关于 Mei 对称性与 Noether 对称性的关系——以 Birkhoff 系统为例 *

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摘要 文章以 Birkhoff 系统为例研究 Mei 对称性与 Noether 对称性之间的关系. 研究了基于无限小生成元向量作用下 Birkhoff 函数和 Birkhoff 函数组的变分问题,建立了该变分问题的 Birkhoff 方程与 Noether 对称性及其守恒量. 研究表明:该变分问题得到的 Birkhoff 方程、Noether 等式和 Noether 守恒量分别与经典 Birkhoff 系统 Mei 对称性的判据方程、结构方程和 Mei 守恒量完全一致. 文末以著名的 Emden 方程等为例来说明结果的应用.

关键词 Birkhoff 系统, Mei 对称性, Noether 对称性, 守恒量, 关系

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引言

动力学系统的对称性与守恒量的研究是分析力学发展的一个重要方面. 对称性方法是研究动力学系统守恒量的一个近代方法,主要有三种概念不同的方法:一是基于 Hamilton 作用量在无限小变换下的不变性的 Noether 对称性[1-9];二是基于微分方程在无限小变换下的不变性的 Lie 对称性[10-18];三是基于动力学方程中出现的动力学函数(如 Lagrange 函数,动能,势能,广义力,广义约束力等)在经历群的无限小变换后仍然满足原方程的一种不变性的 Mei 对称性[19-25]. 梅凤翔教授在其著作[25]中系统地研究了约束力学系统的上述三种对称性与三种守恒量(Noether 守恒量, Hojman 守恒量以及 Mei 守恒量), 及其相互之间的关系. 与以往的研究不同, 本文从变分问题及其不变性角度, 以 Birkhoff 系统为例研究 Mei 对称性与 Noether 对称性及其守恒量之间的关系,主要结果为文中给出的 4 个定理.

1 基于无限小生成元向量作用的 Birkhoff 函数的变分问题

Birkhoff 系统的运动微分方程为

$$\Omega_{\mu\nu}\dot{a}^{\nu}-\frac{\partial B}{\partial a^{\mu}}-\frac{\partial R_{\mu}}{\partial t}=0\quad(\mu,\nu=1,2,\cdots,2n)\tag{1}$$

其中 $R_{\mu}=R_{\mu}(t,a)$ 为 Birkhoff 函数组, $B=B(t,a)$ 为 Birkhoff 函数,而

$$\Omega_{\mu\nu}=\frac{\partial R_{\nu}}{\partial a^{\mu}}-\frac{\partial R_{\mu}}{\partial a^{\nu}}\tag{2}$$

称为 Birkhoff 张量. 当 $\det(\Omega_{\mu\nu})\neq 0$ 时,系统非奇异,可解得

$$\dot{a}^{\mu}=\Omega^{\mu\nu}\left(\frac{\partial B}{\partial a^{\nu}}+\frac{\partial R_{\nu}}{\partial t}\right)\tag{3}$$

其中 $\Omega^{\mu\nu}\Omega_{\nu\sigma}=\delta_{\mu\sigma}$, 这里 $\Omega^{\mu\nu}$ 称为 Birkhoff 逆变张量.

引进时间 t 和变量 a^{μ} 的无限小变换

$$\bar{t}=t+\Delta t,\bar{a}^{\mu}(\bar{t})=a^{\mu}(t)+\Delta a^{\mu},\quad(\mu=1,2,\cdots,2n)\tag{4}$$

或其展开式

$$\bar{t}=t+\varepsilon\tau(t,a),\bar{a}^{\mu}(\bar{t})=a^{\mu}(t)+\varepsilon\xi_{\mu}(t,a),\quad(\mu=1,2,\cdots,2n)\tag{5}$$

其中 ε 为无限小参数, τ, ξ_{μ} 为无限小变换的生成函数或生成元. 取无限小生成元向量为

$$X^{(0)}=\tau\frac{\partial}{\partial t}+\xi_{\mu}\frac{\partial}{\partial a^{\mu}}\tag{6}$$

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对于 Birkhoff 系统(1),构建基于无限小生成元向量作用下的 Birkhoff 函数和 Birkhoff 函数组的变分问题:

求积分泛函

$$A(\gamma)=\int_{t_1}^{t_2}\{X^{(0)}[R_\mu(t,a^v(t))]\dot{a}^\mu(t)-X^{(0)}[B(t,a^v(t))]\}dt \tag{7}$$

在给定边界条件

$$a^\mu(t)\big|_{t=t_1}=a_1^\mu, a^\mu(t)\big|_{t=t_2}=a_2^\mu, \quad (\mu=1,2,\cdots,2n) \tag{8}$$

下的极值问题.

