粘弹性地基上损伤弹性 Timoshenko 梁动力学行为

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摘要 建立了粘弹性地基上损伤弹性 Timoshenko 梁在有限变形情况下的运动微分方程，这是一组非线性偏微分方程. 为了便于分析，首先利用 Galerkin 方法对该方程组进行简化，得到一组非线性常微分方程. 然后利用 Matlab 软件进行数值模拟，考察了载荷参数、地基粘性参数和弹性参数、损伤对梁振动的影响. 采用非线性动力学中的各种数值方法，如时程曲线、相平面图、Poincare 截面和分叉图，发现增大地基的粘弹性参数，有利于增强结构运动的稳定性，而损伤会降低结构运动的稳定性.

关键词 粘弹性地基， 损伤， Timoshenko 梁， 非线性动力学

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引言

材料在使用过程中会发生损伤，损伤累积到一定程度会造成材料的失效破坏，对结构的安全和可靠性的研究早已引起国内外力学工作者的普遍重视. Pellicano 和 Vestroni[1] 用 Galerkin 截断法分析了带有几何非线性项的稳定运动梁在亚临界及超临界速度时的动态特性，发现了超临界速度状态下系统存在稳定区域的现象. Nunziato J W 和 Cowin S C[2] 提出了带孔隙的弹性材料的非线性理论，建立了带孔隙材料的理论框架，后经线性化发展成为可以用于工程计算的线性理论. 盛冬发[3] 从考虑损伤的粘弹性材料的一种卷积本构关系出发，建立了在有限变形下损伤粘弹性 Timoshenko 梁的运动微分方程. 孟红磊[4] 研究了含损伤非线性粘弹性本构模型及数值仿真应用，提出了一种含累积损伤的非线性粘弹性本构方程. 李晶晶[5] 对有限变形条件下，Timoshenko 粘弹性梁非线性分析的数学模型应用微分求积方法进行空域的离散，得到了简支粘弹性梁的简化模型. 唐有绮[6] 研究了轴向加速粘弹性 Timoshenko 梁的非线性参数振动，描述了各参数对稳态响应的影响.

本文从损伤线弹性理论出发，建立了粘弹性地基上损伤弹性 Timoshenko 梁的运动微分方程. 应用 Galerkin 方法和非线性动力学数值分析方法，在数值上分析了粘弹性地基上损伤弹性 Timoshenko 梁丰富的动力学行为. 分析比较了载荷参数，地基粘性参数和弹性参数，损伤对梁的动力学行为的影响，以及地基粘弹性参数对结构损伤增量的影响.

1 损伤弹性 Timoshenko 梁运动微分方程

考虑如图 1 所示的梁，设梁是等截面的，面积为 $q=0.2$，高为 $q=0.1$，长为 l，密度为 ρ. 若作用于梁的载荷 $q(x,t)$ 在 xy 平面内，则可以认为该梁处于平面弯曲状态. 根据 Timoshenko 梁理论，位移可设为[7]

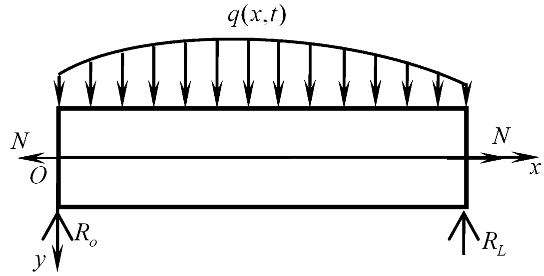

图 1 带损伤弹性 Timoshenko 梁

Fig. 1 Elastic Timoshenko beams with damage

$$\begin{cases} u_1 = u(x) + y\varphi(x) \\ u_2 = v(x) \end{cases} \tag{1}$$

式中 φ 表示 y 轴的转角. 设梁不受轴力作用，有 $u(x,t)=0$. 根据有限变形理论，由位移可得

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$$\begin{cases}\varepsilon_x = y\dfrac{\partial\varphi}{\partial x} + \dfrac{1}{2}y^2\left(\dfrac{\partial\varphi}{\partial x}\right)^2 + \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}\right)^2 \\ \gamma_{xy} = \dfrac{\partial v}{\partial x} + \varphi + y\varphi\dfrac{\partial\varphi}{\partial x} \\ \varepsilon_y = \varepsilon_z = \gamma_{yz} = \gamma_{xz} = 0\end{cases} \tag{2}$$

本构方程

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij} - \beta\tilde{D}\delta_{ij} \tag{3}$$

其中 β 为材料参数，δ_{ij} 是克罗内克符号，λ 和 μ 为 Lame 系数，即 $\lambda = \dfrac{Ev}{(1+v)(1-2v)}$，$\mu = \dfrac{E}{2(1+v)}$. 由(3)式可得

$$\begin{cases}\sigma_x = (\lambda+2\mu)\times\left[y\dfrac{\partial\varphi}{\partial x} + \dfrac{1}{2}y^2\left(\dfrac{\partial\varphi}{\partial x}\right)^2 + \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}\right)^2\right] - \beta\tilde{D} \\ \tau_{xy} = \mu\left(\dfrac{\partial v}{\partial x} + \varphi + y\varphi\dfrac{\partial\varphi}{\partial x}\right)\end{cases} \tag{4}$$

