

轮-带驱动系统稳态周期响应谐波平衡分析*

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摘要 研究含有单向离合器、两滑轮及附件的轮-带驱动系统稳定稳态周期响应. 通过单向离合器连接从动轮与附属系统, 并计入传送带的横向振动的影响, 导出了由偏微分-积分方程与分段常微分方程组成的连续-离散型非线性耦合方程组. 利用 Galerkin 方法将连续非线性方程组截断为一组非线性常微分方程组, 再运用谐波平衡法得到轮-带驱动系统耦合非线性振动的稳态响应. 通过比较有无单向离合器装置的系统稳定稳态幅频响应曲线, 研究了单向离合器对驱动系统以及轮-带系统非线性动态特性的影响. 并首次研究了高频激励下轮-带系统的稳态响应. 最后, 运用 Runge-Kutta 方法对比验证了基于谐波平衡法得到的稳态响应.

关键词 非线性, 单向离合器, 轮带系统, 谐波平衡法, 稳态响应

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引言

单向离合器已经广泛应用于众多的动力系统, 其主要功能是将因轮-带系统振动而引起的在非驱动方向上动力的传递解耦. 轮-带传动系统在许多机械系统中起着非常重要的作用. 因而这样的系统一直广受研究者关注^[1-3]. 为了确保含单向离合器的皮带传动系统能够稳定工作, 研究系统的振动特性是很有必要的.

Zhu 和 Parker^[4-5] 建立双自由度模型研究了带有单向离合器的两轮皮带系统的非线性动力学模型. 单向离合器简化为一个不连续刚度的非线性弹簧模型, 且离合器扭矩仅在驱动轮与附件轴正向相对运动时传递. 他们发现单向离合器的非线性弹簧类似减震器可以减轻振动. Mocksturm 和 Balaji^[6] 研究了咬合状态时一个自由度、分离状态时两个自由度的含单向离合器动力系统. Cheon^[7] 使用了离散系统研究分析了带有一单向离合器齿轮副的非线性行为. 上官文斌等^[8] 建立了含有单向离合器装置的三轮-多楔带系统非线性振动模型, 并进行了参数优化设计, 使系统中张紧臂角度、单向离合器扭矩等波动的幅值得到了降低.

以上的这些研究工作都忽略了驱动带的横向

振动, 将带模型化为无质量的弹簧. Ding 和 Zu^[9-10] 建立了一个描述驱动轮、从动轮和附件轴旋转的模型, 同时考虑了传送带的横向振动, 通过 Galerkin 截断^[11-12] 结合 Runge-Kutta 方法, 研究了低频激励下, 单向离合器对系统耦合振动的影响.

本文采用基于傅里叶变换的谐波平衡法^[13] 求解轮-带驱动系统耦合非线性振动的稳态响应. 通过比较系统有无单向离合器装置的稳态幅频响应曲线, 研究了单向离合器对动力系统的影响. 并考察高频激励下轮-带驱动系统的稳态幅频响应. 最后, 运用 Runge-Kutta 算法对比验证基于谐波平衡法得到的稳态响应结果.

1 振动控制方程

本文采用两个滑轮和一个单向离合器的皮带驱动系统模型^[14], 如图 1 所示. 从动滑轮和附件轴由刚度为 K_a 的环绕弹簧连接. 当从动滑轮与附件轴分离时, 弹簧被断开, 与轴刚性的连接在一起的附属部分起负载作用. 假设传送带属性(带密度 ρ , 带模量 EA , 其中 E 是杨氏模量; A 是横截面的面积)是均匀的. 皮带的轴向平移速度 c 也被假设是恒定和均匀的. $\theta_i(t)$ 、 J_i 、 r_i 以及 M_i 分别是角位移、转动惯量、半径以及从动滑轮和驱动滑轮的预力

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矩. J_a 和 $\theta_a(t)$ 分别是附属部分转动惯量和角位移. 当从动滑轮的角位移小于附属的角位移时, 单向离合器分离. $l_i (i=1, 2)$ 为带段 i 的长度. 中性轴的坐标 x 和时间 t , $w_i(x, t)$ 和 $u_i (i=1, 2)$ 分别为带跨度 i 的横向位移和纵向位移. 假设驱动滑轮与从动滑轮具有相同大小, 即 $l_1 = l_2 = l$, $J_1 = J_2 = J$, $r_1 = r_2 = r'$. 此外, 在传动系统的建模中作如下假设: (I) 皮带的弯曲刚度很小可以忽略不计, 因此皮带可以简化为一个弹簧模型; (II) 忽略滑轮上的皮带打滑情况.