积分泛函(7)也可称为作用量积分或作用量. 泛函(7)在 $a^\mu=a^\mu(t)$ 上取得极值的必要条件是其变分为零,即 $\delta A=0$,因此有

$$\delta A=\int_{t_1}^{t2}\Big[\frac{\partial X^{(0)}(R_\mu)}{\partial a^v}\dot{a}^\mu\delta a^v+X^{(0)}(R_\mu)\delta\dot{a}^\mu-\frac{\partial X^{(0)}(B)}{\partial a^v}\delta a^v\Big]dt=0 \tag{9}$$

利用边界条件(8)以及交换关系

$$d\delta a^\mu=\delta da^\mu(\mu=1,2,\cdots,2n) \tag{10}$$

方程(9)可表为

$$\begin{aligned}\delta A&=\int_{t_1}^{t2}\Big[\frac{\partial X^{(0)}(R_\mu)}{\partial a^v}\dot{a}^\mu-\frac{d}{dt}X^{(0)}(R_v)-\frac{\partial X^{(0)}(B)}{\partial a^v}\Big]\delta a^v dt=\\&\int_{t_1}^{t2}\Big\{\Big[\frac{\partial X^{(0)}(R_v)}{\partial a^\mu}-\frac{\partial X^{(0)}(R_\mu)}{\partial a^v}\Big]\dot{a}^v-\frac{\partial X^{(0)}(B)}{\partial a^\mu}-\frac{\partial X^{(0)}(R_\mu)}{\partial t}\Big\}\delta a^\mu dt=0\end{aligned} \tag{11}$$

由积分区间的任意性和 $\delta a^\mu(\mu=1,2,\cdots,2n)$ 的相互独立性,得

$$\Big[\frac{\partial X^{(0)}(R_v)}{\partial a^\mu}-\frac{\partial X^{(0)}(R_\mu)}{\partial a^v}\Big]\dot{a}^v-\frac{\partial X^{(0)}(B)}{\partial a^\mu}-\frac{\partial X^{(0)}(R_\mu)}{\partial t}=0(\mu=1,2,\cdots,2n) \tag{12}$$

方程(12)是变分问题(7)(8)的 Birkhoff 方程.

2　Noether 对称性

在无限小变换(4)作用下,泛函(7)变为

$$A(\bar{\gamma})=\int_{\bar{t}_1}^{\bar{t}_2}\{X^{(0)}[R_\mu(\bar{t},\bar{a}^v(\bar{t}))]\dot{\bar{a}}^\mu(\bar{t})-X^{(0)}[B(\bar{t},\bar{a}^v(\bar{t}))]\}d\bar{t} \tag{13}$$

其中 $\bar{\gamma}$ 为邻近曲线,则变换前后的差 $A(\bar{\gamma})-A(\gamma)$ 相对 ε 的主线性部分为

$$\begin{aligned}\Delta A=\int_{t_1}^{t2}\Big\{&[X^{(0)}(R_\mu)\dot{a}^\mu-X^{(0)}(B)]\frac{d}{dt}\Delta t+\Big[\frac{\partial X^{(0)}(R_\mu)}{\partial t}\dot{a}^\mu-\frac{\partial X^{(0)}(B)}{\partial t}\Big]\Delta t+\\&\Big[\frac{\partial X^{(0)}(R_\mu)}{\partial a^v}\dot{a}^\mu-\frac{\partial X^{(0)}(B)}{\partial a^v}\Big]\Delta a^v+X^{(0)}(R_\mu)\Delta\dot{a}^\mu\}dt\end{aligned} \tag{14}$$

注意到关系

$$\delta a^\mu=\Delta a^\mu-\dot{a}^\mu\Delta t, \Delta\dot{a}^\mu=\frac{d}{dt}\Delta a^\mu-\dot{a}^\mu\frac{d}{dt}\Delta t \tag{15}$$

式(14)可表为

$$\Delta A=\int_{t_1}^{t2}\varepsilon\{X^{(0)}[X^{(0)}(R_\mu)]\dot{a}^\mu-X^{(0)}[X^{(0)}(B)]-X^{(0)}(B)\dot{\tau}+X^{(0)}(R_\mu)\dot{\xi}_\mu\}dt \tag{16}$$

Noether 对称性是作用量积分在无限小变换下的一种不变性. 如果对于每一个无限小变换(4),始终成立

$$\Delta A=-\int_{t_1}^{t2}\frac{d}{dt}(\Delta G)dt \tag{17}$$

其中 $\Delta G=\varepsilon G$, $G=G(t,a)$ 为规范函数,则这种不变性称为变分问题(7)(8)的 Noether 准对称性. 如 $G=0$,则为 Noether 对称性.

