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宽翼 T 形梁桥动力学理论与特性分析*

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摘要 考虑了剪滞翘曲应力自平衡条件、剪切变形和剪力滞后效应等因素的影响,本文提出了一种对宽翼薄壁 T 形梁动力学特性的分析方法. 分析中为了准确反应 T 形梁翼板的动位移变化,三个广义动位移被引入,且以能量变分原理为基础建立了 T 形梁动力反应的控制微分方程和自然边界条件,据此对 T 形梁的动力反应特性进行了分析,揭示了 T 形梁桥动力反应的规律. 算例中,对比了考虑和不考虑剪滞翘曲应力自平衡条件对 T 形梁动力反应的影响,结果显示考虑剪滞翘曲应力自平衡条件的计算方法与有限元数值解吻合更好.

关键词 T 形梁, 剪力滞后, 自平衡条件, 动力反应, 能量变分原理

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引言

随着材料科学的发展,在桥梁工程中宽翼薄壁 T 形梁桥的应用前景更加广阔 . 但是在对称弯曲状态下,宽翼薄壁 T 形梁存在剪力滞后效应现象 ,设计中如对其考虑不周,往往会在结构中产生一些横向、竖向和斜裂纹等病害 . 在对 T 形梁桥的力学分析中,由于国内外学者的不懈努力该类结构静力分析的解析法日趋完善,但是考虑剪滞翘曲应力自平衡条件、剪力滞后和剪切变形效应等因素,T 形梁桥动力反应分析的解析法尚不充分. 运营中,由于 T 形梁桥可能受到各种动荷载的作用,因而其动力学特性的研究更具理论和工程实际意义. 但是,T 形梁桥受剪力滞后效应的影响,经典强迫振动理论已不适用,因而动力学分析难度加大 . 本文以能量变分原理为基础,运用直接解法对 T 形梁桥的动力反应进行了分析,本文理论为揭示 T 形梁桥动力反应规律奠定了一定的理论基础.

1 控制微分方程和自然边界条件

1.1 不计剪滞翘曲应力自平衡条件影响

1.1.1 体系的动能和势能

图 1 力系作用下图 2 所示的 T 形截面梁,在对称弯曲状态下,若结构的跨度为 L,截面上的竖向动挠度为 $w_1(x,t)$,轴向动位移 $u(x,y,z,t)$ 满足下式[3,10]:

$$u(x,y,z,t) = -z[\theta_1 + \omega_s(y)u_1(x,t)] \quad (1)$$

$$\omega_s(y) = 1-((b-y)/b)^2;\ 0\leqslant y\leqslant b \quad (2)$$

T 形梁稳态振动时的各项势能

T 形截面梁的荷载势能 V_p 为:

$$V_p = -\int_0^L q(x,t)w_1(x,t)dx - [Q(x,t)w_1(x,t)]\Big|_0^L + [M_{1x}(x,t)u_1(x,t) + M_{xA}(x,t)\theta_1(x,t)]\Big|_0^L \quad (3)$$

腹板和翼板的动应变能 V_y 为[2,6]:

$$V_y = \frac{1}{2}\int_0^L EI(\frac{\partial\theta_1}{\partial x})^2dx + \frac{1}{2}\int_0^L kGA(\theta_1 - \frac{\partial w_1}{\partial x})^2dx + \frac{1}{2}\int_0^L I_s\frac{4G}{3b^2}u_1^2dx + \frac{1}{2}EI_s\int_0^L\Big[\frac{4}{3}\frac{\partial\theta_1}{\partial x}\frac{\partial u_1}{\partial x} + \frac{8}{15}(\frac{\partial u_1}{\partial x})^2\Big]dx \quad (4)$$

总势能为:$V_A = V_p + V_y$ (5)

结构总动能 T_A 为[1,11]:

$$T = \frac{1}{2}\int_0^l(\frac{\partial w_1}{\partial t})^2\rho Adx + \frac{1}{2}\rho I_s\int_0^l(\dot{u}_1)^2dx + \frac{1}{2}\rho I\int_0^l(\dot{\theta}_1)^2dx \quad (6)$$

式中:x,z,y 分别为通过截面形心的轴向、竖向和横向坐标;$\theta_1(x,t)$ 为 T 形截面绕 y 轴动转角;$u_1(x,t)$

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为振动时剪力滞后效应引起翼板的纵向动位移差函数;I 为全截面对中性轴的惯性矩;I_s 为翼缘板对中性轴的惯性矩;E,G 分别为材料的杨氏弹性模量和剪切弹性模量;$M_{1x}(x,t)$ 为翼板剪滞效应产生的动弯矩;$M_{xA}(x,t)$ 为梁段端产生竖向转角 $\theta_1(x,t)$ 的动弯矩;k 为截面形状系数;A 为 T 形梁截面面积;ρ 为材料的质量密度;$q(x,t)$ 为分布简谐力.

