 记作 $(\varepsilon_1 \sigma_2, \varepsilon_1 \sigma_2)$.

根据推论 1.1, $n = 2$.

根据定义 1.1, 在 $\mathbf{T}(h, \mathbf{1})$ 中, 存在 2 个线性无关的向量补足 $\langle h \rangle$. 根据定理 1.3, 当选择 $(\varepsilon_1 p_1, \varepsilon_1 p_1)$ 和 $(-\gamma, \gamma)$ 作为 $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补基时, 它们是 $(\sigma_1 a_2, 0)$ 和 $(0, \sigma_2 a_1)$ 或者 $(\sigma_2 a_2, 0)$ 和 $(0,$

$\sigma_1 a_2$);当选择 $(\varepsilon_1 \sigma_1, \varepsilon_1 \sigma_1)$ 和 $(-\gamma, \gamma)$ 作为 $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补基时,它们是 $(p_1 a_2, 0)$ 和 $(0, \sigma_2 a_1)$ 或者 $(\sigma_2 a_2, 0)$ 和 $(0, p_1 a_1)$;当选择 $(\varepsilon_1 \sigma_2, \varepsilon_1 \sigma_2)$ 和 $(-\gamma, \gamma)$ 作为 $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补基时,它们是 $(p_1 a_2, 0)$ 和 $(0, \sigma_1 a_1)$ 或者 $(\sigma_1 a_2, 0)$ 和 $(0, p_1 a_1)$.

这样,存在两个辅助参数 β_1 和 β_2 ,当选择 $(\varepsilon_1 p_1, \varepsilon_1 p_1)$ 和 $(-\gamma, \gamma)$ 作为 $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补基时,使得 $(\beta_1 \sigma_1 a_2, 0)$ 和 $(0, \beta_2 \sigma_2 a_1)$ 或者 $(\beta_1 \sigma_2 a_2, 0)$ 和 $(0, \beta_2 \sigma_1 a_2)$ 发生;当选择 $(\varepsilon_1 \sigma_1, \varepsilon_1 \sigma_1)$ 和 $(-\gamma, \gamma)$ 作为 $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补基时, $(\beta_1 p_1 a_2, 0)$ 和 $(0, \beta_2 \sigma_2 a_1)$ 或者 $(\beta_1 \sigma_2 a_2, 0)$ 和 $(0, \beta_2 p_1 a_1)$ 发生;当选择 $(\varepsilon_1 \sigma_2, \varepsilon_1 \sigma_2)$ 和 $(-\gamma, \gamma)$ 作为 $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补基时, $(\beta_1 p_1 a_2, 0)$ 和 $(0, \beta_2 \sigma_1 a_1)$ 或者 $(\beta_1 \sigma_1 a_2, 0)$ 和 $(0, \beta_2 p_1 a_1)$ 发生.

根据引理 1.3,当选择 $(\varepsilon_1 p_1, \varepsilon_1 p_1)$ 和 $(-\gamma, \gamma)$ 作为 $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补基时, $(\varepsilon_2 \sigma_1 a_2, 0)$ 和 $(0, \varepsilon_3 \sigma_2 a_1)$ 或者 $(\varepsilon_2 \sigma_2 a_2, 0)$ 和 $(0, \varepsilon_3 \sigma_1 a_2)$ 发生;当选择 $(\varepsilon_1 \sigma_1, \varepsilon_1 \sigma_1)$ 和 $(-\gamma, \gamma)$ 作为 $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补基时, $(\varepsilon_2 p_1 a_2, 0)$ 和 $(0, \varepsilon_3 \sigma_2 a_1)$ 或者 $(\varepsilon_2 \sigma_2 a_2, 0)$ 和 $(0, \varepsilon_3 p_1 a_1)$ 发生;当选择 $(\varepsilon_1 \sigma_2, \varepsilon_1 \sigma_2)$ 和 $(-\gamma, \gamma)$ 作为 $\mathbf{T}(h, \mathbf{1})$ 在 $\vec{\mathbf{E}}_{z,\lambda}$ 中的补基时, $(\varepsilon_2 p_1 a_2, 0)$ 和 $(0, \varepsilon_3 \sigma_1 a_1)$ 或者 $(\varepsilon_2 \sigma_1 a_2, 0)$ 和 $(0, \varepsilon_3 p_1 a_1)$ 发生.