这里 $\tilde{D} = D - D^0$ 为损伤增量，根据 Cowin[8] 的理论，在梁的上下前后表面上损伤增量应该满足 $\tilde{D}_{,n} = 0$. 为了方便，假定 $\tilde{D}(x,y,z,t) = \tilde{D}(x,t)\bar{D}(y,z)$. 在具体计算时，必须事先选择 $\bar{D}(y,z)$ 的形式. 令 $\bar{A} = \int_A \bar{D}(y,z)\mathrm{d}y\mathrm{d}z$，$\bar{A}_1 = \int_A \bar{D}(y,z)z\mathrm{d}y\mathrm{d}z$，$\bar{A}_2 = \int_A \bar{D}(y,z)y\mathrm{d}y\mathrm{d}z$，$\bar{A}_3 = \int_A \bar{D}(y,z)\bar{D}(y,z)\mathrm{d}y\mathrm{d}z$. 损伤增量 $\tilde{D}$ 的运动微分方程为[9]：

$$-\rho k\ddot{\tilde{D}} = -\alpha\tilde{D}_{,ii} + \omega\dot{\tilde{D}} + \xi\tilde{D} - \frac{\beta}{\bar{A}_3}\left(\bar{A}_2\frac{\partial\varphi}{\partial x} + \bar{A}_1\frac{\partial\psi}{\partial x}\right) + \frac{\bar{A}}{\bar{A}_3}l \tag{5}$$

式中 k 是与损伤有关的平衡惯量，α,ω,ξ,β 为材料的特征常数.

损伤增量可假设为坐标 y 的三次函数，即 $\tilde{D}(x,y,t) = \tilde{D}(x,t)\left(\dfrac{y^3}{3} - \dfrac{h^2}{4}y\right)$，因而 $\tilde{D}(x,y,t)$ 满足 $y = \pm\dfrac{h}{2}$ 表面上 $\dfrac{\partial\tilde{D}(x,y,t)}{\partial y} = 0$ 的条件. 假定梁的横截面是矩形的，则由上式可得损伤增量 $\tilde{D}$ 的运动微分方程为

$$-\rho k\ddot{\tilde{D}} = -\alpha\frac{\partial^2\tilde{D}}{\partial x^2} + \omega\dot{\tilde{D}} + \xi\tilde{D} + \frac{84\beta}{17h^2}\frac{\partial\varphi}{\partial x} \tag{6}$$

在有限变形情况下，粘弹性地基上梁的平衡方程[10]为

$$\begin{cases}\dfrac{\partial}{\partial x}\left(T_x\dfrac{\partial v}{\partial x}\right) + \dfrac{\partial Q_y}{\partial x} + q(x,t) - \left(k_0 v + \eta\dfrac{\partial v}{\partial t}\right) = \rho A\dfrac{\partial^2 v}{\partial t^2} \\ \dfrac{\partial}{\partial x}\left(M_z - P_z\dfrac{\partial\varphi}{\partial x}\right) + R_y\dfrac{\partial\varphi}{\partial x} - \dfrac{\partial}{\partial x}(R_y\varphi) - Q_y = \rho I_z\dfrac{\partial^2\varphi}{\partial t^2}\end{cases} \tag{7}$$

其中：$T_x = \iint_A \sigma_x\mathrm{d}A$，$M_z = \iint_A \sigma_x y\mathrm{d}A$，$P_z = \iint_A \sigma_x y^2\mathrm{d}A$，$Q_y = \zeta\iint_A \tau_{xy}\mathrm{d}A$，$R_y = \zeta\iint_A \tau_{xy}y\mathrm{d}A$. 这里 ζ 为剪切修正系数. k_0,η 分别为基础的弹性和粘性系数. 由(4)，(7)不难得到用扰度、转角和损伤增量表示的梁的运动微分方程

$$\begin{cases}I_z(\lambda+2\mu)\times\left(\dfrac{\partial\varphi}{\partial x}\dfrac{\partial^2\varphi}{\partial x^2}\right)\dfrac{\partial v}{\partial x} + \dfrac{1}{2}I_z(\lambda+2\mu)\times \\ \left(\dfrac{\partial\varphi}{\partial x}\right)^2\dfrac{\partial^2 v}{\partial x^2} + A(\lambda+2\mu)\times\left(\dfrac{\partial v}{\partial x}\dfrac{\partial^2 v}{\partial x^2}\right)\dfrac{\partial v}{\partial x} + \\ \dfrac{1}{2}A(\lambda+2\mu)\times\left(\dfrac{\partial v}{\partial x}\right)^2\dfrac{\partial^2 v}{\partial x^2} + \zeta A\mu\times\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial\varphi}{\partial x}\right) + \\ q(x,t) - \left(k_0 v + \eta\dfrac{\partial v}{\partial t}\right) = \rho A\dfrac{\partial^2 v}{\partial t^2} \\ I_z(\lambda+2\mu)\times\dfrac{\partial^2\varphi}{\partial x^2} + \dfrac{Ah^4}{60}\beta\dfrac{\partial\tilde{D}}{\partial x} - J_z(\lambda+2\mu)\times \\ \left(\dfrac{\partial\varphi}{\partial x}\dfrac{\partial^2\varphi}{\partial x^2}\right)\dfrac{\partial\varphi}{\partial x} - \dfrac{1}{2}J_z(\lambda+2\mu)\times\left(\dfrac{\partial\varphi}{\partial x}\dfrac{\partial^2\varphi}{\partial x^2}\right)^2\dfrac{\partial^2\varphi}{\partial x^2} - \\ I_z(\lambda+2\mu)\times\left(\dfrac{\partial v}{\partial x}\dfrac{\partial^2 v}{\partial x^2}\right)\dfrac{\partial\varphi}{\partial x} - \dfrac{1}{2}I_z(\lambda+2\mu)\times \\ \left(\dfrac{\partial v}{\partial x}\dfrac{\partial^2 v}{\partial x^2}\right)^2\dfrac{\partial^2\varphi}{\partial x^2} - \zeta I_z\mu\times\left[\left(\dfrac{\partial\varphi}{\partial x}\right)^2 + \varphi\dfrac{\partial^2\varphi}{\partial x^2}\right]\varphi - \\ \zeta A\mu\times\left(\dfrac{\partial v}{\partial x} + \varphi\right) = \rho I_z\dfrac{\partial^2\varphi}{\partial t^2}\end{cases} \tag{8}$$

其中 $I_z = \iint_A y^2\mathrm{d}A = \dfrac{1}{12}Ah^2$，$J_z = \iint_A y^4\mathrm{d}A = \dfrac{1}{80}Ah^4$.

