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复合材料悬臂外伸板的非线性动力学建模及数值研究*

吕书锋　张伟†

（北京工业大学机电学院，北京　100124）

摘要　研究了横向气动载荷和参数激励联合作用下复合材料悬臂外伸矩形板在伸出过程中的非线性动力学问题. 根据 Reddy 的高阶剪切层合板理论，应用 Hamilton 原理建立了外伸板在横向气动力和参数激励作用下的非线性动力学方程，其中横向气动力采用一阶活塞气动力. 然后应用 Galerkin 方法对系统偏微分形式的非线性方程进行离散，得到了一组时变系数的非线性动力学方程. 在此方程的基础上，对复合材料悬臂外伸板进行了数值模拟分析，讨论了外伸速度对悬臂外伸板非线性动力学特性的影响.

关键词　复合材料悬臂外伸板，　高阶剪切理论，　活塞理论，　Hamilton 原理，　非线性动力学

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引言

近年来，轴向悬臂外伸结构在工程中的应用越来越常见，如新型可变形机翼、太阳帆板和天线、机械手臂等. 这类结构因其沿轴向是可运动的，相比于不可移动、静止的结构，当其沿轴向运动或受到外载荷的作用时，可能带来一些新的、影响结构稳定性的动力学问题. 因此研究其运动过程中的非线性动力学特性对工程应用具有很重要的价值.

近年来，轴向悬臂可外伸结构的研究已引起了学者们的关注. Tabarrok 等[1]推导了长度随时间变化梁的运动方程，得到四个非线性偏微分形式运动方程和一个几何关系方程. Taleb 和 Misra[2]研究了不可压缩流体中，以恒定速度外伸的等环形截面悬臂梁的小变形横向振动. Wang 和 Wei[3]将一个柔性机械手臂建模为细长的可移动悬臂梁模型，研究了梁在外伸过程中的运动特性. Fung 等[4]利用四种不同的梁理论建立了带尖端质量移动梁的非线性动力学模型. Theodore 等[5]根据 Euler-Bernoulli 梁理论推导了轴向可外伸柔性梁的运动方程，并通过数值模拟的方法研究了其横向振动. Behdinan 和 Tabarrok 等[6]应用 Hamilton 原理推导了可外伸柔性梁的运动方程. Gosselin 等[7]研究了等环形截面柔细悬臂梁在不可压缩稠密流体中以恒定速度伸展的稳定性. 研究类似模型的还有，Poivan 和 Sampaio[8]研究了轴向运动功能梯度柔性梁的振动问题. Wang 等[9]推导了一种可外伸悬臂板模型线性振动的动力学方程，分析了运动过程中的频率和系统稳定性. 张谦等[10]设计了一种可外伸机翼结构，实验研究了该结构在不同外伸速度下的动力学响应. 本文选取沿轴向可外伸的复合材料悬臂矩形板作为研究对象，对其在活塞气动力[11]和面内参数激励联合作用下的非线性振动特性进行分析.

1　基本方程

考虑一个沿轴向可外伸的复合材料层合矩形悬臂板，该板受到面内均布简谐激励和横向气动载荷共同作用. 板以初始长度 l_0 开始沿 x 轴方向悬臂向外伸出，板宽为 b，厚度为 h，笛卡尔坐标系 Oxy 位于板的中面，如图 1 所示. 外伸速度考虑了一个小扰动的影响，形式为 $V=V_0+V_d\cos(\Omega_2 t)$，面内激励的形式为 $f_y=f_0+f_1\cos\Omega_1 t$，横向的气动力载荷采用一阶活塞气动力，记为 Δp.