由式(16)和(17),容易得到:

如果存在规范函数 $G=G(t,a)$ 使得无限小生成元 τ, ξ_μ 满足

$$X^{(0)}[X^{(0)}(R_\mu)]\dot{a}^\mu-X^{(0)}[X^{(0)}(B)]-X^{(0)}(B)\dot{\tau}+X^{(0)}(R_\mu)\dot{\xi}_\mu+\dot{G}=0 \tag{18}$$

则相应不变性为变分问题(7)(8)的 Noether 准对称性.

方程(18)可称为变分问题(7)(8)的 Noether 等式.

由 Noether 准对称性可直接导出一类守恒量,有如下结果:

定理 1　对于变分问题(7)(8),如果存在规范函数 $G=G(t,a)$ 使得无限小变换的生成元 τ, ξ_μ 满足 Noether 等式(18),则其 Noether 准对称性直接导致守恒量,形如

$$I = X^{(0)}(R_\mu)\xi_\mu - X^{(0)}(B)\tau + G = \text{const.} \quad (19)$$

守恒量(19)可称为变分问题(7)(8)的Noether守恒量.

3 Noether对称性与Mei对称性的关系

Birkhoff系统的Mei对称性是指Birkhoff方程(1)中的Birkhoff函数B和Birkhoff函数组R_μ在经历无限小变换后仍然满足原方程的一种不变性[25].

关于变分问题(7)(8)的Noether对称性与Birkhoff系统(1)的Mei对称性之间的关系,有如下结果:

定理2 变分问题(7)(8)的Birkhoff方程即为Birkhoff系统(1)的Mei对称性的判据方程.

定理3 变分问题(7)(8)的Noether等式(18)即为Birkhoff系统(1)的Mei对称性的结构方程.

定理4 变分问题(7)(8)的Noether守恒量(19)即为Birkhoff系统(1)的Mei对称性直接导致的Mei守恒量.

4 算例

例1. 考虑天体物理学著名的Emden方程[26],它可化为Birkhoff系统,其Birkhoff函数和Birkhoff函数组为[27]

$$B = \frac{1}{2t^2}(a^2)^2 + \frac{1}{6}t^2(a^1)^6,$$

$$R_1 = -\frac{1}{2}a^2, R_2 = -\frac{3}{2}a^1 \quad (20)$$

其中$t \neq 0$. Birkhoff方程(1)给出

$$-\dot{a}^2 - t^2(a^1)^5 = 0, \dot{a}^1 - \frac{1}{t^2}a^2 = 0 \quad (21)$$

泛函(7)给出

$$A = \int_{t_1}^{t2}\Big\{X^{(0)}\Big(-\frac{1}{2}a^2\Big)\dot{a}^1 + X^{(0)}\Big(-\frac{3}{2}a^1\Big)\dot{a}^2 - X^{(0)}\Big[\frac{1}{2t^2}(a^2)^2 + \frac{1}{6}t^2(a^1)^6\Big]\Big\}dt \quad (22)$$

如取无限小变换的生成元向量为

$$X^{(0)} = t\frac{\partial}{\partial t} - \frac{1}{2}a^1\frac{\partial}{\partial a^1} + \frac{1}{2}a^2\frac{\partial}{\partial a^2} \quad (23)$$

则有

$$A = \int_{t_1}^{t2}\Big\{-\frac{1}{4}a^2\dot{a}^1 + \frac{3}{4}a^1\dot{a}^2 + \frac{1}{2t^2}(a^2)^2 + \frac{1}{6}t^2(a^1)^6\Big\}dt \quad (24)$$

于是有

$$\delta A = \int_{t_1}^{t2}\Big\{-\frac{1}{4}\dot{a}^1\delta a^2 - \frac{1}{4}a^2\delta\dot{a}^1 + \frac{3}{4}\dot{a}^2\delta a^1 + \frac{3}{4}a^1\delta\dot{a}^2 + \frac{1}{t^2}a^2\delta a^2 + t^2(a^1)^5\delta a^1\Big\}dt \quad (25)$$

对式(25)等号右边第二项和第四项进行分部积分,并考虑到边界条件

$$\delta a^1|_{t_1} = \delta a^1|_{t_2} = \delta a^2|_{t_1} = \delta a^2|_{t_2} = 0 \quad (26)$$

以及Birkhoff方程(21),有

$$\delta A = \int_{t_1}^{t2}\Big\{[\dot{a}^2 + t^2(a^1)^5]\delta a^1 + \Big(-\dot{a}^1 + \frac{1}{t^2}a^2\Big)\delta a^2\Big\}dt = 0 \quad (27)$$