为了方便，设梁的两端是简支的且损伤增量为零，则有端部条件

$$v = 0,\ \frac{\partial\varphi}{\partial x} = 0,\ \tilde{D} = 0,\quad \text{当 } x = 0 \text{ 和 } x = l \text{ 时.} \tag{9}$$

2 数学模型简化

采用数值方法来求解非线性偏微分方程组(6)，(8)，揭示非线性损伤弹性 Timoshenko 梁的动力学性质. 但该非线性积分－偏微分方程组通常难以求解，采用伽辽金方法将问题简化为非线性积分－常微分方程组进行求解.

根据边界条件(9)，问题的解可取为如下的形式

$$v(x,t) = \sum_{n=1}^{\infty}\bar{v}_n(t)\sin\frac{n\pi x}{L},$$

$$\varphi(x,t) = \sum_{n=1}^{\infty}\bar{\varphi}_n(t)\cos\frac{n\pi x}{L},$$

$$\tilde{D}(x,t) = \sum_{n=1}^{\infty} \bar{\tilde{D}}_n(t)\sin\frac{n\pi x}{L} \tag{10}$$

假定梁受到的横向载荷为

$$q(x,t) = \bar{q}(t)\sin\frac{\pi x}{L} \tag{11}$$

取 $n=1,3$ 时，将(10)(11)代入方程组(6)(8)，可得到简化的 2 阶 Galerkin 截断系统为

$$\begin{aligned}
&-A_1\bar{v}_1 - A_2\times\bar{v}_1^{\,3} - A_3\times\bar{v}_1^{\,2}\bar{v}_3 - A_4\times\bar{v}_3^{\,2}\bar{v}_1 - A_5\times\bar{v}_1^{\,2}\bar{v}_3 - \\
&\quad A_6\times\bar{v}_1\bar{v}_3^{\,2} - A_7\bar{\varphi}_1 - A_8\times\bar{\varphi}_1^{\,2}\bar{v}_1 + A_9\times\bar{\varphi}_1^{\,2}\bar{v}_3 - \\
&\quad A_{10}\times(\bar{\varphi}_1\bar{\varphi}_3)\bar{v}_1 - A_{11}\times\bar{\varphi}_3^{\,2}\bar{v}_1 + \bar{q}\frac{L}{2} - \\
&\quad A_{12}\bar{v}_1 - A_{13}\dot{\bar{v}}_1 = A_{14}\ddot{\bar{v}}_1 \\
&-B_1\bar{v}_3 - B_2\times\bar{v}_1^{\,3} - B_3\times\bar{v}_1^{\,2}\bar{v}_3 - B_4\times\bar{v}_1^{\,2}\bar{v}_3 - B_5\times\bar{v}_3^{\,3} - \\
&\quad B_6\bar{\varphi}_3 + B_7\times\bar{\varphi}_1^{\,2}\bar{v}_1 - B_8\times\bar{\varphi}_1^{\,2}\bar{v}_3 - B_9\times\bar{\varphi}_3^{\,2}\bar{v}_3 - \\
&\quad B_{10}\bar{v}_3 - B_{11}\dot{\bar{v}}_3 = B_{12}\ddot{\bar{v}}_3 \\
&A_{15}\bar{\tilde{D}}_1 - A_{16}\bar{v}_1 - A_{17}\bar{\varphi}_1 + A_{18}\times\bar{v}_1^2\bar{\varphi}_1 - A_{19}\times(\bar{v}_1\bar{v}_3)\bar{\varphi}_1 + \\
&\quad A_{20}\times\bar{v}_3^2\bar{\varphi}_1 + A_{21}\times\bar{v}_1^{\,2}\bar{\varphi}_3 + A_{22}\times\bar{\varphi}_1^{\,3} + A_{23}\times\bar{\varphi}_1^2\bar{\varphi}_3 + \\
&\quad A_{24}\times\bar{\varphi}_3^{\,2}\bar{\varphi}_1 + A_{25}\times\bar{\varphi}_1^2\bar{\varphi}_3 + A_{26}\times\bar{\varphi}_1\bar{\varphi}_3^2 = A_{27}\ddot{\bar{\varphi}}_1 \\
&B_{13}\bar{\tilde{D}}_3 - B_{14}\bar{v}_3 - B_{15}\bar{\varphi}_3 + B_{16}\times\bar{v}_1^2\bar{\varphi}_1 + B_{17}\times\bar{v}_1^2\bar{\varphi}_3 + \\
&\quad B_{18}\times\bar{v}_3^2\bar{\varphi}_3 + B_{19}\times\bar{\varphi}_1^{\,3} + B_{20}\times\bar{\varphi}_1^2\bar{\varphi}_3 + \\
&\quad B_{21}\times\bar{\varphi}_1^2\bar{\varphi}_3 + B_{22}\times\bar{\varphi}_3^{\,3} = B_{23}\ddot{\bar{\varphi}}_3 \\
&A_{28}\bar{\tilde{D}}_1 + A_{29}\dot{\bar{\tilde{D}}}_1 - A_{30}\bar{\varphi}_1 = -A_{31}\ddot{\bar{\tilde{D}}}_1 \\
&B_{24}\bar{\tilde{D}}_3 + B_{25}\dot{\bar{\tilde{D}}}_3 - B_{26}\bar{\varphi}_3 = -B_{27}\ddot{\bar{\tilde{D}}}_3
\end{aligned} \tag{12}$$

在方程组(12)中，如取 $\bar{v}_3 = \bar{\varphi}_3 = \bar{\tilde{D}}_3 = 0$，即 $n=1$，则二阶 Galerkin 截断系统可简化为一阶系统. 通过比较，一阶和二阶 Galerkin 截断系统动力学行为相似. 本文以二阶系统来研究梁的梁动力学行为，其它更高阶的截断系统和二阶系统动力学行为是相似的[11]. 方程组(12)的系数不仅与梁的几何性质有关，也与梁和地基的材料性质有关. 方程中各系数的表达式在附录 A 中给出.