根据 Reddy 的高阶剪切层合板理论[12]，位移

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† 通讯作者 E-mail：sandyzhang0@yahoo.com

场可以写为

$$u(x,y,t)=u_0(x(t),y,t)+z\varphi_x(x(t),y,t)-z^3\frac{4}{3h^2}\left(\varphi_x+\frac{\partial w_0}{\partial x}\right),\tag{1a}$$

$$v(x,y,t)=v_0(x(t),y,t)+z\varphi_y(x(t),y,t)-z^3\frac{4}{3h^2}\left(\varphi_y+\frac{\partial w_0}{\partial y}\right),\tag{1b}$$

$$w(x,y,t)=w_0(x(t),y,t),\tag{1c}$$

其中，u_0,v_0,w_0 为中面上任意一点分别沿 x,y,z 方向的位移，φ_x 和 φ_y 分别为绕 y 和 x 轴的转角.

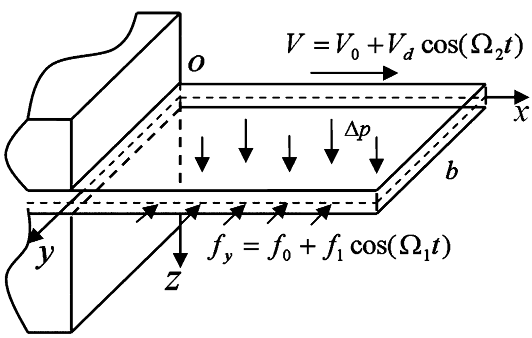

图 1　悬臂外伸板的力学模型

Fig. 1　The model of deploying cantilever plate

采用 von Karman 的大变形几何关系，可以得到应变-位移关系 $\varepsilon_i(i=xx,yy)$ 和曲率-位移关系 $\gamma_i(i=xy,yz,zx)$ 的表达式

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,\ \varepsilon_{yy}=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,$$

$$\gamma_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right),$$

$$\gamma_{yz}=\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right),\ \gamma_{zx}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right).\tag{2}$$

根据 Hamilton 原理，可推得可伸缩悬臂板的非线性动力学方程为

$$A_{11}\frac{\partial^2u_0}{\partial x^2}+A_{66}\frac{\partial^2u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2v_0}{\partial x\partial y}+(A_{12}+A_{66})\cdot\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial x\partial y}+A_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x^2}+A_{66}\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial y^2}=I_0\frac{d^2x}{dt^2}+I_0\ddot{u}_0+(I_1-c_1I_3)\ddot{\varphi}_x-c_1I_3\frac{\partial\ddot{w}_0}{\partial x},\tag{3a}$$

$$A_{66}\frac{\partial^2v_0}{\partial x^2}+A_{22}\frac{\partial^2v_0}{\partial y^2}+(A_{21}+A_{66})\frac{\partial^2u_0}{\partial x\partial y}+A_{22}\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial y^2}+(A_{21}+A_{66})\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x\partial y}+A_{66}\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial x^2}=I_0\ddot{v}_0+(I_1-c_1I_3)\ddot{\varphi}_y-c_1I_3\frac{\partial\ddot{w}_0}{\partial y},\tag{3b}$$

