悬臂式蜂窝夹层板的非线性动力学建模及分析*

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摘要　蜂窝夹层结构因其良好的力学特性,在众多工程领域具有非常广泛的应用.本文建立了悬臂边界条件下,蜂窝夹层板的动力学模型并研究其非线性动力学行为.选取文献中更加接近实体有限元解的等效弹性参数公式对蜂窝芯层进行等效简化,得到六角形蜂窝芯的等效弹性参数.基于 Reddy 高阶剪切变形理论,应用 Hamilton 原理建立悬臂式蜂窝夹层板在受到面内激励和横向激励联合作用下的偏微分运动方程.然后利用 Galerkin 方法得到两自由度非自治常微分形式运动方程.在此基础上,通过对悬臂式蜂窝夹层板进行数值模拟分析系统的非线性动力学.结果表明面内激励和横向激励对系统的动力学特性有着重要影响,在不同激励作用下系统会出现周期运动、概周期运动以及混沌运动等复杂的非线性动力学响应.

关键词　蜂窝夹层板,　悬臂,　非线性动力学,　周期,　混沌

DOI:　10.6052/1672-6553-2017-5

引言

蜂窝夹层结构具有重量轻、高比刚度和高比强度以及良好的结构稳定性和能量吸收性等优越的性能,因而广泛应用于现代工业制造的各个方面.在航空航天工业中,蜂窝夹层结构大量用于飞机的机翼、雷达罩、机舱、尾翼、升降舵和储物箱等部位.蜂窝芯夹层板由较薄的上下蒙皮和较厚中间芯层组成.它的芯层是由金属材料、纸质材料或者其它材料制成的六边形孔格,蒙皮在芯层的上下两面胶结或焊接.蜂窝夹层板的结构如图 1 所示.

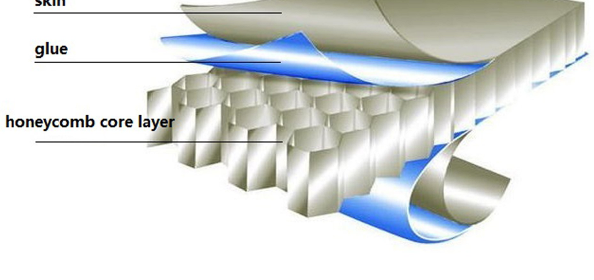

图 1　蜂窝夹板结构示意图
Fig. 1　Structure of honeycomb sandwich plate

蜂窝芯层等效是蜂窝夹层结构相关研究的前提和基础.1969 年,Allen[1]针对蜂窝芯层等效提出了一种忽略芯层面内刚度和弯曲刚度的假设,认为芯层仅能抵抗横向剪切力,极大地简化了受力分析,这种假设在早期的工程中应用非常广泛.1982 年,Gibson[2]等对采用欧拉伯努利梁理论,利用材料力学公式推导出等壁厚正六角形蜂窝芯层的二维等效弹性参数公式.1999 年,富明慧[3]等考虑蜂窝壁版的伸缩变形对面内刚度的影响,提出了一种考虑蜂窝芯层面内刚度的简化方案,克服了 Gibson 公式的缺陷.2001 年,Kim[4]等开发了一个非均匀支撑的柱状结构的常规三维各向异性模型,通过该模型来研究二维六角形、三维六角形和菱形蜂窝材料柱状结构的力学特性.2008 年,祝涛[5]等考虑面内载荷对蜂窝芯层等效弹性模量的影响,拟合了非线性等效弹性参数.同年,孙德强[6]等将铝制蜂窝孔壁视为纤细梁,在考虑弯曲和伸缩变形的基础上利用 Timoshenko 梁理论处理蜂窝孔壁的剪切变形,推导出了与有限元结果更加接近的双壁厚一般六角形蜂窝芯层的面内等效弹性参数公式.2011 年,陈玳珩[7]等提出了蜂窝芯层和蒙皮在位移连续

2016-10-08 收到第 1 稿,2016-11-03 收到修改稿.
* 国家自然科学基金资助项目(11472057),北京市教委科技计划面上项目(KM201711232002)
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性条件下的等效弹性模量的理论分析的新的计算方法,并与有限元数值分析相比较验证其准确性.2012年,陈梦成[8]等提出了以蜂窝芯正六角形胞元壁板弯曲和扭转为基础的蜂窝夹层板的计算方法.2015年,富明慧[9]等基于Timoshenko梁理论,利用文献[6]的计算方法,推导出了一般六角形等壁厚蜂窝芯的面内等效弹性参数公式.