根据上面的分析,得到:

$$G_1 = (a_1^2 + 2\varepsilon_{12} \sigma_1 a_2 + \varepsilon_{11} p_1 - \gamma, a_2^2 + 2\varepsilon_{13} \sigma_2 a_1 + \varepsilon_{11} p_1 + \gamma) \quad (60a)$$

$$G_2 = (a_1^2 + 2\varepsilon_{22} \sigma_2 a_2 + \varepsilon_{21} p_1 - \gamma, a_2^2 + 2\varepsilon_{23} \sigma_1 a_1 + \varepsilon_{21} p_1 + \gamma) \quad (60b)$$

$$G_3 = (a_1^2 + 2\varepsilon_{32} \sigma_2 a_2 + \varepsilon_{31} \sigma_1 - \gamma, a_2^2 + 2\varepsilon_{33} p_1 a_1 + \varepsilon_{31} \sigma_1 + \gamma) \quad (60c)$$

$$G_4 = (a_1^2 + 2\varepsilon_{42} p_1 a_2 + \varepsilon_{41} \sigma_1 - \gamma, a_2^2 + 2\varepsilon_{43} \sigma_2 a_1 + \varepsilon_{41} \sigma_1 + \gamma) \quad (60d)$$

$$G_5 = (a_1^2 + 2\varepsilon_{52} \sigma_1 a_2 + \varepsilon_{51} \sigma_2 - \gamma, a_2^2 + 2\varepsilon_{53} p_1 a_1 + \varepsilon_{51} \sigma_2 + \gamma) \quad (60e)$$

$$G_6 = (a_1^2 + 2\varepsilon_{62} p_1 a_2 + \varepsilon_{61} \sigma_2 - \gamma, a_2^2 + 2\varepsilon_{63} \sigma_1 a_1 + \varepsilon_{61} \sigma_2 + \gamma) \quad (60f)$$

根据引理 1.3,当所有隐辅助参数和辅助参数 γ 等于零,即: $\varepsilon_{ij} = 0$ ($i = 1, \dots, 6; j = 1, 2, 3$)和 $\gamma = 0$,我们可以得到:

$$G_i(a_1, a_2, p_1, 0, 0, 0) = h(a_1, a_2, p_1) \quad (61)$$

根据定理 1.3,当 $\varepsilon_{ij} \neq 0$,对于方程(60)有:

$$\det \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 2\varepsilon_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\varepsilon_{13} & 0 \\ \varepsilon_{11} & 0 & 0 & \varepsilon_{11} & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \neq 0 \quad (62a)$$

$$\det \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2\varepsilon_{23} & 0 \\ 0 & 0 & 2\varepsilon_{22} & 0 & 0 & 0 \\ \varepsilon_{21} & 0 & 0 & \varepsilon_{21} & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \neq 0 \quad (62b)$$

$$\det \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ \varepsilon_{31} & 0 & 0 & \varepsilon_{31} & 0 & 0 \\ 0 & 0 & 2\varepsilon_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\varepsilon_{33} & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \neq 0 \quad (62c)$$

$$\det \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ \varepsilon_{41} & 0 & 0 & \varepsilon_{41} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\varepsilon_{43} & 0 \\ 0 & 0 & 2\varepsilon_{42} & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \neq 0 \quad (62d)$$

$$\det \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 2\varepsilon_{52} & 0 & 0 & 0 \\ \varepsilon_{51} & 0 & 0 & \varepsilon_{51} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\varepsilon_{53} & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \neq 0 \quad (62e)$$

$$\det \begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2\varepsilon_{63} & 0 \\ \varepsilon_{61} & 0 & 0 & \varepsilon_{61} & 0 & 0 \\ 0 & 0 & 2\varepsilon_{62} & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \neq 0 \quad (62f)$$

证明完成.