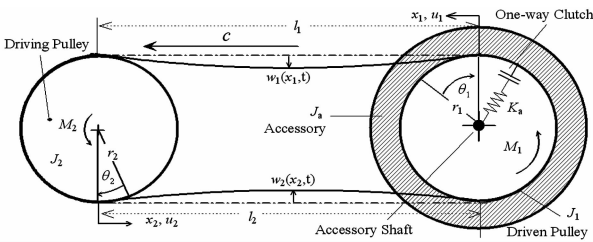


图1 含两滑轮和单向离合器的带传动系统

Fig. 1 Belt-drive system with two pulleys and a one-way clutch

传送带的运动方程为

$$\rho A(w_{i,tt} + 2cw_{i,x_t} + c^2w_{i,x_t x_t}) - P_0w_{i,x_t x_t} + c_s Aw_{i,t} = \left\{ \frac{EA}{2l_i} \int_0^l w_{i,x_1}^2 dx + \frac{EA}{l_i} [u_i(l_i, t) - u_i(0, t)] \right\} w_{i,x_t x_t}, \quad (i=1, 2) \quad (1)$$

其中下标 x 和 t 是分别指对 x 和 t 的偏微分, P_0 为传送带的轴向初始静态张力. 传送带的边界条件为

$$w_i(0, t) = 0, \quad w_i(l_i, t) = 0, \quad (i=1, 2) \quad (2)$$

主动带轮的运动方程为

$$J_2 \ddot{\theta}_2 + c_b \dot{\theta}_2 = r' \left[\frac{EA}{2l_2} \int_0^l w_{2,x_2}^2 dx + \frac{EA r'}{l_2} (\theta_1 - \theta_2) \right] - r' \left[\frac{EA}{2l_1} \int_0^l w_{1,x_1}^2 dx + \frac{EA r'}{l_1} (\theta_2 - \theta_1) \right] + M_2 \quad (3)$$

从动滑轮和附件轴的运动方程为

$$J_1 \ddot{\theta}_p + c_b \dot{\theta}_p = r' \left[\frac{EA}{2l_1} \int_0^l w_{1,x_1}^2 dx + \frac{EA r'}{l_1} (\theta_2 - \theta_p) \right] - r' \left[\frac{EA}{2l_2} \int_0^l w_{2,x_2}^2 dx + \frac{EA r'}{l_2} (\theta_p - \theta_2) \right] + M_1 - T_i(\delta\theta) \quad (4)$$

$$J_a \ddot{\theta}_a + c_a \dot{\theta}_a = M_1 + T_i(\delta\theta)$$

其中 c_a 为离合器阻尼系数, $\delta\theta = \theta_p - \theta_a$, $T_i(\delta\theta)$ 为离合器扭矩. 单向离合器的咬合与分离基于从动滑轮与附件轴间的相对角位移. 当从动滑轮的旋转角位移 θ_p 大于附件轴的旋转角位移 θ_a 时, 离合器咬合. 相应的, 当从动滑轮旋转的角位移小于附件轴旋转的角位移时附件轴的运动从动滑轮中解耦出来, 此时, 从动滑轮

与附件轴之间的没有了机械连接, 即仅当从动滑轮与附件轴咬合时, 从动滑轮才向附件轴传递力矩. 通过离合器传递的扭矩可用分段函数表示

$$T_i(\delta\theta) = f(\delta\theta) K_a(\delta\theta); \quad f(\delta\theta) = \begin{cases} 1 & \delta\theta > 0 \\ 0 & \delta\theta \leq 0 \end{cases} \quad (5)$$

在本文中, 通过下列无量纲变量和参数将方程(1)、(3)、(4)表示为无量纲化形式

$$\begin{aligned} w_i &\leftrightarrow \frac{w_i}{l}, \quad t \leftrightarrow t \sqrt{\frac{P_0}{\rho A l^2}}, \quad c \leftrightarrow c \sqrt{\frac{\rho A}{P_0}}, \\ c_a &\leftrightarrow \frac{c_a}{l r_a} \sqrt{\frac{1}{\rho A P_0}}, \quad x_i \leftrightarrow \frac{x_i}{l}, \quad c_s \leftrightarrow c_s \sqrt{\frac{A l^2}{\rho P_0}}, \\ M_i &\leftrightarrow \frac{M_i}{P_0 r'}, \quad k_1 = \frac{EA}{P_0}, \quad K_a \leftrightarrow \frac{K_a}{P_0 r_a}, \quad J \leftrightarrow \frac{J}{\rho A r' l^2}, \\ J_a &\leftrightarrow \frac{J_a}{\rho A r_a l^2}, \quad c_b \leftrightarrow \frac{c_b}{l r'} \sqrt{\frac{1}{\rho A P_0}}, \end{aligned} \quad (6)$$