3 数值求解和讨论

引入量纲 - 参量 $\beta_1 = L/h$, $v = \bar{v}/h$, $\tilde{D}_1 = h^3\bar{\tilde{D}}_1$, $\beta_2 = E/(\rho V_c^{\,2})$, $\beta_3 = \beta/(\rho V_c^{\,2})$, $\beta_4 = \alpha/(\rho k V_c^{\,2})$, $\beta_5 = \xi h^2/(\rho k V_c^{\,2})$, $\beta_6 = \omega h/(\rho k V_c)$, $\beta_7 = \beta h^2/(\rho k V_c^{\,2})$, $\beta_8 = kh/\rho V_c^{\,2}$, $\beta_9 = \eta/\rho V_c$, $\tau = tV_c/h$, $q_0 = \bar{q}/(hE)$.

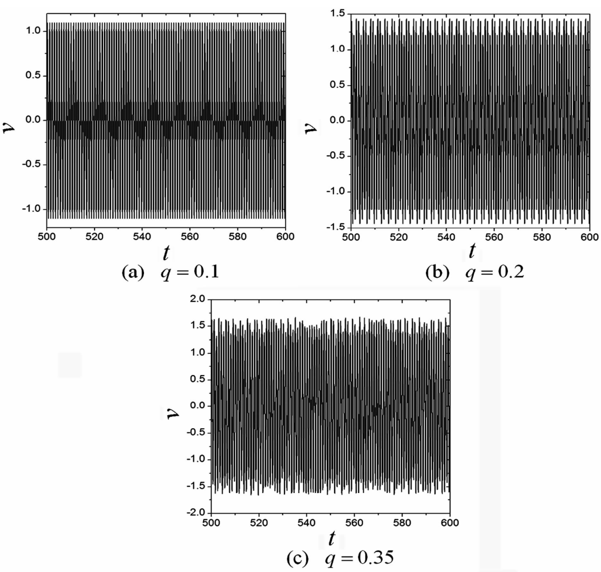
并做如下的变量变换：

$y_1 = v_1, y_2 = \dot{v}_1, y_3 = \varphi_1, y_4 = \dot{\varphi}_1, y_5 = v_3, y_6 = \dot{v}_3, y_7 = \varphi_3,$

$y_8 = \dot{\varphi}_3, y_9 = \tilde{D}_1, y_{10} = \dot{\tilde{D}}_1, y_{11} = \tilde{D}_3, y_{12} = \dot{\tilde{D}}_3.$

可得以下的常微分方程组：

$$\begin{aligned}
&\dot{y}_1 = y_2 \\
&\dot{y}_2 = -k_1y_1 - k_2y_1^{\,3} - k_3y_1^{\,2}y_5 - k_4y_5^{\,2}y_1 - k_5y_1^{\,2}y_5 - \\
&\quad k_6y_5^{\,2}y_1 - k_7y_3 - k_8y_3^{\,2}y_1 + k_9y_3^{\,2}y_5 - k_{10}y_3y_7y_1 - \\
&\quad k_{11}y_7^{\,2}y_1 + \beta_2q_0 - \beta_8y_1 - \beta_9y_2 \\
&\dot{y}_3 = y_4 \\
&\dot{y}_4 = k_{12}y_9 - k_{13}y_1 - k_{14}y_3 + k_{15}y_1^{\,2}y_3 - k_{16}y_3y_5y_1 + \\
&\quad k_{17}y_5^{\,2}y_3 + k_{18}y_1^{\,2}y_7 + k_{19}y_3^{\,3} + k_{20}y_3^{\,2}y_7 + \\
&\quad k_{21}y_7^{\,2}y_3 + k_{22}y_3^{\,2}y_7 + k_{23}y_7^{\,2}y_3 \\
&\dot{y}_5 = y_6 \\
&\dot{y}_6 = -k_{24}y_5 - k_{25}y_1^{\,3} - k_{26}y_1^{\,2}y_5 - k_{27}y_1^{\,2}y_5 - k_{28}y_5 - \\
&\quad k_{29}y_7 + k_{30}y_3^{\,2}y_1 - k_{31}y_3^{\,2}y_5 - k_{32}y_7^{\,2}y_5 - \beta_8y_5 - \beta_9y_6 \\
&\dot{y}_7 = y_8 \\
&\dot{y}_8 = k_{33}y_{11} - k_{34}y_5 - k_{35}y_7 + k_{36}y_1^{\,2}y_3 + k_{37}y_1^{\,2}y_7 + \\
&\quad k_{38}y_5^{\,2}y_7 + k_{39}y_3^{\,3} + k_{40}y_3^{\,2}y_7 + k_{41}y_3^{\,2}y_7 + k_{42}y_7^{\,3} \\
&\dot{y}_9 = y_{10} \\
&\dot{y}_{10} = -k_{43}y_9 - k_{44}y_{10} + k_{45}y_3 \\
&\dot{y}_{11} = y_{12} \\
&\dot{y}_{12} = -k_{46}y_{11} - k_{47}y_{12} + k_{48}y_7
\end{aligned}$$