1.3.6 转迁集

在下面的分析中,我们讨论方程(53)的转迁集.

对于方程(53a),分叉集满足的条件是:

$$a_1^2 + 2\varepsilon_{12}\sigma_1 a_2 + \varepsilon_{11}p_1 - \gamma = 0 \quad (63a)$$

$$a_2^2 + 2\varepsilon_{13}\sigma_2 a_1 + \varepsilon_{11}p_1 + \gamma = 0 \quad (63b)$$

$$\frac{a_1}{\varepsilon_{13}\sigma_2} = \frac{\varepsilon_{12}\sigma_1}{a_2} = \frac{\varepsilon_{12}a_2}{0} \quad \text{或}$$

$$\frac{a_1}{\varepsilon_{13}\sigma_2} = \frac{\varepsilon_{12}\sigma_1}{a_2} = \frac{0}{\varepsilon_{13}a_1} \quad \text{或}$$

$$\frac{a_1}{\varepsilon_{13}\sigma_2} = \frac{\varepsilon_{12}\sigma_1}{a_2} = \frac{\varepsilon_{11}}{\varepsilon_{11}} \quad (63c)$$

由方程(63c)的第三个等式,可得分叉集的表达式:

$$p_1 = -\frac{3}{2\varepsilon_{11}}\sigma_1^2 - \frac{3}{2\varepsilon_{11}}\sigma_2^2 \quad (64)$$

方程(63c)的第一和第二个等式,表明分叉发生时:

$$(a_1, a_2, \sigma_1, \sigma_2, p_1, \gamma) = (0, 0, 0, 0, 0, 0) \quad (65)$$

滞后集:

$$G=0, \det(dG)=0 \text{ 和 } d^2G(v, v) \in \text{range}(dG), \quad (66)$$

任意非零 $v \in \ker(dG)$.

滞后集满足的条件:

$$a_1^2 + 2\varepsilon_{12}\sigma_1 a_2 + \varepsilon_{11}p_1 - \gamma = 0 \quad (67a)$$

$$a_2^2 + 2\varepsilon_{13}\sigma_2 a_1 + \varepsilon_{11}p_1 + \gamma = 0 \quad (67b)$$

$$a_1 a_2 - \varepsilon_{12}\varepsilon_{13}\sigma_1 \sigma_2 = 0 \quad (67c)$$

$$(v_1^2, v_2^2) \in \text{range}(dG) \quad (67d)$$

式中 $v = (v_1, v_2)$.

我们假设 $\sigma_1 \neq 0$ 和 $\sigma_2 \neq 0$, 则:

$$v = (a_1, \varepsilon_{12}\sigma_1) \quad (68)$$

注意到 $(v_1^2, v_2^2) \in \text{range}(dG)$ 当且仅当:

$$(v_1^2, v_2^2) \cdot (\varepsilon_{12}\sigma_1, a_2) = 0 \quad (69)$$

计算方程(69),可得:

$$\varepsilon_{12}a_1^2 + \sigma_1 a_2 = 0 \quad (70)$$

让 a_1 乘以方程(70)的两端,然后代入方程(67c)中,得到:

$$a_1 = -\varepsilon_{13}\sigma_1^{2/3}\sigma_2^{1/3} \quad (71)$$

由方程(67c)和方程(71),可得:

$$a_2 = -\varepsilon_{12}\sigma_1^{1/3}\sigma_2^{2/3} \quad (72)$$

方程(67a)加方程(67b),可得:

$$a_1^2 + 2\varepsilon_{12}\sigma_1 a_2 + a_2^2 + 2\varepsilon_{13}\sigma_2 a_1 + 2\varepsilon_{11}p_1 = 0 \quad (73)$$

把方程(71)和方程(72)代入方程(73)中,得到滞后集的表达式:

$$p_1 = \frac{1}{2\varepsilon_{11}}\sigma_1^{4/3}\sigma_2^{2/3} + \frac{1}{2\varepsilon_{11}}\sigma_1^{2/3}\sigma_2^{4/3} \quad (74)$$