计算泛函(24)的变分ΔA,有

$$\Delta A = \int_{t_1}^{t2}\varepsilon\Big\{\frac{d}{dt}\Big[\frac{1}{2t}(a^2)^2 + \frac{1}{2}a^1a^2 + \frac{1}{6}t^3(a^1)^6\Big] + [\dot{a}^2 + t^2(a^1)^5]\Big(-\frac{1}{2}a^1 - \dot{a}^1t\Big) + \Big(-\dot{a}^1 + \frac{1}{t^2}a^2\Big)\Big(\frac{1}{2}a^2 - \dot{a}^2t\Big)\Big\}dt = 0 \quad (28)$$

因此,生成元向量(23)是与泛函(24)相应的变分问题的Noether对称性. 利用方程(21),由式(28)可得

$$I = \frac{1}{2t}(a^2)^2 + \frac{1}{2}a^1a^2 + \frac{1}{6}t^3(a^1)^6 = \text{const.} \quad (29)$$

式(29)是泛函(24)相应的变分问题的Noether守恒量. 同时,由文献[27],生成元向量(23)也是Birkhoff系统(20)的Mei对称性,而式(29)是相应的Mei守恒量.

例2. 已知四阶Birkhoff系统为

$$B = \frac{1}{2}[(a^3)^2 + (a^4)^2] + a^2,$$

$$R_1 = a^3, R_2 = a^4, R_3 = 0, R_4 = 0 \quad (30)$$

Birkhoff方程(1)给出

$$-\dot{a}^3 = 0, -\dot{a}^4 - 1 = 0, \dot{a}^1 - a^3 = 0, \dot{a}^2 - a^4 = 0 \quad (31)$$

泛函(7)给出

$$A = \int_{t_1}^{t2}\Big\{X^{(0)}(a^3)\dot{a}^1 + X^{(0)}(a^4)\dot{a}^2 - X^{(0)}\Big[\frac{1}{2}(a^3)^2 + \frac{1}{2}(a^4)^2 + a^2\Big]\Big\}dt \quad (32)$$

取无限小变换的生成元向量

$$X^{(0)} = \frac{\partial}{\partial a^1} + (a^3)^2 \frac{\partial}{\partial a^2} - 2a^3 \frac{\partial}{\partial a^3} \tag{33}$$

则有

$$A = \int_{t_1}^{t_2} [-2a^3\dot{a}^1 + (a^3)^2] \mathrm{d}t \tag{34}$$

于是有

$$\delta A = \int_{t_1}^{t_2} [2\dot{a}^3\delta a^1 + 2(-\dot{a}^1 + a^3)\delta a^3] \mathrm{d}t = 0 \tag{35}$$

以及

$$\Delta A = \int_{t_1}^{t_2} \varepsilon\left\{\frac{d}{\mathrm{d}t}(-2a^3) + (-2\dot{a}^1 + 2a^3)(-2a^3)\right\}\mathrm{d}t = 0 \tag{36}$$

因此,生成元向量(33)是与泛函(34)相应的变分问题的 Noether 对称性. 利用方程(31),由式(36)得到

$$I = -2a^3 = \text{const.} \tag{37}$$

式(37)是泛函(34)相应的变分问题的 Noether 守恒量. 同时, 生成元向量(33)也是 Birkhoff 系统(30)相应的 Mei 对称性,而式(37)是其 Mei 守恒量.

5　结论

研究对称性与守恒量及其相互之间的关系对于深入理解力学系统的动力学行为及其内在的物理本质具有重要意义. 本文以 Birkhoff 系统为例从变分不变性角度研究 Mei 对称性与 Noether 对称性及其守恒量之间的关系, 结果表明:变分问题(7)(8)导致的动力学方程, Noether 等式以及 Noether 守恒量就是 Birkhoff 系统(1)的 Mei 对称性的判据方程, 结构方程和 Mei 守恒量.

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RELATION BETWEEN THE MEI SYMMETRY AND THE NOETHER SYMMETRY —TAKING THE BIRKHOFF SYSTEM AS AN EXAMPLE*

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Abstract This paper focuses on studying the relation between the Mei symmetry and the Noether symmetry, takes the Birkhoff system as a example. The variational problem for Birkhoffian and Birkhoff's functions under action of infinitesimal generator vectors is studied. The Birkhoff's equations for the variational problem are established. The Noether symmetry for the variational problem is studied and corresponding conserved quantity is given. The studies show that the Birkhoff equations the Noether identity and the Noether conserved quantity of the variational problem are exactly the same with the criterion equation and structural equation and the conserved quantity for Mei symmetry of classical Birkhoff system. In the end of the paper, we take the well-known Emden equation as example to illustrate the application of the results.

Key words Birkhoff system, Mei symmetry, Noether symmetry, conserved quantity, relation

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