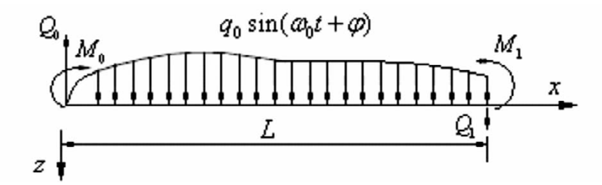

图 1 坐标及荷载系统

Fig. 1 Coordinate and load system

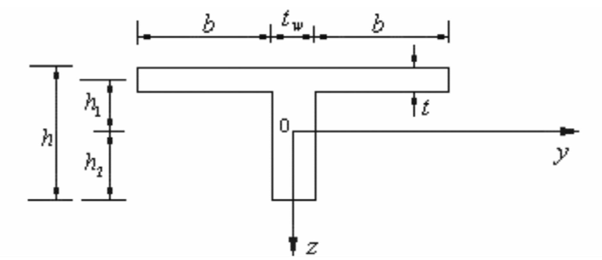

图 2 T 形梁截面

Fig. 2 Cross section of T-beam

1.1.2 T 形梁强迫振动微分方程及自然边界条件

由哈密顿原理 $\delta\int_0^L(T_A-V_A)dt=0$,可推导出 T 形梁动力反应控制微分方程和自然边界条件为[12]:

$$-\rho A\ddot{w}_1-kGA(\theta_1'-w_1'')+q(x,t)=0 \tag{7}$$

$$-\rho I\ddot{\theta}_1+EI\theta_1''+\frac{2}{3}EI_su_1''-kGA(\theta_1-w_1')=0 \tag{8}$$

$$-\rho I_s\ddot{u}_1+\frac{2}{3}EI_s\theta_1''+\frac{8}{15}EI_su_1''-\frac{4GI_s}{3b^2}u_1=0 \tag{9}$$

$$\left[-EI\theta_1'-\frac{2}{3}EI_su_1'+M_{xA}\right]\delta\theta_1\Big|_0^L=0 \tag{10}$$

$$\left[-kGA(\theta_1-w_1')-Q(x,t)\right]\delta w_1\Big|_0^L=0 \tag{11}$$

$$\left[\frac{2}{3}EI_s\theta_1'+\frac{8}{15}EI_su_1''-M_{1x}\right]\delta u_1\Big|_0^L=0 \tag{12}$$

式(6)~(12)中,符号“. ”和“′”分别表示对时间 t 和对坐标 x 求偏导数.

1.1.3 T 形梁强迫振动微分方程的求解

若 T 形梁的强迫振动频率为 ω_0,即

$q(x,t)=q_0(x)\sin(\omega_0t+\varphi)$,

可令:$w_1(x,t)=W_x(x)\sin(\omega_0t+\varphi)$;

$u_1(x,t)=U_1(x)\sin(\omega_0t+\varphi)$;

$\theta_1(x,t)=\psi_1(x)\sin(\omega_0t+\varphi)$,

由方程(8)得 $u_1^{(3)}$、$u_1^{(5)}$ 的表达式,对方程(9)三次求导,将 $u_1^{(3)}$、$u_1^{(5)}$ 代入可得关于 w_1 和 θ_1 项的微分方程,将方程(7)代入,最后得新微分方程为:

$$W_1^{(6)}+W_1^{(4)}\left[\frac{\rho\omega_0^2}{kG}-n\left(\rho\omega_0^2-\frac{4G}{3b^2}\right)\frac{3I}{2I_s}-\frac{4\rho nI\omega_0^2}{5I_s}\right]+$$
$$W_1^{(2)}n\left[\frac{4}{5}\frac{\rho\omega_0^2(kGA-\rho\omega_0^2)}{kGI_s}-\right.$$
$$\left.\left(\rho\omega_0^2-\frac{4G}{3b^2}\right)\frac{3\rho I\omega_0^2}{2I_s}\frac{kG+E}{kGE}\right]+$$
$$W_1n\left(\rho\omega_0^2-\frac{4G}{3b^2}\right)\frac{3\rho\omega_0^2(kGA-\rho I\omega_0^2)}{2EI_skG}$$
$$=n\left(\rho\omega_0^2-\frac{4G}{3b^2}\right)\frac{3(kGA-\rho I\omega_0^2)}{2kGAEI_s}q_0 \tag{13}$$

式中:$n=\frac{2}{3}E-\frac{4}{5}\frac{EI}{I_s}$.