2013年,Motley[10]等研究了全部和部分浸没的悬臂复合板的边界条件对自由振动响应的影响以及这些影响是如何根据材料属性而变化的.同年,Hao[11]等研究了热环境下受到横向和静态面内预加激励的功能梯度悬臂圆柱壳的非线性动力学行为.同年,杜长城[12]等采用Galerkin法和平均法研究了四边简支条件下仅受到横向简谐激励作用的功能梯度薄壁板的非线性动力学响应.同年,Wang[13]等研究了热环境下置于弹性地基上的具有功能梯度表层的复合板的非线性动力学响应.2014年,Zhang[14]等研究了同时受到横向激励和面内激励作用的简支边界条件下,空间构架点阵夹芯层合板的非线性动力学响应.同年,Zhang[15]等研究了横向气动载荷和参数激励联合作用下复合材料悬臂外伸矩形板在伸出过程中的非线性动力学问题.2015年,Ta[16]等利用改进板理论分析了置于弹性基地上的功能梯度板的动力学响应.2016年,Azarboni[17]等研究了非理想矩形板在六种边界条件下激励频率对非线性动态脉冲屈曲的影响.同年,Parandvar[18]等使用有限元方法研究了受到热和谐波负载下功能梯度扁壳的非线性动力学响应.

综上所述,国内外很多学者对不同边界条件下蜂窝夹层板的非线性动力学特性进行了大量研究,但对于复杂载荷作用下悬臂式蜂窝夹层板的非线性动力学响应的研究相对较少.另外,多数文献在蜂窝芯层等效时采用了文献[2]给出的Gibson公式,本文采用了更加接近有限元实体单元的等效弹性参数公式对蜂窝芯层进行等效简化,以悬臂式矩形蜂窝夹层板为研究对象,考虑面内激励和横向外激励的联合作用以及阻尼等对系统的影响,基于Reddy高阶剪切变形理论,应用Hamilton原理建立悬臂蜂窝板的动力学控制方程.利用Galerkin方法得到该系统的常微分形式的非线性动力学方程,根据工程实际背景选取不同的参数,直接对所得系统进行数值模拟和对比分析.

1 蜂窝夹层板的力学模型

以飞机的机翼振动为实际工程背景,考虑悬臂边界条件下矩形蜂窝芯夹层板,模型如图2所示,矩形蜂窝夹层板的ob边被固定,其余三边自由,x方向边长为a,y方向边长为b,板总厚为H,平面直角坐标系xOy位于蜂窝夹层板的中性面内,z轴竖直向下并垂直于xOy面.假设蜂窝板受到横向的简谐外激励为$F=F_0\cos\Omega t$以及面内简谐激励为$P=P_0+P_1\cos\Omega_1 t$,并且考虑横向阻尼γ的影响.蜂窝夹层板的上下蒙皮厚度均为h_f,正六角形的蜂窝芯层厚度为h_c.

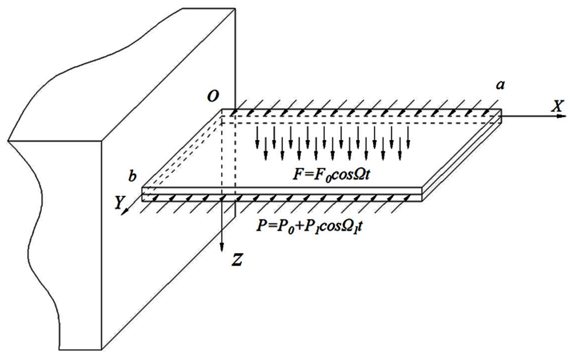


图2 悬臂式蜂窝夹层板模型示意图

Fig. 2 Model of cantilever honeycomb sandwich plate

由于蜂窝夹层板的蒙皮很薄并且与蜂窝芯层紧密粘结,为了计算方便,我们忽略蒙皮厚度,将蜂窝芯层进行等效简化.一般六角形蜂窝芯层的结构单元结构如图3所示.其中d为蜂窝单元壁板的厚度,h、l分别为蜂窝单元的直壁板和斜壁板长度.

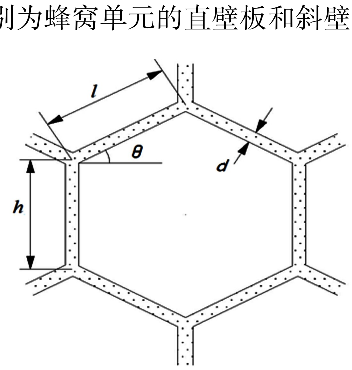


图3 一般六角形蜂窝芯层结构单元

Fig. 3 Unit cell of general hexagonal core layer

文献[6,9]推导出的更加接近实体有限元解的等壁厚一般六角形蜂窝芯层的面内等效弹性参数公式为如下形式:

$$E_1=E_s\frac{d^3}{l^3}\frac{(\cos\theta+d/l)}{(h/l+\sin\theta)\sin^2\theta[1+(2.4+1.5v_s+\cot^2\theta)d^2/l^2]} \tag{1a}$$