其中无量纲参数 c 为轴向速度. 无量纲参数 c_s 、 c_b 和 c_a 分别为皮带横向运动的阻尼系数、皮带与滑轮间的摩擦阻尼系数和附件的阻尼系数. 无量纲参数 k_1 被称为非线性系数, 表征传动带的非线性影响. 轴向传动带的横向运动和驱动皮带滑轮、从动滑轮与附件轴的无量纲方程可由方程(6)定义的新参数表示. 带的无量纲边界条件与有量纲条件相同, 由方程(2)描述. 驱动滑轮是基于引擎特性设定的, 驱动滑轮以 $\theta_2 = A_m \cos \omega t$ 的角速度运动. 无量纲参数 A_m (本文取 $A_m = 0.01$) 表示驱动轮带运动的振幅. 保留方程(7)和(8)中所有包含 θ_2 的项作为激励项. 激励频率 ω 为驱动滑轮转动速度的 s 倍, 其中 s 表示 $2s$ 汽缸发动机 (本文采用 $s=3$). 带的转动与引擎速度关系式为 $c = r_1 \omega / s l$. 引用无量纲参数 $\omega \sqrt{\rho A l^2 / P_0}$ 代替激励频率 ω .

$$\begin{aligned} w_{1,u} + 2cw_{1,x_t} + (c^2 - 1)w_{1,x_1 x_1} + c_s w_{1,t} = & \left[\frac{r'}{l} k_1 (\theta_2 - \theta_1) + \frac{k_1}{2} \int_0^l w_{1,x_1}^2 dx_1 \right] w_{1,x_1 x_1}, \\ w_{2,u} + 2cw_{2,x_2 t} + (c^2 - 1)w_{2,x_2 x_2} + c_s w_{2,t} = & \left[\frac{r'}{l} k_1 (\theta_1 - \theta_2) + \frac{k_1}{2} \int_0^l w_{2,x_2}^2 dx_2 \right] w_{2,x_2 x_2} \quad (7) \end{aligned}$$

$$\begin{aligned} J \ddot{\theta}_p = -c_b \dot{\theta}_p + k_1 \left[\frac{1}{2} \int_0^l w_{1,x_1}^2 dx_1 - \right. & \left. \frac{1}{2} \int_0^l w_{2,x_2}^2 dx_2 + 2 \frac{r'}{l} (\theta_2 - \theta_p) \right] + M_1 - \\ f(\delta\theta) \frac{r_a}{r'} K_a (\theta_p - \theta_a), & \quad (8) \end{aligned}$$

$$J_a \ddot{\theta}_a = -c_a \dot{\theta}_a - M_1 + f(\delta\theta) K_a (\theta_p - \theta_a)$$

2 Galerkin 截断

在本研究中,为了减少谐波平衡法中因为傅里叶变换与逆变换次数的增加带来的误差的影响,在不影响结果准确性的前提下减小方程运算复杂程度,故只需采用2阶 Galerkin 截断.此外,选用边界条件下静态张力弹性弹簧的本征函数为试函数,假设方程(7)的解采用如下形式

$$\begin{aligned} w_1(x_1, t) &= \sin(\pi x_1)q_1(t) + \sin(2\pi x_1)q_2(t), \\ w_2(x_2, t) &= \sin(\pi x_2)q_3(t) + \sin(2\pi x_2)q_4(t) \end{aligned} \quad (9)$$

其中 $q_i(t)$ ($i=1, 2$) 和 $q_i(t)$ ($i=3, 4$) 分别为带1和带2一系列广义位移.将方程(9)代入方程(7)得到下列一系列二阶常微分方程

$$\begin{aligned} &\sum_{i=1}^2 \sin(i\pi x_1) \ddot{q}_i + 2c \sum_{i=1}^2 [i\pi \cos(i\pi x_1)] \dot{q}_i + \\ &c_s \sum_{i=1}^2 \sin(i\pi x_1) \dot{q}_i = -[c^2 - 1 - I_1 - \frac{k_1 r'}{l}(\theta_2 - \theta_p)] \cdot \\ &\sum_{i=1}^2 [-i^2 \pi^2 \sin(i\pi x_1)] q_i, \\ &\sum_{i=3}^4 \sin[(i-2)\pi x_2] \ddot{q}_i + 2c \sum_{i=3}^4 [(i-2)\pi \cos[(i-2)\pi x_2]] \dot{q}_i + \\ &c_s \sum_{i=3}^4 \sin[(i-2)\pi x_2] \dot{q}_i = \\ &-[c^2 - 1 - I_2 - \frac{k_1 r'}{l}(\theta_2 - \theta_p)] \cdot \\ &\sum_{i=3}^4 [-(i-2)^2 \pi^2 \sin[(i-2)\pi x_2]] q_i \end{aligned} \quad (10)$$

$$\begin{aligned} I_1 &= \frac{k_1}{2} \int_0^1 [w_1(x_1, t),_{x_1}]^2 dx_1 = \frac{\pi^2 k_1}{4} (q_1^2 + 4q_2^2), \\ I_2 &= \frac{k_1}{2} \int_0^1 [w_2(x_2, t),_{x_2}]^2 dx_2 = \frac{\pi^2 k_1}{4} (q_3^2 + 4q_4^2) \end{aligned} \quad (11)$$

