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位形空间中约束力学系统的 Lagrange 对称性与守恒量*

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摘要 研究位形空间中约束力学系统的 Lagrange 对称性,给出位形空间中约束力学系统的统一动力学方程,给出位形空间中约束力学系统统一方程的 Lagrange 对称性的判据,得到位形空间中约束力学系统统一方程的 Lagrange 对称性导致的守恒量及其存在的条件,并举例说明结果的应用.

关键词 位形空间, 力学系统, Lagrange 对称性, 守恒量

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引言

对称性是数学、物理等学科领域内非常重要的法则之一. 对称性理论也是近代分析力学研究的主要方向之一,Noether 对称性、Lie 对称性和 Mei 对称性一直是对称性理论研究的主要对象. 近年来,人们对 Noether、Lie 和 Mei 对称性的研究成果丰硕,理论体系已经比较完善,寻求和研究新型对称性是对称性理论发展的要求. Lagrange 对称性作为新型对称性的一种,相对于 Noether、Lie 和 Mei 对称性,Lagrange 对称性理论不够完善. 20 世纪六七十年代 Currie 等对不同自由度[9,10] Lagrange 函数等价问题的研究是人们对 Lagrange 对称性的最早探索,上世纪 70 年代末到 90 年代,Lutzky 等对力学系统的 Lagrange 函数等价问题做了一系列的研究[11-14],Hojman 将这种 Lagrange 函数等价关系称为 Lagrange 对称性[14,15],Lagrange 对称性现已被推广到 Hamilton、Birkhoff 等系统[15-26]. 赵跃宇等人是我国最早研究 Lagrange 对称性的学者[15]. 本文根据力学系统运动微分方程的特点,将广义非完整约束反力,广义反推力等系统可能受到的力看做一合力,然后研究两个系统运动微分方程的 Lagrange 对称性. 以便讨论位形空间中任意两种系统的微分方程满足 Lagrange 对称性的定义和判据,以及 Lagrange 对称性导致守恒量的条件和守恒量形式.

1 位形空间中力学系统的统一动力学方程

设某一系统的 Lagrange 函数为 L,系统所受非势力的合力为 $\boldsymbol{F}$,则系统动力学方程为

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_s}-\frac{\partial L}{\partial q_s}=f_s \tag{1}$$

另一不同于上述系统的 Lagrange 函数为$\bar{L}$,系统所受非势力合力为$\bar{\boldsymbol{F}}$,则系统动力学方程为

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \bar{L}}{\partial \dot{q}_s}-\frac{\partial \bar{L}}{\partial q_s}=\bar{f}_s \tag{2}$$

其中 f_s 和 $\bar{f}_s$ 分别为系统的广义非势力,广义约束反力等力的广义合力在广义坐标 q_s 方向上的分量,即

$$f_s=\frac{\partial \boldsymbol{F}}{\partial x}\frac{\partial x}{\partial q_s}+\frac{\partial \boldsymbol{F}}{\partial y}\frac{\partial y}{\partial q_s}+\frac{\partial \boldsymbol{F}}{\partial z}\frac{\partial z}{\partial q_s} \tag{3}$$

$$\bar{f}_s=\frac{\partial \bar{\boldsymbol{F}}}{\partial x}\frac{\partial x}{\partial q_s}+\frac{\partial \bar{\boldsymbol{F}}}{\partial y}\frac{\partial y}{\partial q_s}+\frac{\partial \bar{\boldsymbol{F}}}{\partial z}\frac{\partial z}{\partial q_s} \tag{4}$$