$$v_{12}=\frac{\cos\theta(\cos\theta+d/l)[1+(1.4+1.5v_s)d^2/l^2]}{(h/l+\sin\theta)\sin\theta[1+(2.4+1.5v_s+\cot^2\theta)d^2/l^2]} \tag{1b}$$

$$E_2=E_s\frac{d^3}{l^3}\frac{(h/l+\sin\theta)}{\cos^2\theta(\cos\theta+d/l)[1+(2.4+1.5v_s+\tan^2\theta+2(h/l)\sec^2\theta)d^2/l^2]} \tag{1c}$$

$$v_{21}=\frac{(h/l+\sin\theta)\sin\theta[1+(1.4+1.5v_s)d^2/l^2]}{\cos\theta(\cos\theta+d/l)[1+(2.4+1.5v_s+\tan^2\theta+2(h/l)\sec^2\theta)d^2/l^2]} \tag{1d}$$

其中 E_1、E_2 分别为蜂窝芯层在 x、y 方向上的弹性模量;v_{12}、v_{21} 分别为蜂窝芯层在 x、y 方向上的泊松比;E_s 为蜂窝芯基体材料的弹性模量,v_s 为蜂窝芯基体材料本身的泊松比.由于这组公式同时考虑剪切、拉伸和弯曲的影响,并且与实体有限元模型更加接近,因此本文选择公式(1)作为六角形蜂窝夹层板的等效弹性参数公式.

2　臂蜂窝夹层板的动力学方程

根据 Reddy 的三阶剪切变形理论,蜂窝夹层板的位移场可以写为如下形式[19]:

$$u(x,y,z,t)=u_0(x,y,t)+z\varphi_x(x,y,t)-\frac{4}{3h^2}z^3\left(\varphi_x+\frac{\partial w_0}{\partial x}\right) \tag{2a}$$

$$v(x,y,z,t)=v_0(x,y,t)+z\varphi_y(x,y,t)-\frac{4}{3h^2}z^3\left(\varphi_y+\frac{\partial w_0}{\partial y}\right) \tag{2b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{2c}$$

其中 u_0、v_0、w_0 为板中性面在 x、y、z 方向的位移,φ_x、φ_y 分别为中性面的法线对于 x、y 轴的转角,h 为板的厚度.

非线性应变位移关系如下:

$$\varepsilon_{xx}=\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2$$

$$\varepsilon_{yy}=\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2$$

$$\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right)$$

$$\varepsilon_{yz}=\frac{1}{2}\left(\frac{\partial v_0}{\partial z}+\frac{\partial w_0}{\partial y}\right)$$

$$\varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial u_0}{\partial z}+\frac{\partial w_0}{\partial x}\right)$$

$$\varepsilon_{zz}=\frac{\partial w_0}{\partial z} \tag{3}$$

将(3)式代入(2)式可以得到位移形式的应变表达式为如下(4)式:

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix}+z\begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix}+z^3\begin{Bmatrix}\varepsilon_{xx}^{(3)}\\ \varepsilon_{yy}^{(3)}\\ \gamma_{xy}^{(3)}\end{Bmatrix}$$

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\begin{Bmatrix}\gamma_{yz}^{(0)}\\ \gamma_{xz}^{(0)}\end{Bmatrix}+z^2\begin{Bmatrix}\gamma_{yz}^{(2)}\\ \gamma_{xz}^{(2)}\end{Bmatrix} \tag{4}$$

上式中:

$$\begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\\ \frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\end{Bmatrix}$$

$$\begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial \varphi_x}{\partial x}\\ \frac{\partial \varphi_y}{\partial y}\\ \frac{\partial \varphi_x}{\partial y}+\frac{\partial \varphi_y}{\partial x}\end{Bmatrix},\ \begin{Bmatrix}\gamma_{yz}^{(0)}\\ \gamma_{xz}^{(0)}\end{Bmatrix}=\begin{Bmatrix}\varphi_y+\frac{\partial w_0}{\partial y}\\ \varphi_x+\frac{\partial w_0}{\partial x}\end{Bmatrix}$$

$$\begin{Bmatrix}\varepsilon_{xx}^{(3)}\\ \varepsilon_{yy}^{(3)}\\ \gamma_{xy}^{(3)}\end{Bmatrix}=-c_1\begin{Bmatrix}\frac{\partial \varphi_x}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\\ \frac{\partial \varphi_y}{\partial y}+\frac{\partial^2 w_0}{\partial y^2}\\ \frac{\partial \varphi_x}{\partial y}+\frac{\partial \varphi_y}{\partial x}+2\frac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}$$

$$\begin{Bmatrix}\gamma_{yz}^{(2)}\\ \gamma_{xz}^{(2)}\end{Bmatrix}=-c_2\begin{Bmatrix}\varphi_y+\frac{\partial w_0}{\partial y}\\ \varphi_x+\frac{\partial w_0}{\partial x}\end{Bmatrix},\ c_1=\frac{4}{3h^2},\ c_2=\frac{4}{h^2} \tag{5}$$