将方程(10)的第一个等式乘以 $\sin(k\pi x_1)$ ($k=1, 2$),第二个等式乘以 $\sin(k\pi x_2)$ ($k=1, 2$)并在区间 $[0, 1]$ 积分,得

$$\begin{aligned} \ddot{q}_1 &= -c_s \dot{q}_1 + \frac{16}{3} c \dot{q}_2 + \pi^2 [c^2 - 1 - I_1 - \frac{k_1 r'}{l}(\theta_2 - \theta_p)] q_1, \\ \ddot{q}_2 &= -c_s \dot{q}_2 - \frac{16}{3} c \dot{q}_1 + 4\pi^2 [c^2 - 1 - I_1 - \frac{k_1 r'}{l}(\theta_2 - \theta_p)] q_2, \\ \ddot{q}_3 &= -c_s \dot{q}_3 + \frac{16}{3} c \dot{q}_4 + \pi^2 [c^2 - 1 - I_2 - \frac{k_1 r'}{l}(\theta_p - \theta_2)] q_3, \\ \ddot{q}_4 &= -c_s \dot{q}_4 - \frac{16}{3} c \dot{q}_3 + 4\pi^2 [c^2 - 1 - I_2 - \frac{k_1 r'}{l}(\theta_p - \theta_2)] q_4, \end{aligned} \quad (12)$$

$$J\dot{\theta}_p + c_b \dot{\theta}_p = M_1 + I_1 - I_2 + \frac{2k_1 r'}{l}(\theta_2 - \theta_p) +$$

$$\begin{aligned} &f(\delta\theta) \frac{r_a K_a}{r'} (\theta_p - \theta_a) J_a \ddot{\theta}_a + c_a \dot{\theta}_a = \\ &f(\delta\theta) K_a (\theta_p - \theta_a) - \frac{r' M_1}{r_a} \end{aligned} \quad (13)$$

3 谐波平衡法

考虑到该系统响应具有周期性,故可以将所有解展开为 R 次谐波傅里叶级数^[11]:

$$\begin{aligned} q_x(t) &= u_{x,1} + \sum_{r=1}^R (u_{x,2r} \cos r\Omega t + u_{x,2r+1} \sin r\Omega t), \\ x &= 1, 2, 3, 4 \\ \theta_p(t) &= u_{p,1} + \sum_{r=1}^R (u_{p,2r} \cos r\Omega t + u_{p,2r+1} \sin r\Omega t) \\ \theta_a(t) &= u_{a,1} + \sum_{r=1}^R (u_{a,2r} \cos r\Omega t + u_{a,2r+1} \sin r\Omega t) \end{aligned} \quad (14)$$

由于方程的相似性可将式(12)分解后统一写为:

$$\begin{aligned} \ddot{q}_m + c_s \dot{q}_m + C_n \dot{q}_n + a q_m + A q_m \cos \Omega t + \\ f(q_m, q_n, \theta_p) q_m = 0 \end{aligned} \quad (15)$$

由周期性可知 $q_x(\theta) = q_x(\theta + T)$, $\theta_p = \theta_p(\theta + T)$,故 $f(q_x, \theta_p) = f(\theta + T)$,周期函数 $f(q_x, \theta_p)$ 进行 Fourier 级数展开

$$\begin{aligned} f(q_m, q_n, \theta_1) = \\ f_1 + \sum_{r=1}^R [f_{2r} \cos(r\Omega t) + f_{2r+1} \sin(r\Omega t)] \end{aligned} \quad (16)$$

将展开为 Fourier 级数的各项(14)、(16)代入(15)中,令方程两边的常数项、各次正弦谐波、余弦谐波的系数相等,得到由 $(2R+1)N$ 个方程构成的代数方程组 $s_i = 0$.