2 系统的 Lagrange 对称性

对于给定系统(1)和(2)的两组动力学函数 $L,\boldsymbol{f}$ 和$\bar{L},\bar{\boldsymbol{f}}$,定义 L_r 和$\bar{L}_r$ 分别为

$$L_r=\frac{\partial^2 L}{\partial \dot{q}_r\partial \dot{q}_k}\ddot{q}_k+\frac{\partial^2 L}{\partial \dot{q}_r\partial q_k}\dot{q}_k+\frac{\partial^2 L}{\partial \dot{q}_r\partial t}-\frac{\partial L}{\partial q_r}-f_r=$$
$$W_{rk}\ddot{q}_k+\frac{\partial^2 L}{\partial \dot{q}_r\partial q_k}\dot{q}_k+\frac{\partial^2 L}{\partial \dot{q}_r\partial t}-\frac{\partial L}{\partial q_r}-f_r, \tag{5}$$

$$\bar{L}_r=\frac{\partial^2 \bar{L}}{\partial \dot{q}_r\partial \dot{q}_k}\ddot{q}_k+\frac{\partial^2 \bar{L}}{\partial \dot{q}_r\partial q_k}\dot{q}_k+\frac{\partial^2 \bar{L}}{\partial \dot{q}_r\partial t}-\frac{\partial \bar{L}}{\partial q_r}-\bar{f}_r=$$

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$$\overline{W}_{rk}\ddot{q}_k+\frac{\partial^2\overline{L}}{\partial\dot{q}_r\partial q_k}\dot{q}_k+\frac{\partial^2\overline{L}}{\partial\dot{q}_r\partial t}-\frac{\partial\overline{L}}{\partial q_r}-\bar{f}_r. \tag{6}$$

其中

$$W_{rk}=\frac{\partial^2 L}{\partial\dot{q}_r\partial\dot{q}_k},\quad \overline{W}_{rk}=\frac{\partial^2\overline{L}}{\partial\dot{q}_r\partial\dot{q}_k} \tag{7}$$

引入

$$\Diamond_r=\frac{\partial^2}{\partial\dot{q}_r\partial\dot{q}_k}\ddot{q}_k+\frac{\partial^2}{\partial\dot{q}_r\partial q_k}\dot{q}_k+\frac{\partial^2}{\partial\dot{q}_r\partial t}-\frac{\partial}{\partial q_r} \tag{8}$$

并称之为Lagrange算子，则

$$L_r=\Diamond_r(L)-f_r, \tag{9}$$

$$\overline{L}_r=\Diamond_r(\overline{L})-\bar{f}_r \tag{10}$$

定义 对于系统（1）和（2），如果由动力学函数L和$\boldsymbol{f}$确定的

$$L_r=0 \tag{11}$$

的每一个解都满足由动力学函数$\overline{L}$和$\bar{\boldsymbol{f}}$确定的

$$\overline{L}_r=0 \tag{12}$$

反之亦然，则表明两系统之间具有Lagrange对称性.

由式（10）和（12）得

$$\ddot{q}_k=\overline{U}^{kr}(\bar{f}_r+\frac{\partial\overline{L}}{\partial q_r}-\frac{\partial^2\overline{L}}{\partial\dot{q}_r\partial q_k}\dot{q}_k-\frac{\partial^2\overline{L}}{\partial\dot{q}_r\partial t}), \tag{13}$$

其中

$$\overline{W}_{rk}\overline{U}^{kr}=\delta_r^k \tag{14}$$

把（13）式代入（11）式，并考虑到（9）式得

$$W_{rk}\overline{U}^{kl}(\bar{f}_l+\frac{\partial\overline{L}}{\partial q_l}-\frac{\partial^2\overline{L}}{\partial\dot{q}_l\partial q_k}\dot{q}_k-\frac{\partial^2\overline{L}}{\partial\dot{q}_l\partial t})=$$
$$f_r+\frac{\partial L}{\partial q_r}-\frac{\partial^2 L}{\partial\dot{q}_r\partial q_k}\dot{q}_k-\frac{\partial^2 L}{\partial\dot{q}_r\partial t} \tag{15}$$

由定义和（15）式得

判据：对于系统（1）和系统（2），如果两组动力学函数$\overline{L}$，$\bar{\boldsymbol{f}}$和L，$\boldsymbol{f}$满足方程（15），则两系统之间具有Lagrange对称性.