$$(A_{55}+c_2^2F_{55}-2c_2D_{55})\frac{\partial^2w_0}{\partial x^2}+(A_{44}+c_2^2F_{44}-2c_2D_{44})\frac{\partial^2w_0}{\partial y^2}+A_{11}\frac{\partial u_0}{\partial x}\frac{\partial^2w_0}{\partial x^2}+A_{21}\frac{\partial u_0}{\partial x}\frac{\partial^2w_0}{\partial y^2}+2A_{66}\frac{\partial u_0}{\partial y}\frac{\partial^2w_0}{\partial x\partial y}+A_{11}\frac{\partial^2u_0}{\partial x^2}\frac{\partial w_0}{\partial x}+A_{66}\frac{\partial^2u_0}{\partial y^2}\frac{\partial w_0}{\partial x}+(A_{21}+A_{66})\frac{\partial^2u_0}{\partial x\partial y}\frac{\partial w_0}{\partial y}+\frac{3}{2}A_{11}\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2w_0}{\partial x^2}+\frac{3}{2}A_{22}\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2w_0}{\partial y^2}+\left(\frac{1}{2}A_{21}+A_{66}\right)\left(\frac{\partial w_0}{\partial x}\right)^2\cdot\frac{\partial^2w_0}{\partial y^2}+A_{22}\frac{\partial v_0}{\partial y}\frac{\partial^2w_0}{\partial y^2}+(A_{66}+\frac{1}{2}A_{21})\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2w_0}{\partial x^2}+2A_{66}\frac{\partial v_0}{\partial x}\frac{\partial^2w_0}{\partial x\partial y}+(A_{12}+A_{21}+4A_{66})\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial x\partial y}-c_1^2H_{11}\frac{\partial^4w_0}{\partial x^4}+(A_{55}-2c_2D_{55}+c_2^2F_{55})\frac{\partial\varphi_x}{\partial x}+(c_1F_{22}-c_1^2H_{22})\frac{\partial^3\varphi_y}{\partial y^3}+(c_1F_{11}-c_1^2H_{11})\frac{\partial^3\varphi_x}{\partial x^3}-c_1^2H_{22}\frac{\partial^4w_0}{\partial y^4}+A_{66}\frac{\partial^2v_0}{\partial x^2}\frac{\partial w_0}{\partial y}+c_1(F_{21}+2F_{66}-c_1H_{21}-2c_1H_{66})\frac{\partial^3\varphi_x}{\partial x\partial y^2}+A_{22}\frac{\partial^2v_0}{\partial y^2}\frac{\partial w_0}{\partial y}+(A_{44}-2c_2D_{44}+c_2^2F_{44})\frac{\partial\varphi_y}{\partial y}+(A_{12}+A_{66})\frac{\partial^2v_0}{\partial x\partial y}\frac{\partial w_0}{\partial x}+c_1(F_{12}+2F_{66}-c_1H_{12}-2c_1H_{66})\frac{\partial^3\varphi_y}{\partial x^2\partial y}+A_{12}\frac{\partial v_0}{\partial y}\frac{\partial^2w_0}{\partial x^2}-2c_1^2(H_{21}-2H_{66})\frac{\partial^4w_0}{\partial x^2\partial y^2}-f_y\frac{\partial^2w_0}{\partial y^2}+\Delta p-\delta\frac{\partial w_0}{\partial t}=I_0\ddot{w}_0-c_1^2I_6\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)+c_1I_3\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)+c_1(I_4-c_1I_6)\left(\frac{\partial\ddot{\varphi}_x}{\partial x}+\frac{\partial\ddot{\varphi}_y}{\partial y}\right),\tag{3c}$$

$$(D_{11}-2c_1F_{11}+c_1^2H_{11})\frac{\partial^2\varphi_x}{\partial x^2}-(c_1F_{11}-c_1^2H_{11})\frac{\partial^3w_0}{\partial x^3}+(D_{66}-2c_1F_{66}+c_1^2H_{66})\frac{\partial^2\varphi_x}{\partial y^2}-(A_{55}-2c_2D_{55}+c_2^2F_{55})\frac{\partial w_0}{\partial x}+c_1(c_1H_{12}-F_{12}-2F_{66}+2c_1H_{66})\frac{\partial^3w_0}{\partial x\partial y^2}+(D_{12}-2c_1F_{12}-2c_1F_{66}+D_{66}+c_1^2H_{12}+c_1^2H_{66})\frac{\partial^2\varphi_y}{\partial x\partial y}-(A_{55}-2c_2D_{55}+c_2^2F_{55})\varphi_x=(I_1-c_1I_3)\ddot{u}_0+(I_2-2c_1I_4+c_1^2I_6)\ddot{\varphi}_x-c_1(I_4-c_1I_6)\frac{\partial\ddot{w}_0}{\partial x},\tag{3d}$$