$$\begin{aligned} s_1 &= a u_{m,1} + f_1 u_1 + \frac{1}{2} A u_{m,2} + \\ &\frac{1}{2} \sum_{i=1}^R (u_{m,2i} f_{2i} + u_{m,2i+1} f_{2i+1}), \\ s_{2r} &= -(\Omega)^2 u_{m,2r} + r\Omega (c_s u_{m,2r+1} + C_n u_{n,2r+1}) + \\ &a u_{m,2r} + A u_{m,1} + \frac{1}{2} \sum_{i=2}^R A [u_{m,2(i+1)} + u_{m,2(i-1)}] + \\ &f_1 u_{m,2r} + f_{2r} u_{m,1} + \frac{1}{2} \sum_{i=1}^R u_{m,2i} [f_{2(i-r)} + \\ &f_{2(i+r)} - f_{2(r-i)}] + \frac{1}{2} \sum_{i=1}^R u_{m,2i+1} [f_{2(i-r)+1} + \\ &f_{2(i+r)+1} - f_{2(r-i)+1}], \quad r=1, 2, 3, \dots, R \\ s_{2r+1} &= -(\Omega)^2 u_{m,2r+1} - r\Omega (c_s u_{m,2r} + C_n u_{n,2r}) + \\ &a u_{m,2r+1} + \frac{1}{2} \sum_{i=1}^R A [u_{m,2(i+1)+1} + u_{m,2(i-1)+1}] + \\ &f_1 u_{m,2r+1} + f_{2r+1} u_{m,1} + \frac{1}{2} \sum_{i=1}^R u_{m,2i} [-f_{2(i-r)+1} + \\ &f_{2(i+r)+1} + f_{2(r-i)+1}] + \frac{1}{2} \sum_{i=1}^R u_{m,2i+1} [f_{2(i-r)} - \end{aligned}$$

$$f_{2(i+r)} + f_{2(r-i)}], \quad r=1,2,3,\dots,R \quad (17)$$

对于方程(13),为了方便对系统动态响应进行迭代求解,将单向离合器弹簧扭矩中分段函数部分用双曲正切函数表达成连续函数形式

$$g(\delta\theta) = \begin{cases} f(\delta\theta)(\theta_p - \theta_a)\delta & \theta > 0 \\ 0 & \delta\theta \leq 0 \end{cases} \quad (18)$$

$$G(\delta\theta) = 0.5[1 + \tanh(\varepsilon\delta\theta)]\delta\theta, \quad \delta\theta = \theta_p - \theta_a \quad (19)$$

其中 ε 取 10000^[4]. 由于响应的周期性,方程(13)同样可以通过傅里叶变换谐波平衡方法得出相对应的平衡方程,此时 q_x 用傅里叶展开形式代入,分别令这两个方程两边的常数项、各次正弦谐波、余弦谐波的系数相等得到如下方程组:

$$\begin{aligned} s_{p,1} &= \frac{2k_1 r'}{l} u_{p,1} - \frac{r_a K_a}{r} F_1 - M_1 - \\ &\frac{\pi^2 k_1^2}{4} (u_{1,1}^2 + 4u_{2,1}^2 - u_{3,1}^2 - 4u_{4,1}^2) - \\ &\frac{1}{2} \frac{k_1 \pi^2}{4} \sum_{i=1}^R \sum_{m=1}^4 h(m) (u_{m,2i}^2 + u_{m,2i+1}^2), \\ s_{p,2r} &= -J(r\Omega)^2 u_{p,2r} + r\Omega c_b u_{p,2r+1} - \frac{2k_1 r'}{l} A + \\ &\frac{2k_1 r'}{l} u_{p,2r} - \frac{1}{2} h(m) \frac{k_1 \pi^2}{4} \sum_{i=1}^R u_{m,2i} [u_{m,2(i-r)} + \\ &u_{m,2(i+r)} + u_{m,2(r-i)}] - \frac{1}{2} h(m) \frac{k_1 \pi^2}{4} \cdot \\ &\sum_{i=1}^R u_{m,2i+1} [u_{m,2(i-r)+1} + u_{m,2(i+r)+1} - u_{m,2(r-i)+1}] - \\ &\frac{r_a K_a}{r} F_{2r} - 2 \sum_{m=1}^4 h(m) u_{m,1} u_{m,2r}, \quad r=1,2,3,\dots,R \\ s_{p,2r+1} &= -J(r\Omega)^2 u_{p,2r+1} - r\Omega c_b u_{m,2r} + \frac{2k_1 r'}{l} u_{p,2r+1} - \\ &\frac{1}{2} h(m) \frac{k_1 \pi^2}{4} \sum_{i=1}^R u_{m,2i} [-u_{m,2(i-r)+1} + u_{m,2(i+r)+1} + \\ &u_{m,2(r-i)+1}] - \frac{1}{2} h(m) \frac{k_1 \pi^2}{4} \sum_{i=1}^R u_{m,2i+1} [u_{m,2(i-r)} - \\ &u_{m,2(i+r)} + u_{m,2(r-i)}] - \frac{r_a K_a}{r} F_{2r+1} - \\ &2 \sum_{m=1}^4 h(m) u_{m,1} u_{m,2r+1}, \quad r=1,2,3,\dots,R \end{aligned} \quad (20)$$

$$s_{a,1} = \frac{2k_1 r'}{l} u_{a,1} - \frac{r'}{r_a} M_1 - K_a F_1,$$

$$s_{a,2r} = - (r\Omega)^2 J_a u_{a,2r} + r\Omega c_a u_{a,2r+1} - K_a F_{2r},$$

$$r=1,2,3,\dots,R$$

$$s_{a,2r+1} = - (r\Omega)^2 J_a u_{a,2r+1} - r\Omega c_a u_{a,2r} - K_a F_{2r+1},$$