根据 Hamilton 原理建立蜂窝夹层板的非线性

动力学方程为如下形式：

$$\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot{u}_0+(I_1-c_1I_3)\ddot{\varphi}_x-c_1I_3\frac{\partial\ddot{w}_0}{\partial x} \tag{6a}$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=I_0\ddot{v}_0+(I_1-c_1I_3)\ddot{\varphi}_y-c_1I_3\frac{\partial\ddot{w}_0}{\partial y} \tag{6b}$$

$$\begin{aligned}&\frac{\partial N_{xx}}{\partial x}\frac{\partial w_0}{\partial x}+N_{xx}\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial N_{xy}}{\partial x}\frac{\partial w_0}{\partial y}+2N_{xy}\frac{\partial^2 w_0}{\partial x\partial y}+\\&\frac{\partial N_{xy}}{\partial y}\frac{\partial w_0}{\partial x}+\frac{\partial N_{yy}}{\partial y}\frac{\partial w_0}{\partial y}+N_{yy}\frac{\partial^2 w_0}{\partial y^2}+(\frac{\partial Q_x}{\partial x}-c_2\frac{\partial R_x}{\partial x})+\\&(\frac{\partial Q_y}{\partial y}-c_2\frac{\partial R_y}{\partial y})+c_1(\frac{\partial^2 P_{xx}}{\partial x^2}+2\frac{\partial^2 P_{xy}}{\partial x\partial y}+\frac{\partial^2 Pyy}{\partial y^2})+\\&F_0\cos\Omega t-\gamma\dot{w}_0\\&=I_0\ddot{w}_0-c_1^2I_6(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2})+c_1I_3(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y})+\\&\quad c_1(I_4-c_1I_6)(\frac{\partial\ddot{\varphi}_x}{\partial x}+\frac{\partial\ddot{\varphi}_y}{\partial y})\end{aligned} \tag{6c}$$

$$\begin{aligned}&\frac{\partial M_{xx}}{\partial x}+\frac{\partial M_{xy}}{\partial y}-c_1\frac{\partial P_{xx}}{\partial x}-c_1\frac{\partial P_{xy}}{\partial y}-(Q_x-c_2R_x)\\&=(I_1-c_1I_3)\ddot{u}_0+(I_2-2c_1I_4+c_1^2I_6)\ddot{\varphi}_x-\\&\quad c_1(I_4-c_1I_6)\frac{\partial\ddot{w}_0}{\partial x}\end{aligned} \tag{6d}$$

$$\begin{aligned}&\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_{yy}}{\partial y}-c_1\frac{\partial P_{xy}}{\partial x}-c_1\frac{\partial P_{yy}}{\partial y}-(Q_y-c_2R_y)\\&=(I_1-c_1I_3)\ddot{v}_0+(I_2-2c_1I_4+c_1^2I_6)\ddot{\varphi}_y-\\&\quad c_1(I_4-c_1I_6)\frac{\partial\ddot{w}_0}{\partial y}\end{aligned} \tag{6e}$$

其中应力的合力与应变的关系表示为如下形式：

$$\begin{Bmatrix}N_{xx}\\N_{yy}\\N_{xy}\end{Bmatrix}=\begin{bmatrix}A_{11}&A_{12}&0\\A_{12}&A_{22}&0\\0&0&A_{66}\end{bmatrix}\begin{Bmatrix}\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\\\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\\\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\end{Bmatrix} \tag{7a}$$

$$\begin{aligned}\begin{Bmatrix}M_{xx}\\M_{yy}\\M_{xy}\end{Bmatrix}=&\begin{bmatrix}D_{11}&D_{12}&0\\D_{21}&D_{22}&0\\0&0&D_{66}\end{bmatrix}\begin{Bmatrix}\frac{\partial\varphi_x}{\partial x}\\\frac{\partial\varphi_y}{\partial y}\\\frac{\partial\varphi_x}{\partial y}+\frac{\partial\varphi_y}{\partial x}\end{Bmatrix}-\\&c_1\begin{bmatrix}F_{11}&F_{12}&0\\F_{21}&F_{22}&0\\0&0&F_{66}\end{bmatrix}\begin{Bmatrix}\frac{\partial\varphi_x}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\\\frac{\partial\varphi_y}{\partial y}+\frac{\partial^2 w_0}{\partial y^2}\\\frac{\partial\varphi_x}{\partial y}+\frac{\partial\varphi_y}{\partial x}+2\frac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}\end{aligned} \tag{7b}$$