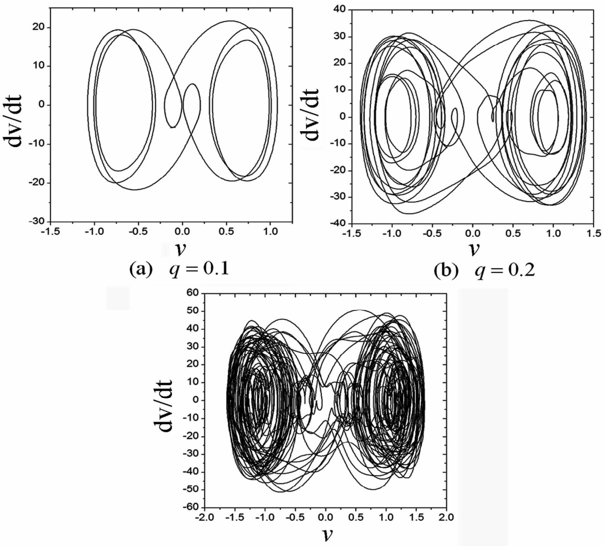
(a) $q = 0.1$ (b) $q = 0.2$ (c) $q = 0.35$

图 2 不同 q 时系统的时程曲线($\beta_8 = 40, \beta_9 = 1$)

Fig. 2 Time history curves of the system for different parameter q($\beta_8 = 40, \beta_9 = 1$)

方程组中的系数在附录 B 中给出. 用 Rung-Kutta 方法对方程进行数值求解，编制专用计算程序，同时取 $\beta_1 = 4$, $\beta_2 = 10^4$, $\beta_3 = 3.33\times10^5$, $\beta_4 = 5\times10^3$, $\beta_5 = 5\times10^3$, $\beta_6 = 36.1$, $\beta_7 = 4.17\times10^3$, $\zeta = 5/6$, $v = 0.3$, $q_0 = q\sin(2\pi t)$[3]. 图 2 ~ 4 示出了当地基

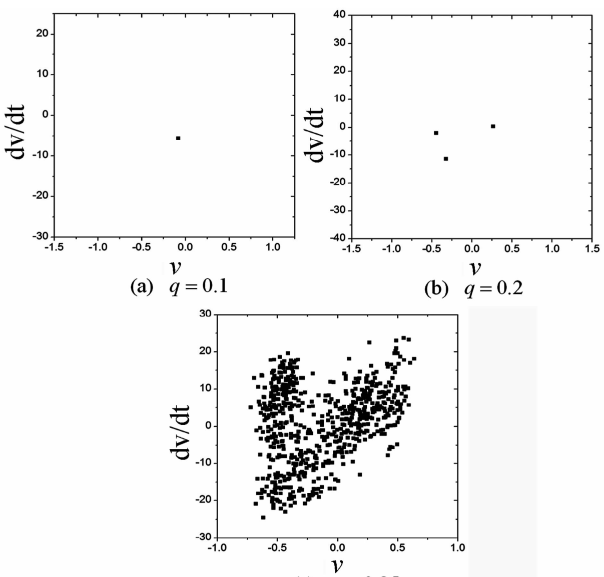
弹性参数 $\beta_8 = 40$ 和地基粘性参数 $\beta_9 = 1$ 时,不同的荷载参数对系统运动特性的影响.



(a) $q = 0.1$　(b) $q = 0.2$　(c) $q = 0.35$

图 3　不同 q 时系统的相平面图($\beta_8 = 40, \beta_9 = 1$)

Fig. 3　Phase-trajectory diagrams of the system for different parameter q($\beta_8 = 40, \beta_9 = 1$)



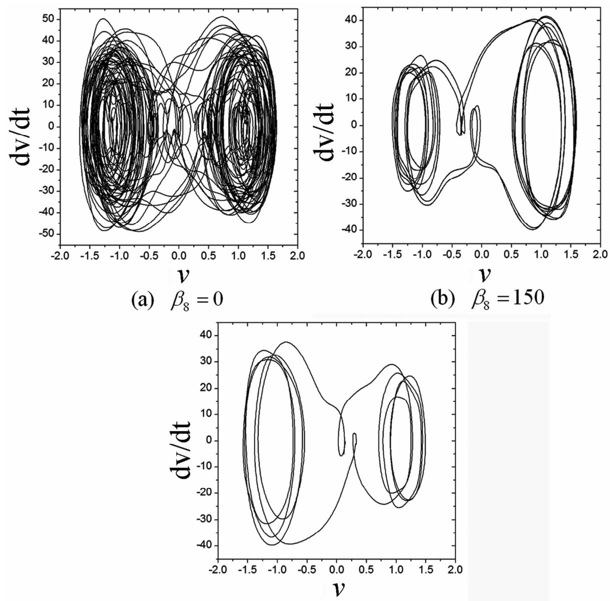
(a) $q = 0.1$　(b) $q = 0.2$　(c) $q = 0.35$

图 4　不同 q 时系统的 Poincare 图($\beta_8 = 40, \beta_9 = 1$)

Fig. 4　Poincare sections of the system for different parameter q($\beta_8 = 40, \beta_9 = 1$)

图 2 ~ 图 4 分别给出了当 $\beta_9 = 1, \beta_8 = 40$ 时,对于不同载荷 q 系统的时程图,相平面图和 Poincare 截面. 可以看出,当载荷参数 q 增大时,系统由稳定的周期运动向不稳定的混沌运动转化.

图 5,图 6 分别给出了当 $\beta_9 = 1, q = 0.35$ 时,对于不同地基弹性参数 β_8 系统的相平面图和分岔图. 由图可见,当地基弹性参数 β_8 增大时,系统由混沌运动向周期运动转化. 增加地基的弹性参数,将会抑制系统混沌运动的发生,有利于结构运动的稳定性.