双极限点集满足的条件:

$$a_1^2 + 2\varepsilon_{12}\sigma_1 a_2 + \varepsilon_{11}p_1 - \gamma = 0 \quad (75a)$$

$$a_2^2 + 2\varepsilon_{13}\sigma_2 a_1 + \varepsilon_{11}p_1 + \gamma = 0 \quad (75b)$$

$$a_1 a_2 - \varepsilon_{12}\varepsilon_{13}\sigma_1 \sigma_2 = 0 \quad (75c)$$

$$(a_{11}, a_{21}) \neq (a_{12}, a_{22}), \text{ 当 } p_1 = \text{const} \quad (75d)$$

根据方程(64)计算,可知:

$$\sigma_1 = 0, p_1 \leq 0 \text{ 和 } \sigma_2 = 0, p_1 \leq 0 \text{ 当 } \varepsilon_{11} = 1, \quad (76a)$$

$$\sigma_1 = 0, p_1 \geq 0 \text{ 和 } \sigma_2 = 0, p_1 \geq 0 \text{ 当 } \varepsilon_{11} = -1 \quad (76b)$$

我们定义 ρ , θ 和 δ 是 $f_T \sigma_2 \sigma_1$ -空间的圆柱坐标的转换,令:

$$\sigma_1 = \rho^3 \cos^3 \theta, \sigma_2 = \rho^3 \sin^3 \theta, p_1 = \rho^6 \delta \quad (77)$$

把方程(77)代入方程(64)和方程(74)中, \mathbf{B} ,

\mathbf{H} 和 \mathbf{D} 有如下形式:

$$\mathbf{B}: \left\{ \delta = -\frac{3}{2\varepsilon_{11}}\cos^6 \theta - \frac{3}{2\varepsilon_{11}}\sin^6 \theta \right\} \quad (78a)$$

$$\mathbf{H}: \left\{ \delta = \frac{1}{8\varepsilon_{11}}\sin^2(2\theta) \right\} \quad (78b)$$

$$\mathbf{D}: \left\{ \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}; \delta \leq 0; \varepsilon_{11} = 1 \right\} \text{ 和}$$

$$\left\{ \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}; \delta \geq 0; \varepsilon_{11} = -1 \right\} \quad (78c)$$

同理,方程(53b), (53c), (53d), (53e) 和 (53f) 的转迁集 Σ 也能够被获得.

2 数值模拟

本节利用 Runge-Kutta 法对方程(53a)进行数值计算,针对分叉表达式,滞后表达式和双极限点集得到关于三个分叉参数 σ_1 , σ_2 和 p_1 的平面转迁集和立体转迁集,同时给出了相应的力-幅图.

根据以上分析结果,对方程(53a)进行数值计算,由方程(64)和方程(74),我们得到方程(53a)关于三个分叉参数 σ_1 , σ_2 和 p_1 的平面转迁集和立体转迁集,当 $\varepsilon_{11} = -1$ 时,如图1和图2所示;当 $\varepsilon_{11} = 1$ 时,如图5和图6所示, B 表示分叉集, H 表示滞后集, D 表示双极限点集. 从方程(64)的表达式可以发现,当 p_1 固定时,在图2(a)中,分叉集是一个圆;当 σ_1 或 σ_2 固定时,在图2(a)中,分叉集是一个抛物线.

图 1 和图 2 将方程(53a)平衡点附近邻域分为不同的区域,分别对应点阵夹芯板不同的振动形式,在区域(1)中方程(53a)有一个零解;在区域(2)中方程(53a)有两个零解;在区域(3)中方程(53a)没有零解。