$$\begin{aligned}\begin{Bmatrix}P_{xx}\\P_{yy}\\P_{xy}\end{Bmatrix}=&\begin{bmatrix}F_{11}&F_{12}&0\\F_{21}&F_{22}&0\\0&0&F_{66}\end{bmatrix}\begin{Bmatrix}\frac{\partial\varphi_x}{\partial x}\\\frac{\partial\varphi_y}{\partial y}\\\frac{\partial\varphi_x}{\partial y}+\frac{\partial\varphi_y}{\partial x}\end{Bmatrix}-\\&c_1\begin{bmatrix}H_{11}&H_{12}&0\\H_{21}&H_{22}&0\\0&0&H_{66}\end{bmatrix}\begin{Bmatrix}\frac{\partial\varphi_x}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\\\frac{\partial\varphi_y}{\partial y}+\frac{\partial^2 w_0}{\partial y^2}\\\frac{\partial\varphi_x}{\partial y}+\frac{\partial\varphi_y}{\partial x}+2\frac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}\end{aligned} \tag{7c}$$

$$\begin{aligned}\begin{Bmatrix}Q_x\\Q_y\end{Bmatrix}=&\begin{bmatrix}A_{44}&0\\0&A_{55}\end{bmatrix}\begin{Bmatrix}\varphi_y+\frac{\partial w_0}{\partial y}\\\varphi_x+\frac{\partial w_0}{\partial x}\end{Bmatrix}-\\&c_2\begin{bmatrix}D_{44}&0\\0&D_{55}\end{bmatrix}\begin{Bmatrix}\varphi_y+\frac{\partial w_0}{\partial y}\\\varphi_x+\frac{\partial w_0}{\partial x}\end{Bmatrix}\end{aligned} \tag{7d}$$

$$\begin{aligned}\begin{Bmatrix}R_x\\R_y\end{Bmatrix}=&\begin{bmatrix}D_{44}&0\\0&D_{55}\end{bmatrix}\begin{Bmatrix}\varphi_y+\frac{\partial w_0}{\partial y}\\\varphi_x+\frac{\partial w_0}{\partial x}\end{Bmatrix}-\\&c_2\begin{bmatrix}F_{44}&0\\0&F_{55}\end{bmatrix}\begin{Bmatrix}\varphi_y+\frac{\partial w_0}{\partial y}\\\varphi_x+\frac{\partial w_0}{\partial x}\end{Bmatrix}\end{aligned} \tag{7e}$$

上式中：

$$(A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij})=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}Q_{ij}^k(1,z,z^2,z^3,z^4,z^6)dz,\ (i,j=1,2,6)$$

$$(A_{ij},D_{ij},F_{ij})=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}Q_{ij}^k(1,z^2,z^4)dz,\ (i,j=4,5)$$

悬臂式蜂窝夹层板的边界条件为：

$$x=0: w=v=u=\varphi_x=\varphi_y=0 \tag{8a}$$

$$x=a: N_{xy}=M_{xx}=M_{xy}-c_1P_{xy}=\overline{Q}_x=0 \tag{8b}$$

$$y=0: N_{yy}=N_{xy}=M_{yy}=M_{xy}-c_1P_{xy}=\overline{Q}_y=0 \tag{8c}$$

$$y=b: N_{yy}=N_{xy}=M_{yy}=M_{xy}-c_1P_{xy}=\overline{Q}_y=0 \tag{8d}$$

$$\int_0^a N_{xx}\mid_{x=0,a} dy=\int_0^a (p_0+p_1\cos\Omega_2 t)dy \tag{8e}$$

其中等效剪力可以表达为：

$$\overline{Q}_x=Q_x+\frac{\partial M_{xy}}{\partial y}-c_2R_x+c_1\left(\frac{\partial P_{xx}}{\partial x}+\frac{\partial P_{xy}}{\partial y}\right)$$

$$\overline{Q}_y=Q_y+\frac{\partial M_{xy}}{\partial x}-c_2R_y+c_1\left(\frac{\partial P_{yy}}{\partial y}+\frac{\partial P_{xy}}{\partial x}\right)$$

取横向位移 w 的模态函数为如下形式：

$$w_0=w_1(t)X_1(x)Y_1(y)+w_2(t)X_2(x)Y_1(y) \tag{9}$$

其中：

$$X_i(x)=\sin\lambda_i x-\sinh\lambda_i x+\alpha_i(\cosh\lambda_i x-\cos\lambda_i x)$$

$$Y_j(y)=\sin\mu_j y+\sinh\mu_j y-\beta_j(\cosh\mu_j y+\cos\mu_j y)$$

$$\cos\lambda_i a\cosh\lambda_i a+1=0,\ \cos\mu_j b\cosh\mu_j b-1=0$$

$$\alpha_i=\frac{\sinh\lambda_i a+\sin\lambda_i a}{\cosh\lambda_i a+\cos\lambda_i a}$$