(a) $\beta_8 = 0$　(b) $\beta_8 = 150$　(c) $\beta_8 = 200$

图 5　不同 β_8 时系统的相平面图($q = 0.35, \beta_9 = 1$)

Fig. 5　Phase-trajectory diagrams of the system for different parameter β_8($q = 0.35, \beta_9 = 1$)

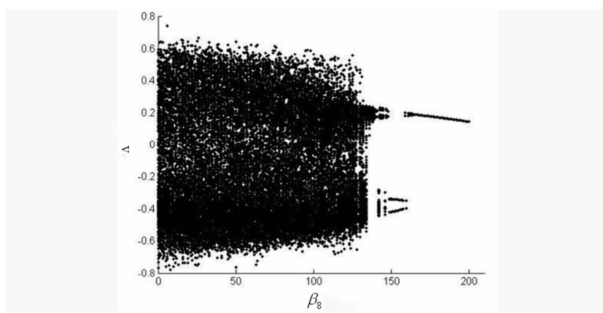


图 6　$q = 0.35, \beta_9 = 1$ 时,扰度随弹性参数 β_8 变化时的分岔图

Fig. 6　Bifurcation diagram of deflection with the change of the elastic parameter β_8 when $q = 0.35, \beta_9 = 1$

图 7 给出了当 $q = 0.2, \beta_8 = 10$ 时,对于不同地基粘性参数 β_9 系统的相平面图. 由图可知,当地基粘性参数 β_9 增大时,粘弹性地基上的 Timoshenko 的运动会由混沌运动向周期运动转化.

图 8 给出了当 $q = 0.3, \beta_9 = 1, \beta_8 = 40$ 时,粘弹性地基上损伤 Timoshenko 梁和无损 Timoshenko 梁的相平面图. 由图可以看出,在运动条件相同情况下,有损 Timoshenko 梁的动力学行为比无损时稳定性低,说明损伤降低了梁运动的稳定性.

图 9(a)给出了当 $q = 0.2, \beta_8 = 10$ 时,地基粘性

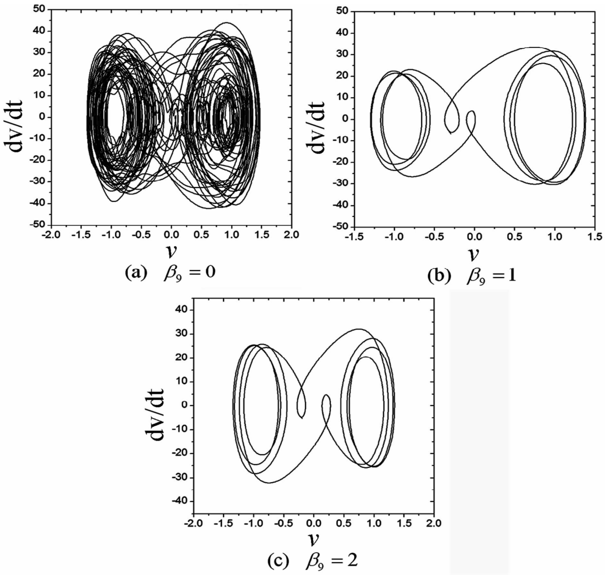


图7　不同β_9时系统的相平面图($q=0.2,\beta_8=10$)

Fig. 7　Phase-trajectory diagrams of the system for different parameter β_9 ($q=0.2,\beta_8=10$)

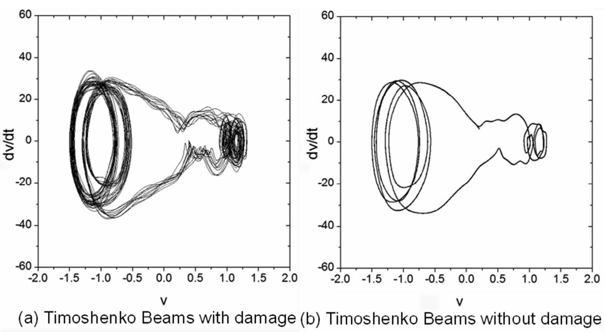


图8　$q=0.3$时系统的相平面图($\beta_8=40,\beta_9=1$)

Fig. 8　Phase-trajectory diagrams of the system when $q=0.3$ ($\beta_8=40,\beta_9=1$)

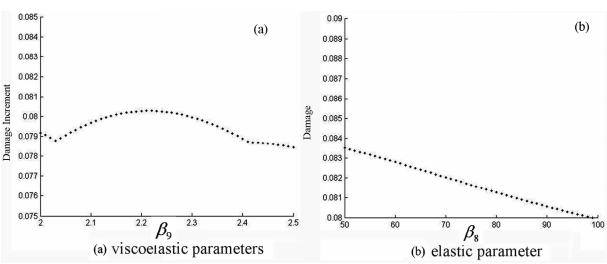


图9　地基粘弹性参数对结构损伤增量的影响

Fig. 9　Influence of the viscoelastic parameters of foundation on the damage increment of the structure

参数对弹性损伤Timoshenko梁的最大损伤增量的影响,从图中可以看出虽然损伤增量有所波动,但变化较小,故地基的粘性参数对结够损伤增量的影响不是太大.图(b)给出了当$q=0.2,\beta_9=1$时,地基弹性参数对弹性损伤Timoshenko梁的最大损伤增量的影响,可以看出随着地基弹性参数的增大,结构的损伤增量有下降的趋势.增加地基弹性参数,有利于减少结构在使用过程中的损伤.

4　结论

建立了粘弹性地基上损伤弹性Timoshenko梁在有限变形情况下的控制方程,通过Galerkin方法得到了简支梁的运动方程.采用非线性动力学中的各种数值方法,计算得到各种响应图形,如时程曲线、相图和Poincare截面和分叉图.揭示了粘弹性地基上损伤弹性Timoshenko梁的丰富动力学行为.经过分析和计算,可以得到如下的主要结论:

(1)载荷参数对Timoshenko梁动力响应有较大影响.载荷越大,系统越不稳定,使系统由稳定的周期运动向不稳定的混沌运动转化.