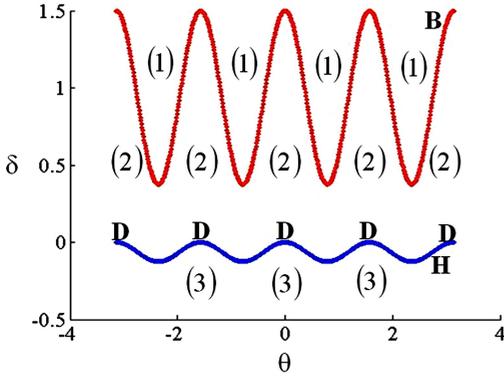


图 1 $\varepsilon_{11} = -1$ 时方程(53a)的平面转迁集

Fig. 1 Plan transition variety for Equation (53a) when $\varepsilon_{11} = -1$

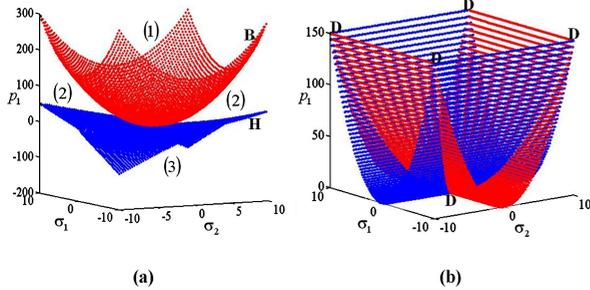


图 2 $\varepsilon_{11} = -1$ 时方程(53a)的三维转迁集

Fig.2 Three-dimensional sketch of Transitions variety for Equation (53a) when $\varepsilon_{11} = -1$

图 3 表示,当 $\varepsilon_{11} = -1, \varepsilon_{12} = 1$ 和 $\varepsilon_{13} = 1$ 时,方程(53a)的第一阶模态的力-幅响应,其对应参数值分别为:

- (a) $\sigma_1 = -3, \sigma_2 = -1, \gamma = 0$
- (b) $\sigma_1 = -1, \sigma_2 = -3, \gamma = 0$
- (c) $\sigma_1 = -3, \sigma_2 = -1, \gamma = 1$
- (d) $\sigma_1 = -1, \sigma_2 = -3, \gamma = 1$

图 4 表示,当 $\varepsilon_{11} = -1, \varepsilon_{12} = 1$ 和 $\varepsilon_{13} = 1$ 时,方程(53a)的第二阶模态的力-幅响应,其对应参数值分别为:

- (a) $\sigma_1 = -1, \sigma_2 = -3, \gamma = 0$
- (b) $\sigma_1 = -3, \sigma_2 = -1, \gamma = 0$
- (c) $\sigma_1 = -1, \sigma_2 = -3, \gamma = 1$
- (d) $\sigma_1 = -3, \sigma_2 = -1, \gamma = 1$

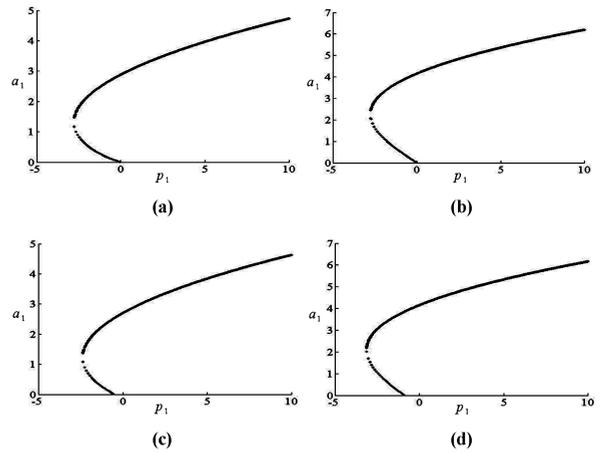


图 3 $\varepsilon_{11} = -1$ 时方程(53a)在不同参数下第一阶模态的力-幅响应曲线

Fig.3 Force-amplitude response curves of the first-order mode with different parameters for Equation (53a) when $\varepsilon_{11} = -1$