$$\beta_j=\frac{\sinh\mu_j b-\sin\mu_j b}{\cosh\mu_j b-\cos\mu_j b}$$

悬臂式蜂窝夹层板的主要振动形式为横向振动，因此很多文献在研究悬臂边界条件下板的振动时仅考虑它的横向位移，本文为了更加准确地描述蜂窝夹层板的非线性振动，综合考虑面内振动和横向振动，引入其他方向的模态函数为如下形式：

$$u_0=u_1(t)\frac{\partial X_1(x)}{\partial x}Y_1(y)+u_2(t)\frac{\partial X_2(x)}{\partial x}Y_1(y) \tag{10a}$$

$$v_0=v_1(t)X_1(x)\frac{\partial Y_1(y)}{\partial y}+v_2(t)X_2(x)\frac{\partial Y_1(y)}{\partial y} \tag{10b}$$

$$\varphi_x=\varphi_{x1}(t)\frac{\partial X_1(x)}{\partial x}Y_1(y)+\varphi_{x2}(t)\frac{\partial X_2(x)}{\partial x}Y_1(y) \tag{10d}$$

$$\varphi_y=\varphi_{y1}(t)X_1(x)\frac{\partial Y_1(y)}{\partial y}+\varphi_{y2}(t)X_2(x)\frac{\partial Y_1(y)}{\partial y} \tag{10d}$$

设横向激励的表达式为如下形式：

$$F_0(t)=F_1(t)X_1(x)Y_1(y)+F_2(t)X_2(x)Y_1(y) \tag{11}$$

根据 Galerkin 法，将所有模态函数式(9)、(10)以及(11)式分别代入相应的偏微分方程(6a)～(6e)，然后在等式的两边乘以相应的模态函数部分并在整个板内积分，并忽略 u,v,φ_x,φ_y 方向的惯性项，可以得到悬臂边界条件下蜂窝夹层板的两自由度非线性动力学常微分方程为如下形式：

$$\ddot{w}_1-a_7\gamma\dot{w}_1-a_5w_1-a_6w_2-a_9(P_0+P_1\cos\Omega_1 t)w_1-a_1w_1^3-a_2w_1^2w_2-a_3w_1w_2^2-a_4w_2^3=a_8F_1\cos\Omega t \tag{12a}$$

$$\ddot{w}_2-b_7\gamma\dot{w}_2-b_5w_1-b_6w_2-b_9(P_0+P_1\cos\Omega_1 t)w_2-b_1w_1^3-b_2w_1^2w_2-b_3w_1w_2^2-b_4w_2^3=b_8F_2\cos\Omega t \tag{12b}$$

3 数值模拟

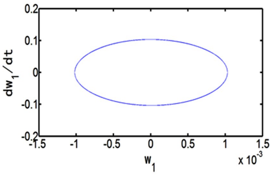
本节利用 Runge-Kutta 方法直接对悬臂式蜂窝夹层板的两自由度非线性动力学方程(12)进行数值模拟，分析激励和阻尼对系统非线性振动的影响. 铝合金矩形蜂窝夹层板的长 $a=5\text{m}$，宽 $b=2\text{m}$，蜂窝芯层厚度 $h_c=0.01\text{m}$，材料基体的泊松比 $v_s=0.33$，芯层基体密度和弹性模量分别为 $\rho_s=2.66\times10^3\text{kg/m}^3$ 和 $E_s=72\times10^9\text{Pa}$，正六角形芯层壁板厚度和边长分别为 $d=0.0008\text{m}$ 和 $l=0.01\text{m}$. 横向激励和面内激励的频率为 $\Omega=\Omega_1=100\text{Hz}$，阻尼为 $\gamma=150\text{N}\cdot\text{s/m}$，经计算得方程(12)中各系数取如下值：$a_1=6.99\times10^9$，$a_2=-3.36\times10^{10}$，$a_3=4.88\times10^{10}$，$a_4=-1.50\times10^{10}$，$a_5=-120.09$，$a_6=-3161.98$，$a_7=-0.41$，$a_8=-0.41$，$a_9=0.93$，$b_1=1.87\times10^8$，$b_2=1.14\times10^{10}$，$b_3=-2.23\times10^{10}$，$b_4=1.09\times10^{10}$，$b_5=731.85$，$b_6=-4948.91$，$b_7=-0.41$，$b_8=-0.41$，$b_9=0.93$.

当横向激励为 $F_1=27\text{Pa}$ 和 $F_2=15\text{Pa}$，面内激励为 $P_0=11\text{Pa}$ 和 $P_1=8\text{Pa}$，系统出现周期运动如图 4 所示.

当横向激励为 $F_1=25\text{Pa}$ 和 $F_2=10\text{Pa}$，面内激励为 $P_0=25\text{Pa}$ 和 $P_1=12\text{Pa}$，系统出现 3 倍周期运动如图 5 所示.