$$(D_{22}-2c_1F_{22}+c_1^2H_{22})\frac{\partial^2\varphi_y}{\partial y^2}-(c_1F_{22}-c_1^2H_{22})\frac{\partial^3 w_0}{\partial y^3}+$$
$$(D_{66}-2c_1F_{66}+c_1^2H_{66})\frac{\partial^2\varphi_y}{\partial x^2}-(A_{44}-2c_2D_{44}+$$
$$c_2^2F_{44})\frac{\partial w_0}{\partial y}+c_1(c_1H_{21}-F_{21}-2F_{66}+2c_1H_{66})\frac{\partial^3 w_0}{\partial x^2\partial y}+$$
$$(D_{21}-2c_1F_{21}-2c_1F_{66}+D_{66}+c_1^2H_{21}+c_1^2H_{66})\frac{\partial^2\varphi_x}{\partial x\partial y}-$$
$$(A_{44}-2c_2D_{44}+c_2^2F_{44})\varphi_y=(I_1-c_1I_3)\ddot{v}_0+$$
$$(I_2-2c_1I_4+c_1^2I_6)\ddot{\varphi}_y-c_1(I_4-c_1I_6)\frac{\partial\ddot{w}_0}{\partial y}. \tag{3e}$$

其中，δ 为阻尼系数，Δp 表示由一阶活塞理论推得的气动载荷，形式为

$$\Delta p=-\frac{4q_d\gamma}{M_\infty}\left(\frac{1}{v_a}\frac{\partial w_0}{\partial x}\frac{dx}{dt}+\frac{\partial w_0}{\partial y}+\frac{1}{v_a}\frac{\partial w_0}{\partial t}\right). \tag{4}$$

其中，动压 $q_d=\frac{1}{2}\rho_a{v_a}^2$，$v_a$ 为机翼上一点的法洗速度，M_∞ 为当地马赫数，$\gamma=M_\infty/\sqrt{{M_\infty}^2-1}$ 为空气动力修正因子.

悬臂外伸板在固定和自由端的边界条件分别为

当 $x=0$，

$$u_0=v_0=w_0=\varphi_x=\varphi_y=0, \tag{5a}$$

当 $x=l(t)$，

$$N_{xx}=N_{xy}=M_{xx}=M_{xy}=P_{xx}=P_{xy}=Q_x=R_x=0, \tag{5b}$$

当 $y=0$ 和 b，

$$N_{yy}=N_{xy}=M_{yy}=M_{xy}=P_{yy}=P_{xy}=Q_y=R_y=0, \tag{5c}$$

2 Galerkin 离散

对方程(3)进行无量纲化，然后应用 Galerkin 方法将偏微分形式的非线性方程离散为常微分形式的非线性动力学方程. 本文选取了系统前两阶振动模态进行二阶 Galerkin 离散，在满足位移边界条件的情况下选取振型函数为

$$u(x,y,t)=u_1(t)\sin\frac{\pi x}{2l}\cos\frac{\pi y}{b}+$$
$$u_2(t)\sin\frac{3\pi x}{2l}\cos\frac{2\pi y}{b}, \tag{6a}$$
$$v(x,y,t)=v_1(t)\sin\frac{\pi x}{2l}\sin\frac{\pi y}{b}+$$
$$v_2(t)\sin\frac{3\pi x}{2l}\sin\frac{2\pi y}{b}, \tag{6b}$$
$$w(x,y,t)=w_1(t)X_1(x)Y_1(y)+$$
$$w_2(t)X_2(x)Y_2(y), \tag{6c}$$
$$\varphi_x(x,y,t)=\varphi_{x1}(t)\sin\frac{\pi x}{2l}\cos\frac{\pi y}{b}+$$
$$\varphi_{x2}(t)\sin\frac{\pi x}{l}\cos\frac{2\pi y}{b}, \tag{6d}$$
$$\varphi_y(x,y,t)=\varphi_{y1}(t)\left(1-\cos\frac{\pi x}{2l}\right)\sin\frac{\pi y}{b}+$$
$$\varphi_{y2}(t)\left(1-\cos\frac{\pi x}{l}\right)\sin\frac{2\pi y}{b} \tag{6e}$$

其中，$X_i(x)$ 取沿 x 方向的固支-自由梁函数，$Y_j(y)$ 取沿 y 方向的自由-自由梁函数.