3 Lagrange对称性导致的守恒量

对于两系统的Lagrange对称性有如下命题：

命题 对于系统（1）和系统（2），如果两组动力学函数L，f和$\overline{L}$，$\bar{f}$满足条件

$$\frac{\partial\bar{f}_s}{\partial\dot{q}_l}=A_s^r\frac{\partial f_r}{\partial\dot{q}_l} \tag{16}$$

则系统的Lagrange对称性可导致守恒量

$$tr(A)=\mathrm{const}, \tag{17}$$

其中A为以A_s^r为元素的矩阵，

$$A_s^r=\overline{W}_{sk}U^{kr},\quad W_{rk}U^{ks}=\delta_r^s \tag{18}$$

证明：将（18）式代入（15）式得

$$\frac{\partial^2\overline{L}}{\partial\dot{q}_l\partial q_k}\dot{q}_k+\frac{\partial^2\overline{L}}{\partial\dot{q}_l\partial t}-\frac{\partial\overline{L}}{\partial q_l}-\bar{f}_l=$$
$$A_l^r(\frac{\partial^2 L}{\partial\dot{q}_r\partial t}+\frac{\partial^2 L}{\partial\dot{q}_r\partial q_k}\dot{q}_k-\frac{\partial L}{\partial q_r}-f_r) \tag{19}$$

对（19）式求关于$\dot{q}_s$的偏导数得

$$\frac{\partial^3\overline{L}}{\partial\dot{q}_l\partial q_k\partial\dot{q}_s}\dot{q}_k+\frac{\partial^2\overline{L}}{\partial\dot{q}_l\partial q_s}+\frac{\partial^3\overline{L}}{\partial\dot{q}_l\partial\dot{q}_s\partial t}-\frac{\partial^2\overline{L}}{\partial q_l\partial\dot{q}_s}-\frac{\partial\bar{f}_l}{\partial\dot{q}_s}=$$
$$\frac{\partial A_l^r}{\partial\dot{q}_s}(\frac{\partial^2 L}{\partial\dot{q}_r\partial q_k}\dot{q}_k+\frac{\partial^2 L}{\partial\dot{q}_r\partial t}-\frac{\partial L}{\partial q_r}-f_r)+$$
$$A_l^r(\frac{\partial^3 L}{\partial\dot{q}_r\partial q_k\partial\dot{q}_s}\dot{q}_k+\frac{\partial^2 L}{\partial\dot{q}_r\partial q_s}+\frac{\partial^3 L}{\partial\dot{q}_r\partial t\partial\dot{q}_s}-$$
$$\frac{\partial^2 L}{\partial q_r\partial\dot{q}_s}-\frac{\partial f_r}{\partial\dot{q}_s}) \tag{20}$$

联立（9）式和（11）式得

$$-W_{rk}\ddot{q}_k=\frac{\partial^2 L}{\partial\dot{q}_r\partial q_k}\dot{q}_k+\frac{\partial^2 L}{\partial\dot{q}_r\partial t}-\frac{\partial L}{\partial q_r}-f_r \tag{21}$$

将（21）式代入（20）式得

$$\frac{\partial\overline{W}_{ls}}{\partial q_k}\dot{q}_k+\frac{\partial\overline{W}_{ls}}{\partial t}+\frac{\partial^2\overline{L}}{\partial\dot{q}_l\partial q_s}-\frac{\partial^2\overline{L}}{\partial q_l\partial\dot{q}_s}-\frac{\partial\bar{f}_l}{\partial\dot{q}_s}=$$
$$-\frac{\partial A_l^r}{\partial\dot{q}_s}W_{rk}\ddot{q}_k+A_l^r\frac{\partial W_{rs}}{\partial q_k}\dot{q}_k+A_l^r(\frac{\partial W_{rs}}{\partial t}+\frac{\partial^2 L}{\partial\dot{q}_r\partial q_s}-$$
$$\frac{\partial^2 L}{\partial q_r\partial\dot{q}_s}-\frac{\partial f_r}{\partial\dot{q}_s}). \tag{22}$$