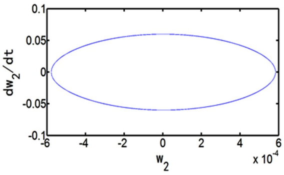
保持横向激励为 $F_1=25\text{Pa}$ 和 $F_2=10\text{Pa}$，减小面内激励为 $P_0=11\text{Pa}$ 和 $P_1=8\text{Pa}$，系统出现概周期运动如图 6 所示.

在保持面内激励为 $P_0=11.2\text{Pa}$ 和 $P_1=4.2\text{Pa}$ 的同时，减小横向激励为 $F_1=-4.9\text{Pa}$ 和 $F_2=1.5\text{Pa}$ 时，系统出现混沌运动如图 7 所示.



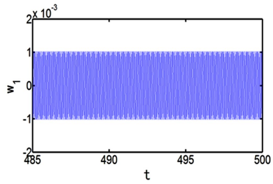
(a) $(w_1, \dot{w}_1)$ 二维相图

(a) Phase portrait in the plane of $(w_1, \dot{w}_1)$



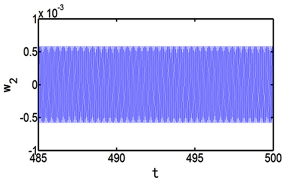
(b) $(w_2, \dot{w}_2)$ 二维相图

(b) Phase portrait in the plane of $(w_2, \dot{w}_2)$



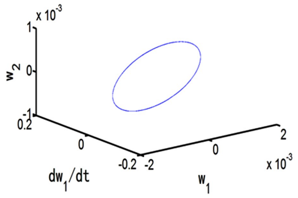
(c) (t, w_1) 波形图

(c) Wave form of w_1



(d) (t, w_2) 波形图

(d) Wave form of w_2

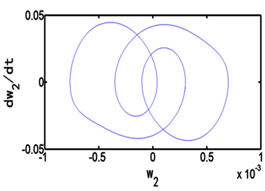


(e) $(w_1, \dot{w}_1, w_2)$ 三维相图

(e) Three-dimensional phase portrait of $(w_1, \dot{w}_1, w_2)$

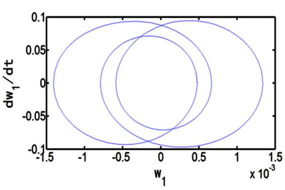
图4 周期运动

Fig. 4 Periodic motion of the cantilever honeycomb sandwich plate



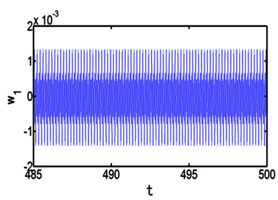
(a) $(w_1, \dot{w}_1)$ 二维相图

(a) Phase portrait in the plane of $(w_1, \dot{w}_1)$



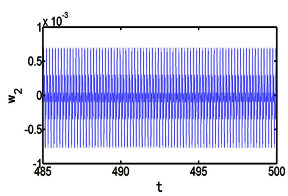
(b) $(w_2, \dot{w}_2)$ 二维相图

(b) Phase portrait in the plane of $(w_2, \dot{w}_2)$



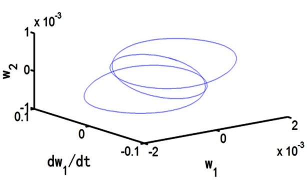
(c) (t, w_1) 波形图

(c) Wave form of w_1



(d) (t, w_2) 波形图

(d) Wave form of w_2

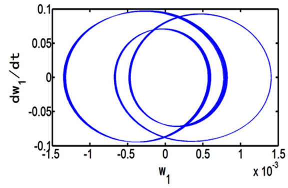


(e) 三维相图

(e) Three-dimensional phase portrait

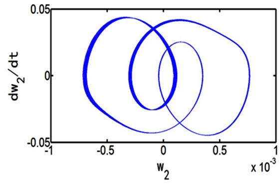
图5 3倍周期运动

Fig. 5 Three-periodic motion of the cantilever honeycomb sandwich plate



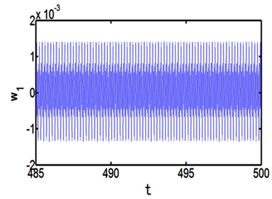
(a) $(w_1, \dot{w}_1)$ 二维相图

(a) Phase portrait in the plane of $(w_1, \dot{w}_1)$



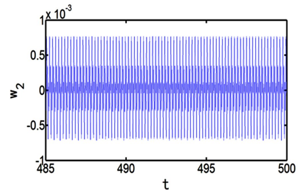
(b) $(w_2, \dot{w}_2)$ 二维相图

(b) Phase portrait in the plane of $(w_2, \dot{w}_2)$



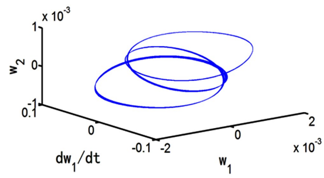
(c) (t, w_1) 波形图

(c) Wave form of w_1



(d) (t, w_2) 波形图

(d) Wave form of w_2

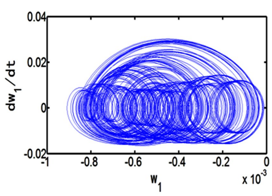


(e) $(w_1, \dot{w}_1, w_2)$ 三维相图

(e) Three-dimensional phase portrait of $(w_1, \dot{w}_1, w_2)$

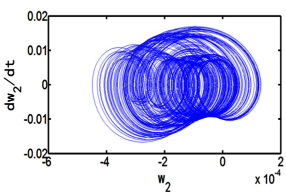
图6 概周期运动

Fig. 6 Quasi-periodic motion of the cantilever honeycomb sandwich plate



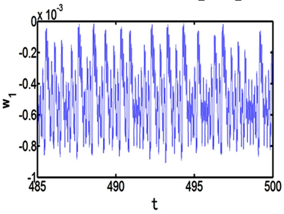
(a) $(w_1, \dot{w}_1)$ 二维相图

(a) Phase portrait in the plane of $(w_1, \dot{w}_1)$



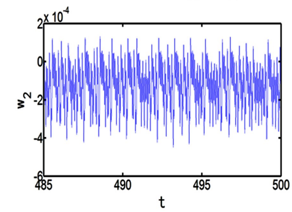
(b) $(w_2, \dot{w}_2)$ 二维相图

(b) Phase portrait in the plane of $(w_2, \dot{w}_2)$



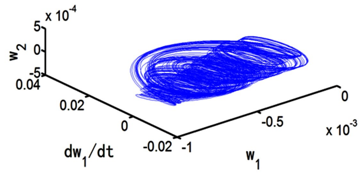
(c) (t, w_1) 波形图