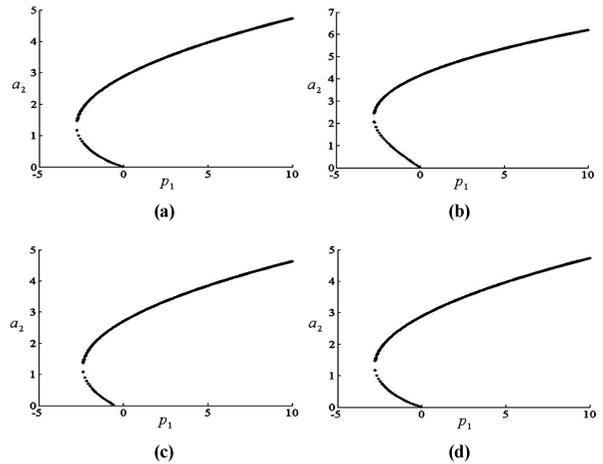


图 4 $\varepsilon_{11} = -1$ 时方程(53a)在不同参数下第二阶模态的力-幅响应曲线

Fig.4 Force-amplitude response curves of the second-order mode with different parameters for Equation (53a) when $\varepsilon_{11} = -1$

同理,图 5 和图 6 将方程(53a)平衡点附近邻域分为不同的区域,分别对应点阵夹芯板不同的振动形式,在区域(1)中方程(53a)有一个零解;在区域(2)中方程(53a)有两个零解;在区域(3)中方程(53a)没有零解。

图 7 表示,当 $\varepsilon_{11} = 1, \varepsilon_{12} = 1$ 和 $\varepsilon_{13} = 1$ 方程(53a)的第一阶模态的力-幅响应,其对应参数值分别为:

- (a) $\sigma_1 = -3, \sigma_2 = -1, \gamma = 0$
- (b) $\sigma_1 = -1, \sigma_2 = -3, \gamma = 0$
- (c) $\sigma_1 = -3, \sigma_2 = -1, \gamma = 1$
- (d) $\sigma_1 = -1, \sigma_2 = -3, \gamma = 1$

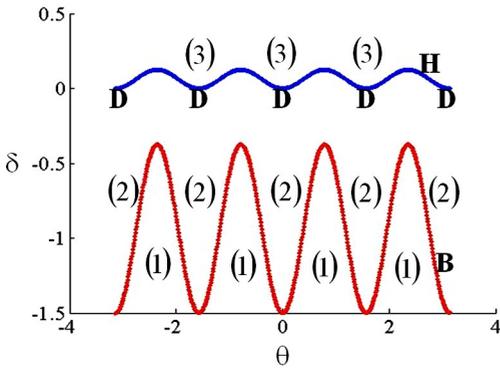


图5 $\varepsilon_{11} = 1$ 时方程(53a)的平面转迁集

Fig.5 Plan transition variety for Equation (53a) when $\varepsilon_{11} = 1$

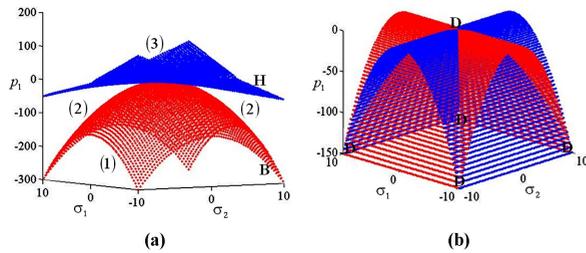


图6 $\varepsilon_{11} = 1$ 时方程(53a)的三维转迁集

Fig.6 Three-dimensional sketch of Transitions variety for Equation (53a) when $\varepsilon_{11} = 1$

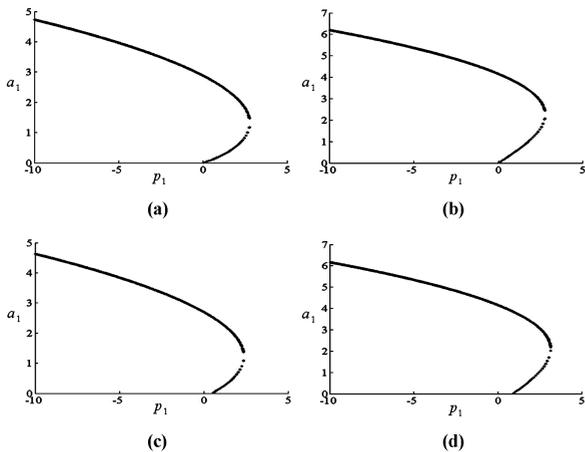


图7 $\varepsilon_{11} = 1$ 时方程(53a)在不同参数下

第一阶模态的力-幅响应曲线

Fig.7 Force-amplitude response curves of the first-order mode with different parameters for Equation (53a) when $\varepsilon_{11} = 1$