对$A_l^r W_{rs}$求关于$\dot{q}_k$的偏导数

$$\frac{\partial(A_l^r W_{rs})}{\partial\dot{q}_k}=\frac{\partial(\overline{W}_{ls}U^{sr}W_{rs})}{\partial\dot{q}_k}=\frac{\partial^3\overline{L}}{\partial\dot{q}_l\partial\dot{q}_s\partial\dot{q}_k}=$$
$$\frac{\partial A_l^r}{\partial\dot{q}_k}W_{rs}+A_l^r\frac{\partial W_{rs}}{\partial\dot{q}_k}, \tag{23}$$

即

$$\frac{\partial A_l^r}{\partial\dot{q}_k}W_{rs}=\frac{\partial\overline{W}_{lk}}{\partial\dot{q}_s}-A_l^r\frac{\partial W_{rk}}{\partial\dot{q}_s}. \tag{24}$$

由（18）式得

$$\overline{W}_{lk}=A_l^r W_{rk}, \tag{25}$$

求（25）式关于$\dot{q}_s$的偏导数得

$$\frac{\partial\overline{W}_{lk}}{\partial\dot{q}_s}=\frac{\partial A_l^r}{\partial\dot{q}_k}W_{rk}+A_l^r\frac{\partial W_{rk}}{\partial\dot{q}_s} \tag{26}$$

由（24）和（26）式得

$$\frac{\partial A_l^r}{\partial\dot{q}_s}W_{rs}=\frac{\partial\overline{W}_{lk}}{\partial\dot{q}_s}-A_l^r\frac{\partial W_{rk}}{\partial\dot{q}_s}=\frac{\partial A_l^r}{\partial\dot{q}_k}W_{rk}, \tag{27}$$

$$-\frac{\partial A_l^r}{\partial q_k}W_{rs}\dot{q}_k=A_l^r\frac{\partial W_{rs}}{\partial q_k}\dot{q}_k-\frac{\partial\overline{W}_{ls}}{\partial q_k}\dot{q}_k. \tag{28}$$

把（27）和（28）式代入（22）式得

$$\frac{\partial^2 \bar{L}}{\partial \dot{q}_l \partial q_s} - \frac{\partial^2 \bar{L}}{\partial q_l \partial \dot{q}_s} - \frac{\partial \bar{f}_l}{\partial \dot{q}_s} = -\frac{\partial A_l^r}{\partial \dot{q}_k} W_{rs} \ddot{q}_k - \frac{\partial A_l^r}{\partial q_k} W_{rs} \dot{q}_k - \frac{\partial \bar{W}_{ls}}{\partial t} + A_l^r \frac{\partial W_{rs}}{\partial t} + A_l^r \left(\frac{\partial^2 L}{\partial \dot{q}_r \partial q_s} - \frac{\partial^2 L}{\partial q_r \partial \dot{q}_s} - \frac{\partial f_r}{\partial \dot{q}_s} \right), \tag{29}$$

即

$$\frac{\partial^2 \bar{L}}{\partial \dot{q}_l \partial q_s} - \frac{\partial^2 \bar{L}}{\partial q_l \partial \dot{q}_s} - \frac{\partial \bar{f}_l}{\partial \dot{q}_s} = -\frac{\mathrm{d} A_l^r}{\mathrm{d}t} W_{rs} + \frac{\partial A_l^r}{\partial t} W_{rs} - \frac{\partial \bar{W}_{ls}}{\partial t} + A_l^r \frac{\partial W_{rs}}{\partial t} + A_l^r \left(\frac{\partial^2 L}{\partial \dot{q}_r \partial q_s} - \frac{\partial^2 L}{\partial q_r \partial \dot{q}_s} - \frac{\partial f_r}{\partial \dot{q}_s} \right). \tag{30}$$