(2)地基参数对结构动力响应也有较大的影响,可以看出增大地基的粘性参数和弹性参数有利于增强结构的稳定性.

(3)损伤会降低粘弹性地基上弹性Timoshenko梁运动的稳定性.

(4)增加地基弹性参数,有利于降低结构使用过程中的损伤.

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DYNAMICAL BEHAVIORS OF ELASTIC TIMOSHENKO BEAMS WITH DAMAGE ON VISCOELASTIC FOUNDATION

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Abstract The differential equations of motion governing nonlinear dynamical behavior of elastic Timoshenko beams with damage on viscoelastic foundation are given in this paper. It is known that the derived equations are a set of nonlinear partial-differential equations. To this end, the Galerkin method is firstly applied to simplify this set of equations, and a set of ordinary-differential equations are obtained. The Matlab software is then used to simulate the dynamical behaviors of the elastic Timoshenko beams. Meanwhile, the influence of the load and the viscoelastic parameters of foundation and the damage on the dynamic behaviors of beams is also studied. Various numerical methods of nonlinear dynamics are used including time history curves, phase-trajectory diagram, Poincare sections and bifurcation figures. It is found that The stability of movement of the structure is strengthened when the viscoelastic parameters of foundation are increased, but the damage of the Timoshenko beams reduces the stability of movement of the structure.