将振型函数(6)代入方程(3)进行 Galerkin 运算的过程中，特别需要注意，因为板的长度沿 x 轴是随时间改变的，所以在推导中要用到如下运算关系

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\frac{dx}{dt}, \tag{7a}$$
$$\frac{D^2}{Dt^2}=\frac{\partial^2}{\partial t^2}+2\frac{\partial^2}{\partial x\partial t}\frac{dx}{dt}+\frac{\partial^2}{\partial x^2}\left(\frac{dx}{dt}\right)^2+\frac{\partial}{\partial x}\frac{d^2x}{dt^2}. \tag{7b}$$

考虑横向振动为系统的主要运动方式，离散后可以得到以横向位移 w_1 和 w_2 为变量的两自由度无量纲非线性动力学方程

$$\alpha_1\ddot{w}_1+\alpha_2\dot{w}_1+a_3\dot{w}_2+\alpha_4w_1+\alpha_5(f_0+$$
$$f_1\cos\Omega_1t)w_1+\alpha_6w_2+\alpha_7w_1^3+\alpha_8w_1^2w_2+$$
$$\alpha_9w_1w_2^2+\alpha_{10}w_2^3=0, \tag{8a}$$
$$\beta_1\ddot{w}_2+\beta_2\dot{w}_2+\beta_3\dot{w}_1+\beta_4w_2+\beta_5(f_0+$$
$$f_1\cos\Omega_1t)w_2+\beta_6w_1+\beta_7w_2^3+\beta_8w_2^2w_1+$$
$$\beta_9w_2w_1^2+\beta_{10}w_1^3=0. \tag{8b}$$

这里，方程中的系数 α_i 和 β_i $(i=1,2,\cdots,10)$ 都是与时间有关的变量，即得到的方程是变系数系统，方程的质量项、阻尼项和刚度项都随时间变化.

3 算例分析

为了研究复合材料悬臂外伸板的非线性动力学特性，选取如下参数进行数值分析. 板的组成为各铺层等厚度、同材料、正交铺设的三层石墨/环氧，其它参数：$l_0=2.0\mathrm{m}$，$b=1.5\mathrm{m}$，$h=0.004\mathrm{m}$，$f_0=2000\mathrm{N/m^2}$，$V_d=0.005\mathrm{m/s}$，$\Omega_1=\Omega_2=15$，$\kappa=1.4$，$M_\infty=3.0$，$V_a=900\mathrm{m/s}$，$\rho_a=0.65\mathrm{kg/m^3}$，$\delta=600\mathrm{N\cdot s/m}$. 根据非线性振动方程(8)，数值分析相关参

数对系统外伸过程中动力学稳定性的影响.

图 2 与图 3 给出了两种不同外伸速度下悬臂板沿轴向外伸过程中第一阶和第二阶振动的时域分析曲线. 首先观察这些图的全局响应特性，尽管外伸速度的取值不同，系统的整个外伸过程呈现出一些相似的振动规律，即初始的时候，系统会发生振幅的跳跃现象，随着板的继续外伸，系统的第一阶振动幅值逐渐增大，第二阶振动幅值先减小再增大，前两阶振动频率逐渐降低，振幅可能再次发生跳跃和发散现象.