由式（28）得

$$A_l^r \frac{\partial W_{rs}}{\partial t} - \frac{\partial \bar{W}_{ls}}{\partial t} + \frac{\partial A_l^r}{\partial t} W_{rs} = 0 \tag{31}$$

将（31）式代入（30）式得

$$\frac{\partial^2 \bar{L}}{\partial \dot{q}_l \partial q_s} - \frac{\partial^2 \bar{L}}{\partial q_l \partial \dot{q}_s} - \frac{\partial \bar{f}_l}{\partial \dot{q}_s} = -\frac{\mathrm{d} A_l^r}{\mathrm{d}t} W_{rs} + A_l^r \left(\frac{\partial^2 L}{\partial \dot{q}_r \partial q_s} - \frac{\partial^2 L}{\partial q_r \partial \dot{q}_s} - \frac{\partial f_r}{\partial \dot{q}_s} \right). \tag{32}$$

定义矩阵 $T, \bar{T}, A, U, W, \bar{W}$,其元素分别为

$$T = \frac{\partial^2 L}{\partial \dot{q}_r \partial q_k} - \frac{\partial^2 L}{\partial q_r \partial \dot{q}_k}, \bar{T} = \frac{\partial^2 \bar{L}}{\partial \dot{q}_r \partial q_k} - \frac{\partial^2 \bar{L}}{\partial q_r \partial \dot{q}_k}, \tag{33}$$

$$A = (A_s^r), U = (U^{sk}), W = (W_{rs}), \bar{W} = (\bar{W}_{rs}). \tag{34}$$

将条件（16）,（33）和（34）式代入（32）式,得

$$\dot{A} = -\bar{T}U + ATU. \tag{35}$$

因为 T 和 $\bar{T}$ 为反对称矩阵,U 和 $\bar{W}$ 为对称矩阵,因此对于任意正整数 m 有

$$\dot{A} = (A)^{m-1} = (-\bar{T}U + \bar{W}UTU)(A)^{m-1} = -\bar{T}U(\bar{W}U)^{m-1} + \bar{W}UTU(\bar{W}U)^{m-1}. \tag{36}$$

根据矩阵 $T, \bar{T}, U$ 和 $\bar{W}$ 的特性及矩阵迹的性质得

$$tr[\bar{T}U(\bar{W}U)^{m-1}] = 0, tr[\bar{W}UTU(\bar{W}U)^{m-1}] = 0 \tag{37}$$

即

$$tr[A(A)^{m-1}] = 0 \tag{38}$$

即

$$\frac{\mathrm{d}}{\mathrm{d}t}[tr(A^m)] = 0 \tag{39}$$

可得（17）式,命题 得证.

4　算例

设某系统的 Lagrange 函数为

$$L = \frac{1}{2} m(t)(\dot{q}_1^2 + \dot{q}_2^2) \ (m(t) = m_0(1 - \alpha t), \alpha = const) \tag{40}$$

系统所受各类广义力合力的分量分别为

$$f_1 = -\alpha m_0 \dot{q}_1 - \frac{1}{1 + b^2 t^2} m(t),$$

$$f_2 = -\alpha m_0 \dot{q}_2 - \frac{1}{1 + b^2 t^2} m(t). \tag{41}$$

试研究系统的 Lagrange 对称性及其导致的守恒量.