$$r=1,2,3,\dots,R \quad (21)$$

其中当 r 不等于 1 时, $-2k_1 r' A/l$ 项为 0, 系数 $h(m)$ 在 $m=1,2,3,4$ 时分别等于 1, 4, -1, -4. 在非线性代数方程组(17)、(20)和(21)中,稳态响应 q_x, θ_p, θ_a 的 Fourier 级数各项系数 u_r 是待求的未知量,由于 $f(q_x, \theta_p), G(\delta\theta)$ 是关于 q_x, θ_p, θ_a 的函数,函数 $f(q_x, \theta_p), G(\delta\theta)$ 对应的 Fourier 级数各次谐波系数 f_r, F_r 也是未知量 u_r 的函数,因此,求解之前必须把 f_r, F_r 表示成 u_r 的表达式. 由于 f_r, F_r 和 u_r 是各自函数的 Fourier 级数系数,采用 Fourier 变换和逆变换实现两者之间的联系,步骤如下:

(1) 离散傅里叶逆变换 (IDFT): 给定变量 q_x, θ_p, θ_a 的 Fourier 系数 u_r , 通过离散傅里叶变换的逆变换得到 q_x, θ_p, θ_a 在一个时域周期之内的离散时间序列 $q_n, \theta_{pn}, \theta_{an}$

$$\begin{aligned} q_{x,n} &= u_{x,1} + \sum_{r=1}^R [u_{x,2r} \cos(\frac{2\pi r n}{N}) + \\ &u_{x,2r+1} \sin(\frac{2\pi r n}{N})], \quad n \in [0, N-1] \\ \theta_{p,n} &= u_{p,1} + \sum_{r=1}^R [u_{p,2r} \cos(\frac{2\pi r n}{N}) + \\ &u_{p,2r+1} \sin(\frac{2\pi r n}{N})], \quad n \in [0, N-1] \\ \theta_{a,n} &= u_{a,1} + \sum_{r=1}^R [u_{a,2r} \cos(\frac{2\pi r n}{N}) + \\ &u_{a,2r+1} \sin(\frac{2\pi r n}{N})], \quad n \in [0, N-1] \end{aligned} \quad (22)$$

由 q_x, θ_p, θ_a 的时间序列 $q_n, \theta_{pn}, \theta_{an}$ 确定 $f_n(q_x, \theta_p)$ 的时间序列 g_n

$$\begin{aligned} g_n &= f_n(q_n, \theta_{pn}, \theta_{pn}) \quad n \in [0, N-1] \\ G_n &= F_n(\theta_p, \theta_a) \quad n \in [0, N-1] \end{aligned} \quad (23)$$

(2) 离散傅里叶变换 (DFT): 对 g_n, G_n 进行傅里叶变换可得到频域内 f, F 的傅里叶系数 f_r, F_r

$$\begin{aligned} f_1 &= \frac{1}{N} \sum_{n=0}^{N-1} g_n, \quad F_1 = \frac{1}{N} \sum_{n=0}^{N-1} G_n, \\ f_{2r} &= \frac{2}{N} \sum_{n=0}^{N-1} g_n \cos(\frac{2\pi r n}{N}), \\ F_{2r} &= \frac{2}{N} \sum_{n=0}^{N-1} G_n \cos(\frac{2\pi r n}{N}), \\ f_{2r+1} &= \frac{2}{N} \sum_{n=0}^{N-1} g_n \sin(\frac{2\pi r n}{N}), \\ F_{2r+1} &= \frac{2}{N} \sum_{n=0}^{N-1} G_n \sin(\frac{2\pi r n}{N}), \quad r \in [1, R] \end{aligned} \quad (24)$$

最终代入式(17)、(20)和(21)中,得到只关于未知变量 u 的非线性代数方程组. 本文采用基于拟牛顿法的 Broyden 方法求解非线性代数方程组. 给

出 u 的一组初值,用迭代方法求解即可得到系统的稳态响应.

4 稳态响应

表 1 列出了皮带传动系统的物理和几何特性^[5-6]和方程(11)中的无量纲参数计算值.