(d) $\sigma_1 = -3, \sigma_2 = -1, \gamma = 1$

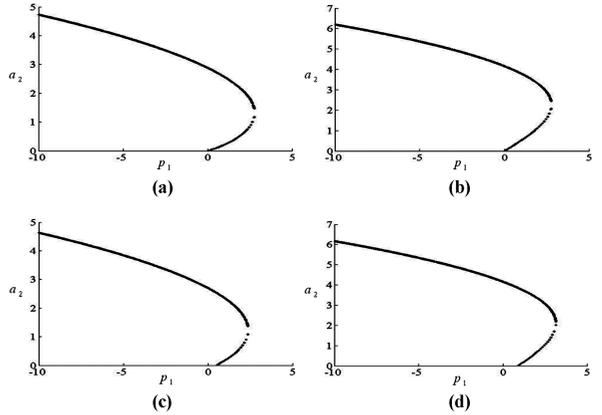


图8 $\varepsilon_{11} = 1$ 时方程(53a)在不同参数下

第二阶模态的力-幅响应曲线

Fig.8 Force-amplitude response curves of the second-order mode with different parameters for Equation (53a) when $\varepsilon_{11} = 1$

同样的方法,也可以对方程(53b)、(53c)、(53d)、(53e)和(53f)进行数值计算,针对分叉表达式,滞后表达式和双极限点集得到关于三个分叉参数 σ_1, σ_2 和 p_1 的平面转迁集和立体转迁集。

3 结论

针对 1:1 内共振情况下点阵夹芯板的非线性动力学分叉方程,推广了对于含有两个状态变量和三个及三个以上分叉参数的一般非线性动力学方程的奇异性理论,得到点阵夹芯板的非线性力学分叉方程余维 4 的 6 个普适开折的表达式,从方程(64)的表达式可以发现:当 p_1 固定时,分叉集是一个圆;当 σ_1 或 σ_2 固定时,分叉集是一个抛物线,同时,在区域(1)中,方程(53)有一个零解;在区域(2)中方程(53)有两个零解;在区域(3)中方程(53)没有零解。研究结果对理解点阵夹芯板结构的稳定性具有指导意义,并为该类构件的工程应用提供参考。

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图8表示,当 $\varepsilon_{11} = 1, \varepsilon_{12} = 1$ 和 $\varepsilon_{13} = 1$ 时,方程(53a)的第二阶模态的力-幅响应,其对应参数值分别为:

- (a) $\sigma_1 = -1, \sigma_2 = -3, \gamma = 0$
- (b) $\sigma_1 = -3, \sigma_2 = -1, \gamma = 0$
- (c) $\sigma_1 = -1, \sigma_2 = -3, \gamma = 1$

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SINGULARITY ANALYSIS ON DYNAMICS OF LIGHT-WEIGHT SANDWICHPLATE UNDER 1:1 RESONANCE*

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Abstract In this paper, the singularity theory is utilized to investigate 1:1 resonant bifurcations of the symmetric cross-ply composite laminated plates with two detuning parameters and an in-plane excitations. Based on the averaged equation, the restricted tangent space is obtained for the bifurcation equations with two detuning parameters and an in-plane excitations. The singularity theory is developed for the general nonlinear dynamic equation with the two state variables and four parameters. The universal unfoldings of bifurcation equation with codimension 4 are then obtained in the case of 1:1 internal resonance. The transition sets in the parameter plane and the bifurcation diagrams are depicted. The relationships among two detuning parameters and an in-plane excitations are determined when the bifurcation, hysteresis and double limit point occur. The numerical results also indicate that the number of solutions in different bifurcated regions is different.

Key words bifurcation equations, singularity theory, universal unfoldings, transition sets

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