则根据（9）式和（10）式得

$$L_1 = \Diamond_1(L) - f_1 = -\alpha m_0 \dot{q}_1 + m(t) \ddot{q}_1 - f_1 = m(t)\left(\ddot{q}_1 + \frac{1}{1 + b^2 t^2}\right) \tag{42}$$

$$L_2 = \Diamond_2(L) - f_2 = m(t)\left(\ddot{q}_2 + \frac{1}{1 + b^2 t^2}\right). \tag{43}$$

假设有另一系统,其性质与系统（40）不同,其 Lagrange 函数为

$$\bar{L} = m(t)\left\{\frac{1}{6}\left(\dot{q}_1 + \frac{1}{b}\arctan bt\right)^3 + \frac{1}{2}\left[\dot{q}_2 + \frac{1}{2b}\ln(1 + b^2 t^2)\right]^2\right\} \tag{44}$$

其各类广义力合力的分量分别为

$$\bar{f}_1 = -\alpha m_0 \left[\frac{1}{2}\left(\dot{q}_1 + \frac{1}{b}\arctan bt\right)^2\right],$$

$$\bar{f}_2 = -\alpha m_0 \left[\dot{q}_2 + \frac{1}{2b}\ln(1 + b^2 t^2)\right]. \tag{45}$$

则由（10）得 $\bar{L}_1$ 和 $\bar{L}_2$ 分别为

$$\bar{L}_1 = \Diamond_1(\bar{L}) - \bar{f}_1 = -\alpha m_0 \left[\frac{1}{2}\left(\dot{q}_1 + \frac{1}{b}\arctan bt\right)^2\right] + m(t)\left[\left(\dot{q}_1 + \frac{1}{b}\arctan bt\right) \times \left(\ddot{q}_1 + \frac{1}{1 + b^2 t^2}\right)\right] - \bar{f}_1 \tag{46}$$

$$\bar{L}_2 = \Diamond_2(\bar{L}) - \bar{f}_2 = -\alpha m_0 \left[\dot{q}_2 + \frac{1}{2b}\ln(1 + b^2 t^2)\right] + m(t)\left(\ddot{q}_2 + \frac{1}{1 + b^2 t^2}\right)] - \bar{f}_2 \tag{47}$$

将（45）式代入（46）式和（47）式,得

$$\bar{L}_1 = m(t)\left[\left(\dot{q}_1 + \frac{1}{b}\arctan bt\right)\left(\ddot{q}_1 + \frac{1}{1 + b^2 t^2}\right)\right] = \left(\dot{q}_1 + \frac{1}{b}\arctan bt\right) L_1,$$

$$\bar{L}_2 = m(t)\left(\ddot{q}_2 + \frac{1}{1 + b^2 t^2}\right) = L_2. \tag{48}$$

并且可以得到

$$A_1^1 = \dot{q}_1 + \frac{1}{b}\arctan bt,$$

$$A_2^2 = 1. \tag{49}$$

易知(45)式和(41)式满足条件(16),故由命题得守恒量

$$I_L = tr(A) = 1 + \dot{q}_1 + \frac{1}{b}\arctan bt = \text{const} \tag{50}$$

5 小结

本文讨论了位形空间中约束力学系统统一动力学方程的 Lagrange 对称性理论,给出了位形空间中约束力学系统统一动力学方程的 Lagrange 对称性判据和导致守恒量的条件以及守恒量形式. 本文给出的结论不仅可以研究同类系统的 Lagrange 对称性,而且可以研究两个不同性质力学微分方程的 Lagrange 对称性. 当系统方程中的 f_s 为非完整约束反力,广义反推力和广义非势力之和时,讨论系统变为变质量非完整系统,即本文结论为文献[25]的结果; 当系统方程中的 f_s 为非完整约束反力和广义非势力之和时,则研究系统为非完整系统,本文结论将与文献[17]的结果相符.

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SYMMETRY AND CONSERVED QUANTITY OF LAGRANGIANS FOR A CONSTRAINED MECHANICAL SYSTEM IN CONFIGURATION SPACE *

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Abstract This paper studied the symmetry of Lagrangians and the conserved quantities for two dynamical equations of a constrained mechanical system in configuration space. The criterion of the symmetry for the two equations was given. The conditions under which there exist a conserved quantity and the form of the conserved quantity were obtained. And an example was presented to illustrate the application of the results.

Key words configuration spcace, mechanical system, symmetry of Lagrangians, conserved quantity

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