表 1 含单向离合器的传送带属性

Table 1 Properties of the belt - drive with one-way clutch

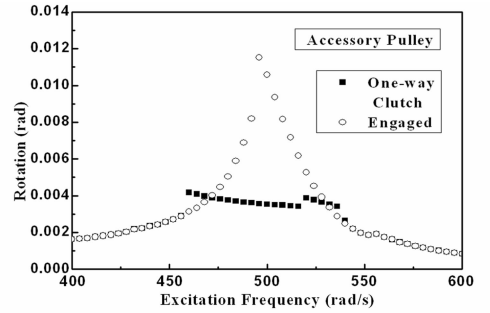
Item	Value	Dimensionless Value
Radius of pulleys r_1, r_2	0.042m	-
radius of accessory r_a	0.0839m	-
rotation inertia J_1	0.001607kg/m ² /rad	0.873761
Rotation inertia of accessory J_a	1.62 J_1	0.70859
Preload on Driven pulley M_0	3 N m	0.230415
Friction coefficient c_b	0.035 Nm s/rad	0.237823
Length of span $i l_i (i = 1, 2)$	m	1
Young's modulus E	2×10^9 N/m ²	-
linear viscous damping c_s	20000 Ns/m	0.18483
Belt cross - section A	10^{-4} m ²	-
Belt density ρ	1150 kg/m ³	-
static tension P_0	310 N	-
Moment of inertia of the belt cross-section I	2.08×10^{-10} m ⁴	-
Nonlinear coefficient k_1	-	645.161
Torsional Stiffness K_a	1200 Nm/rad	46.13767
Friction coefficient c_a	0.05 Nm s/rad	0.126618

4.1 谐波平衡法

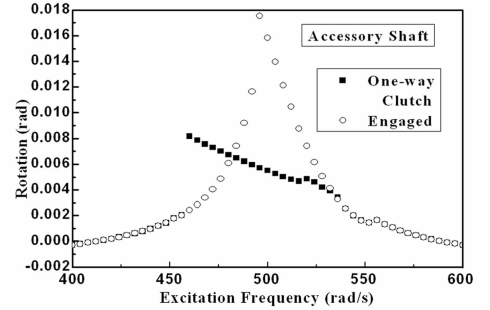
图 2(a)和 2(b)分别表示从动滑轮和附件轴有无离合器时的稳态响应.从图中可以看到跳跃现象及典型的非线性现象;从动滑轮与附件轴的共振区域宽于带段;单向离合器减小了从动滑轮和附件轴的共振.同时带的非线性张力运动作用使从动滑轮和附件轴在不含离合器时均产生了跳跃现象.而在含单向离合器时,在从动滑轮和附件轴图形中均可看到振峰的软化.由此可知,软化非线性是单向离合器作用产生的结果.也就是说,单向离合器使稳态幅频响应曲线向左侧弯曲.由图 2(c)和 2(d)可知,平移带共振区域的减少是由于皮带传动系统含有离合器的作用.此外,不含离合器的带段的稳态响应表明共振高峰的软化和硬化非线性均在同一时间.而含有单向离合器时,平移带的共振高峰的软化非线性消失.

图 3 给出了系统在频率范围为[1000,2000]的共振稳态幅频特性曲线,其中 $A_m = 0.014$, $K_a = 1300$.从图中仍可以看到跳跃现象及典型的非线性现象;从动滑轮的共振区域宽于带段.图 3(a)表

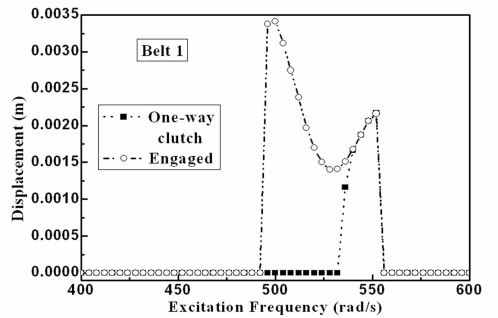
明,单向离合装置能够有效的降低共振时的响应振幅.但是,图 3(b)显示,虽然单向离合装置减小了带的共振响应振幅,却使得带的共振区域增大了.同时联系图 3(a)与图 3(b)发现,轮-带系统综合时段段 1 在频率[1100,1200]区间没有共振而含单向离合器时却发生了振幅较小的共振,这是由于单向离合器对轮-带系统作用时导致系统固有频率发生改变而引起的.