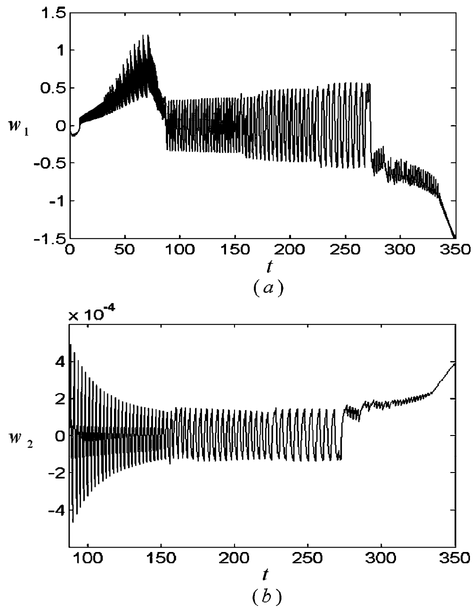

图 2　当 $V_0=0.10$ 时系统的时间历程图

Fig. 2　Time history curves when $V_0=0.10$

通过对两种不同速度下系统时间历程图的比较，可以发现外伸速度对系统外伸过程中动力学稳定性的影响. 首先观察系统的第一阶振动，当外伸速度为 $v_0=0.10\text{m/s}$ 时，如图 2(a)，在无量纲时间 $t=90$ 左右，振幅结束初始跳跃，当外伸速度增大到 $v_0=0.20\text{m/s}$ 时，见图 3(a)，振幅结束跳跃的无量纲时间大约为 $t=80$，显而易见，随着外伸速度的增加，振幅结束初始跳跃的时间逐渐提前. 但与此同时，系统振幅再次发生跳跃的时间会提前，并且外伸速度越快，系统再次发生振幅跳跃的时间就越提前.

对于第二阶振动而言，运动规律与第一阶的运动形式相似，如图 2(b)、3(b). 相比而言，第二阶振动的振幅远小于第一阶振幅，可见第一阶振动为系统的主要振动形式.

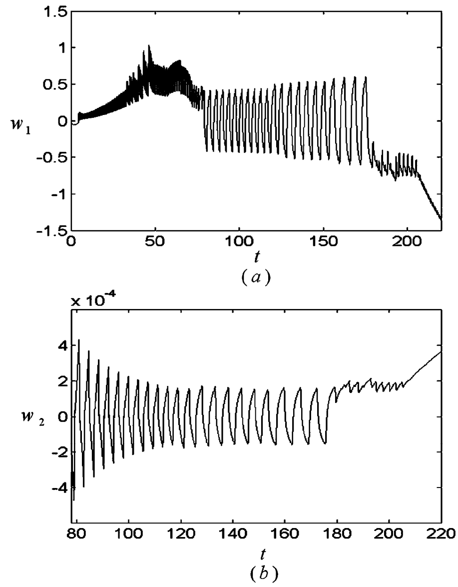

图 3　当 $V_0=0.20$ 时系统的时间历程图

Fig. 3　Time history curves when $V_0=0.20$

4　结论

本文考虑一个沿轴向可外伸的复合材料悬臂板模型，应用高阶剪切理论和 Hamilton 原理建立其动力学方程，根据得到的时变系数方程，应用数值模拟的方法研究了一阶活塞气动力作用下系统的非线性动力学响应. 研究发现：悬臂板在外伸过程中，前两阶振动频率逐渐降低，存在振幅跳跃和发散的现象. 并且随着外伸速度的增大，系统的振幅结束初始跳跃的时间逐渐提前，同时再次发生振幅跳跃的时间也会提前.

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NONLINEAR ANALYSIS OF DEPLOYING LAMINATED COMPOSITE CANTILEVER PLATES *

Lu Shufeng Zhang Wei†

(*College of Mechanical Engineering*, *Beijing University of Technology*, *Beijing* 100124, *China*)

Abstract This paper studied the nonlinear dynamics of deploying cantilever laminated composite plates subjected to transversal aerodynamic pressures and in-plane excitations. The first-order piston theory was employed to model the transversal air pressures. Based on Reddy′s third-order shear deformable plate theory and Hamilton Principal, the nonlinear governing equations of motion were established for the deploying cantilever laminated composite plates. By choosing suitable vibration mode-shape functions, the two-degree-of-freedom nonlinear governing equations of motion with time-varying coefficients were deduced by using Galerkin method. The influences of varying deploying velocities on the nonlinear resonance of the deploying cantilever plate were analyzed.

Key words axially moving cantilever laminated plates, third-order plate theory, piston theory, Hamilton principle, nonlinear dynamics

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† Corresponding author E-mail: sandyzhang0@yahoo.com