(c) Wave form of w_1



(d) (t, w_2) 波形图

(d) Wave form of w_2



(e) $(w_1, \dot{w}_1, w_2)$ 三维相图

(e) Three-dimensional phase portrait of $(w_1, \dot{w}_1, w_2)$

图7 混沌运动

Fig. 7 Chaotic motion of the cantilever honeycomb sandwich plate

4 结论

本文以悬臂边界条件下的矩形蜂窝夹层板作为研究对象,基于 Reddy 高阶剪切变形理论,运用 Hamilton 原理和 Galerkin 方法得到受到面内激励和横向激励联合作用下振动系统的常微分形式的运动方程.通过对悬臂式蜂窝夹层板的非线性振动进行数值模拟,分析在不同激励作用下系统展现出的非线性动力学行为.

在确定悬臂式蜂窝夹层板的材料属性和几何形状等初始参数的情况下,通过数值分析方法得到系统的二维相图、波形图和三维相图.数值模拟表明,随着外激励和面内激励的变化,系统会出现周期运动、多倍周期运动、概周期运动和混沌等多种运动形式.由此可见,激励是影响系统非线性动力学行为的重要因素之一,改变外激励的幅值可以对悬臂式蜂窝夹层板的非线性动力学行为产生较大影响.本文所得结果将对于飞机机翼的减振设计提供一定的指导.

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NONLINEAR DYNAMIC MODELING AND ANALYSIS FOR A CANTILEVER HONEYCOMB SANDWICH PLATE*

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Abstract Honeycomb sandwich structures have been widely used in many engineering fields because of their excellent mechanical properties. The formulas for the cantilever honeycomb sandwich plate are derived, and the nonlinear vibrations of the plate are given in this paper. In order to obtain the equivalent elastic parameters of the hexagonal core layer in the honeycomb sandwich plate, the equivalent elastic parameters that more closer to the finite element solutions for the cores are selected. Based on the Reddy′s third-order shear deformation theory, the nonlinear partial differential equations of motion are derived for the composite laminated cantilever plate subjected to in-plane and transverse excitations by using the Hamilton´s principle. The Galerkin method is then used to transform the nonlinear partial differential equations of motion to a two-degree-of-freedom nonlinear system ordinary differential equation of motion. The numerical method is also utilized to examine the nonlinear dynamic responses of the cantilever honeycomb sandwich plate. The results show that in-plane and transverse excitations have an important influence on nonlinear dynamic characteristics, and periodic, multi-periodic, quasi-periodic motions and chaotic motions all occur for the system with the change of forcing loads.

Key words honeycomb sandwich plate, cantilever, nonlinear dynamics, periodic motions, chaos

Recived 08 October 2016, revised 03 November 2016.

* The project supported by the National Natural Science Foundation of China(11472057) and the Beijing Municipal Education Commission Foundation (KM201711232002).

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