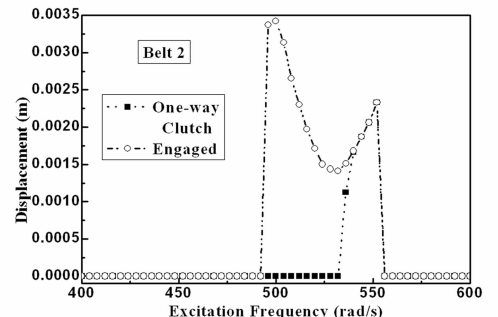
(a) the comparison of the driven pulley



(b) the comparison of the accessory shaft



(c) the comparison of the belt span 1



(d) the comparison of the belt span 2

图 2 比较有无单向离合器的系统稳态响应曲线

Fig. 2 The comparisons of the steady-state response with and without clutch

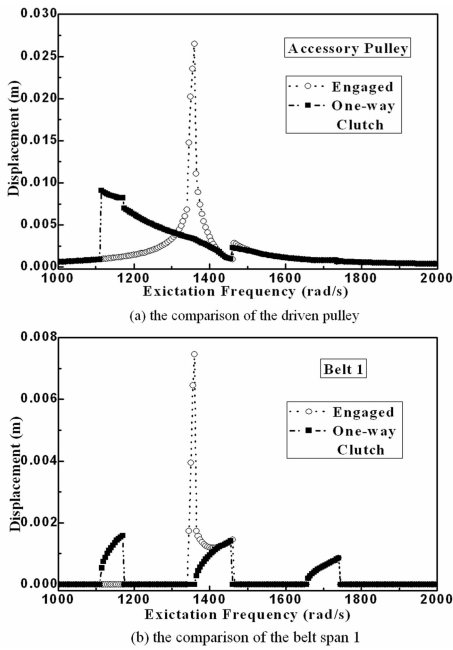


图 3 比较有无单向离合器的系统稳态响应曲线
Fig. 3 The comparisons of the steady-state response with and without one-way clutch

4.2 数值验证

当该轮带动力系统中所有参数均与文献^[10]相同时, 通过对比谐波平衡法与 4 阶 Runge-Kutta 方法在系统有无离合器作用下所得到的稳态响应曲线如图 4 所示, 从图中对比附件轴和带段 1 在有单向离合器作用下的图形, 可知谐波平衡法与 Runge-Kutta 方法所得到的结果是非常的接近的。

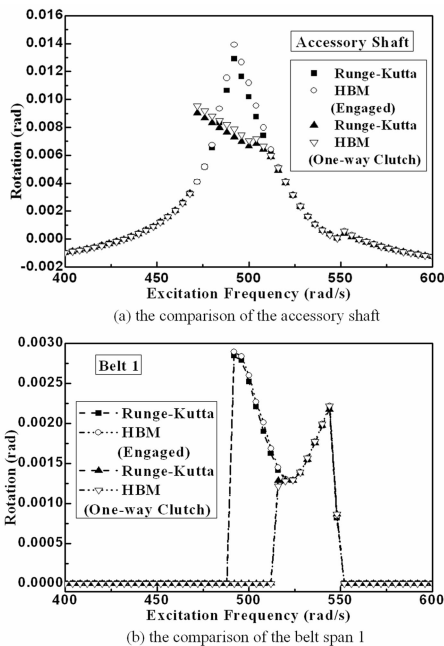


图 4 谐波平衡法与 Runge-Kutta 方法的比较
Fig. 4 The comparisons of the harmonic balance method and the Runge-Kutta algorithm

5 结论

研究了一个非线性离散-连续耦合轮-带驱动系统, 该系统中驱动轮、从动轮和附件轴的旋转运动与传动带的横向振动相耦合. 利用 Galerkin 截断法将非线性连续方程组离散化为一组二阶非线性常微分方程, 并运用谐波平衡法进行求解系统的稳定稳态响应. 通过与 4 阶 Runge-Kutta 方法对比, 验证了本文的谐波平衡法具有精度. 研究发现, 对于低频共振, 单向离合器不仅降低了从动滑轮和附件轴旋转振动共振的振幅, 而且还减小了带横向振动的共振区域. 同时观察到带的非线性张力运动作用使从动滑轮和附件轴在不含离合器时也会产生软特性的跳跃现象. 另外, 高频激励的共振响应分析表明, 单向离合装置在减小了带的共振响应振幅的同时, 增大了带的共振区域。

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THE FREQUENCY-RESPONSE CURVE OF A BELT-DRIVE SYSTEM BY USING THE HARMONIC BALANCE METHOD*

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Abstract This paper studied a belt-drive system containing a translating belt, two pulleys, accessory systems, and a one-way clutch. The driven pulley and the accessory system were coupled by the one-way clutch. Moreover, the transverse vibration of the translating belt was taken into account. Therefore, the derived coupled discrete-continuous nonlinear equations consist of integro-partial-differential equations and piece-wise ordinary differential equations. The continuous nonlinear equations were truncated to a set of nonlinear ordinary differential equations by using the Galerkin method. The steady-state responses of the coupled vibration of the belt-drive system were investigated by the numerical harmonic balance method. Moreover, the effects of the one-way clutch on the power system were studied by comparing the frequency-response curve of the translating belt with and without one-way clutch device. The nonlinear dynamic characteristics of the belt-drive system were studied. Furthermore, the steady-state responses of the belt-drive system were investigated at the higher excitation frequency for the first time. In addition, the steady-state responses based on the harmonic balance method were confirmed by the Runge-Kutta algorithm.

Key words nonlinearity, one-way clutch, belt-drive system, harmonic balance method, steady-state response

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