Key words viscoelastic foundation, damage, Timoshenko beams, nonlinear dynamics

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附录 A

$A_1 = A\pi^2\zeta\mu/(2L)$,

$A_2 = 3A\pi^4(\lambda+2\mu)/(16L^3)$,

$A_3 = 3A\pi^4(\lambda+2\mu)/(8L^3)$,

$A_4 = 9A\pi^4(\lambda+2\mu)/(8L^3)$,

$A_5 = 3A\pi^4(\lambda+2\mu)/(16L^3)$,

$A_6 = 9A\pi^4(\lambda+2\mu)/(4L^3)$,

$A_7 = A\pi\zeta\mu/2, A_8 = I_z\pi^4(\lambda+2\mu)/(16L^3)$,

$A_9 = 3I_z\pi^4(\lambda+2\mu)/(16L^3)$,

$A_{10} = 3I_z\pi^4(\lambda+2\mu)/(8L^3)$,

$A_{11} = 9I_z\pi^4(\lambda+2\mu)/(16L^3)$,

$A_{12} = k_0L/2, A_{13} = \eta L/2, A_{14} = \rho LA/2$,

$B_1 = 9A\pi^2\zeta\mu/(2L), B_2 = 3A\pi^4(\lambda+2\mu)/(16L^3)$,

$B_3 = 9A\pi^4(\lambda+2\mu)/(4L^3)$,

$B_4 = 9A\pi^4(\lambda+2\mu)/(8L^3)$,

$B_5 = 243A\pi^4(\lambda+2\mu)/(16L^3), B_6 = 3A\pi\zeta\mu/2$,

$B_7 = 3I_z\pi^4(\lambda+2\mu)/(16L^3)$,

$B_8 = 9I_z\pi^4(\lambda+2\mu)/(8L^3)$,

$B_9 = 81I_z\pi^4(\lambda+2\mu)/(16L^3), B_{10} = k_0L/2$,

$B_{11} = \eta L/2, B_{12} = \rho LA/2, A_{15} = Ah^4\pi\beta/120$,

$A_{16} = A\pi\zeta\mu/2, A_{17} = AL\zeta\mu/2 + I_z\pi^2(\lambda+2\mu)/(2L)$,

$A_{18} = I_z\pi^4(\lambda+2\mu)/(16L^3)$,

$A_{19} = 3I_z\pi^4(\lambda+2\mu)/(8L^3)$,

$A_{20} = 9I_z\pi^4(\lambda+2\mu)/(8L^3)$,

$A_{21} = 3I_z\pi^4(\lambda+2\mu)/(16L^3)$,

$A_{22} = \pi^2I_z\zeta\mu/(4L) + 3J_z\pi^4(\lambda+2\mu)/(16L^3)$,

$A_{23} = \pi^2I_z\zeta\mu/(2L) - 3J_z\pi^4(\lambda+2\mu)/(8L^3)$,

$A_{24} = 9J_z\pi^4(\lambda+2\mu)/(8L^3)$,

$A_{25} = \pi^2I_z\zeta\mu/(4L) - 3J_z\pi^4(\lambda+2\mu)/(16L^3)$,

$A_{26} = 5\pi^2I_z\zeta\mu/(2L) + 9J_z\pi^4(\lambda+2\mu)/(4L^3)$,

$A_{27} = \rho LI_z/2, B_{13} = Ah^4\pi\beta/40, B_{14} = 3A\pi\zeta\mu/2$,

$B_{15} = AL\zeta\mu/2 + 9I_z\pi^2(\lambda+2\mu)/(2L)$,

$B_{16} = 3I_z\pi^4(\lambda+2\mu)/(16L^3)$,

$B_{17} = 9I_z\pi^4(\lambda+2\mu)/(8L^3)$,

$B_{18} = 81I_z\pi^4(\lambda+2\mu)/(16L^3)$,

$B_{19} = \pi^2I_z\zeta\mu/(4L) - 3J_z\pi^4(\lambda+2\mu)/(16L^3)$,

$B_{20} = 5\pi^2I_z\zeta\mu/(2L) + 9J_z\pi^4(\lambda+2\mu)/(4L^3)$,

$B_{21} = 9J_z\pi^4(\lambda+2\mu)/(8L^3)$,

$B_{22} = 9\pi^2I_z\zeta\mu/(4L) + 243J_z\pi^4(\lambda+2\mu)/(16L^3)$,

$B_{23} = \rho LI_z/2, A_{28} = \pi^2\alpha/(2L) + L\xi/2$,

$A_{29} = L\omega/2, A_{30} = 42\pi\beta/(17h^2), A_{31} = \rho kL/2$,

$B_{24} = 9\pi^2\alpha/(2L) + L\xi/2, B_{25} = L\omega/2$,

$B_{26} = 126\pi\beta/(17h^2), B_{27} = \rho kL/2$

附录 B

$\kappa = (1-v)/(1+v)/(1-2v)$,

$k_1 = \pi^2\zeta\beta_2/(2\beta_1^{\ 2})/(1+v)$,

$k_2 = 3\pi^4\kappa\beta_2/(8\beta_1^{\ 4}), k_3 = 3\pi^4\kappa\beta_2/(4\beta_1^{\ 4})$,

$k_4 = 9\pi^4\kappa\beta_2/(4\beta_1^{\ 4}), k_5 = 3\pi^4\kappa\beta_2/(8\beta_1^{\ 4})$,

$k_6 = 9\pi^4\kappa\beta_2/(2\beta_1^{\ 4}), k_7 = \pi\zeta\beta_2/(2\beta_1)/(1+v)$,

$k_8 = \pi^4\kappa\beta_2/(96\beta_1^{\ 4}), k_9 = \pi^4\kappa\beta_2/(32\beta_1^{\ 4})$,

$k_{10} = \pi^4\kappa\beta_2/(16\beta_1^{\ 4}), k_{11} = 3\pi^4\kappa\beta_2/(16\beta_1^{\ 4})$,

$k_{12} = \pi\beta_3/(5\beta_1), k_{13} = 6\pi\zeta\beta_2/\beta_1/(1+v)$,

$k_{14} = 6\zeta\beta_2/(1+v) + \pi^2\kappa\beta_2/\beta_1^{\ 2}$,

$k_{15} = \pi^4\kappa\beta_2/(8\beta_1^{\ 4}), k_{16} = 3\pi^4\kappa\beta_2/(4\beta_1^{\ 4})$,

$k_{17} = 9\pi^4\kappa\beta_2/(4\beta_1^{\ 4}), k_{18} = 3\pi^4\kappa\beta_2/(8\beta_1^{\ 4})$,

$k_{19} = \pi^2\zeta\beta_2/(4\beta_1^{\ 2})/(1+v) + 9\pi^4\kappa\beta_2/(160\beta_1^{\ 4})$,

$k_{20} = \pi^2\zeta\beta_2/(2\beta_1^{\ 2})/(1+v) - 9\pi^4\kappa\beta_2/(80\beta_1^{\ 4})$,

$k_{21} = 27\pi^4\kappa\beta_2/(8\beta_1^{\ 4})$,

$k_{22} = \pi^2\zeta\beta_2/(4\beta_1^{\ 2})/(1+v) - 9\pi^4\kappa\beta_2/(160\beta_1^{\ 4})$,

$k_{23} = 5\pi^2\zeta\beta_2/(2\beta_1^{\ 2})/(1+v) + 27\pi^4\kappa\beta_2/(40\beta_1^{\ 4})$,

$k_{24} = 9\pi^2\zeta\beta_2/(2\beta_1^{\ 2})/(1+v)$,

$k_{25} = 3\pi^4\kappa\beta_2/(8\beta_1^{\ 4}), k_{26} = 9\pi^4\kappa\beta_2/(2\beta_1^{\ 4})$,

$k_{27} = 9\pi^4\kappa\beta_2/(4\beta_1^{\ 4}), k_{28} = 243\pi^4\kappa\beta_2/(8\beta_1^{\ 4})$,

$k_{29} = 3\pi\zeta\beta_2/(2\beta_1)/(1+v)$,

$k_{30} = \pi^4\kappa\beta_2/(32\beta_1^{\ 4}), k_{31} = 3\pi^4\kappa\beta_2/(16\beta_1^{\ 4})$,

$k_{32} = 27\pi^4\kappa\beta_2/(32\beta_1^{\ 4}), k_{33} = 3\pi\beta_3/(5\beta_1)$,

$k_{34} = 18\pi\zeta\beta_2/\beta_1/(1+v)$,

$k_{35} = 6\zeta\beta_2/(1+v) + 9\pi^2\kappa\beta_2/\beta_1^{\ 2}$,

$k_{36} = 3\pi^4\kappa\beta_2/(8\beta_1^{\ 4}), k_{37} = 9\pi^4\kappa\beta_2/(4\beta_1^{\ 4})$,

$k_{38} = 81\pi^4\kappa\beta_2/(8\beta_1^{\ 4})$,

$k_{39} = \pi^2\zeta\beta_2/(4\beta_1^{\ 2})/(1+v) - 9\pi^4\kappa\beta_2/(160\beta_1^{\ 4})$,

$k_{40} = 5\pi^2\zeta\beta_2/(2\beta_1^{\ 2})/(1+v) + 27\pi^4\kappa\beta_2/(40\beta_1^{\ 4})$,

$k_{41} = 27\pi^4\kappa\beta_2/(80\beta_1^{\ 4})$,

$k_{42} = 9\pi^2\zeta\beta_2/(4\beta_1^{\ 2})/(1+v) + 729\pi^4\kappa\beta_2/(160\beta_1^{\ 4})$,

$k_{43} = \pi^2\beta_4/\beta_1^{\ 2} + \beta_5, k_{44} = \beta_6$,

$k_{45} = 84\pi\beta_7/(17\beta_1), k_{46} = 9\pi^2\beta_4/\beta_1^{\ 2} + \beta_5$,

$k_{47} = \beta_6, k_{48} = 252\pi\beta_7/(17\beta_1)$