对方程(13)分析可知,其特征方程解可为:

$r_{1,2}=\pm(\alpha_1+\beta_1i)$;$r_{3,4}=\pm(\alpha_2+\beta_2i)$;$r_{5,6}=\pm(\alpha_3+\beta_3i)$. 故方程(13)的通解为:

$$W_1(x)=c_1ch(\alpha_1+\beta_1i)x+c_2sh(\alpha_1+\beta_1i)x+$$
$$c_3ch(\alpha_2+\beta_2i)x+c_4sh(\alpha_2+\beta_2i)x+$$
$$c_5ch(\alpha_3+\beta_3i)x+c_6sh(\alpha_3+\beta_3i)x-\frac{1}{\rho A\omega_0^2}q_0 \tag{14}$$

根据常微分方程组性质和方程(14)可以假设 $\psi_1(x)$ 解的形式,将方程(14)和 $\psi_1(x)$ 的假设式代入方程(7),根据恒等式原理求得 $\psi_1(x)$ 的常系数,$\psi_1(x)$ 的解可表示为:

$$\psi_1(x)=c_1B_1sh(\alpha_1+\beta_1i)x+c_2B_1ch(\alpha_1+\beta_1i)x+$$
$$c_3B_3sh(\alpha_2+\beta_2i)x+c_4B_3ch(\alpha_2+\beta_2i)x+$$
$$c_5B_5sh(\alpha_3+\beta_3i)x+c_6B_5ch(\alpha_3+\beta_3i)x \tag{15}$$

式中:$r=\frac{\rho\omega_0^2}{kG}$;$B_1=\frac{r+(\alpha_1+\beta_1i)^2}{\alpha_1+\beta_1i}$;

$B_3=\frac{r+(\alpha_2+\beta_2i)^2}{\alpha_2+\beta_2i}$;$B_4=\frac{r+(\alpha_3+\beta_3i)^2}{\alpha_3+\beta_3i}$.

同样,根据方程(15)和常微分方程组性质可以假设 $U_1(x)$ 解的形式,将方程(15)和 $U_1(x)$ 的假设式代入方程(9),根据恒等式原理求得 $U_1(x)$ 的常系数,那么 $U_1(x)$ 的解可表示为:

$$U_1(x)=c_1\frac{B_1(\alpha_1+\beta_1i)^2}{T_2+T_1(\alpha_1+\beta_1i)^2}sh(\alpha_1+\beta_1i)x+$$

$$c_2\frac{B_1(\alpha_1+\beta_1 i)^2}{T_2+T_1(\alpha_1+\beta_1 i)^2}ch(\alpha_1+\beta_1 i)x+$$
$$c_3\frac{B_3(\alpha_2+\beta_2 i)^2}{T_2+T_1(\alpha_2+\beta_2 i)^2}sh(\alpha_2+\beta_2 i)x+$$
$$c_4\frac{B_3(\alpha_2+\beta_2 i)^2}{T_2+T_1(\alpha_2+\beta_2 i)^2}ch(\alpha_2+\beta_2 i)x+$$
$$c_5\frac{B_5(\alpha_3+\beta_3 i)^2}{T_2+T_1(\alpha_3+\beta_3 i)^2}sh(\alpha_3+\beta_3 i)x+$$
$$c_6\frac{B_3(\alpha_3+\beta_3 i)^2}{T_2+T_1(\alpha_3+\beta_3 i)^2}ch(\alpha_3+\beta_3 i)x \quad (16)$$

式中：$T_1=-\frac{4}{5}$；$T_2=-\frac{3}{2E}(\rho\omega_0^2-\frac{4G}{3b^2})$.

1.2 考虑剪滞翘曲应力自平衡条件影响

1.2.1 体系的动能和势能

同样，图1力系作用下图2所示T形截面梁，在对称弯曲状态下，若结构的跨度为L，截面上的动挠度为$w_2(x,t)$，则剪滞效应引起翼板的翘曲位移为$\eta(x,y,z,t)$，且满足下式[1,10]：

$\eta(x,y,z,t)=[w_{s0}-zw_{sy}]u_2(x,t)$，且

$$w_{sy}=1-((b-y)/b)^2;\ 0\leqslant y\leqslant b \quad (17)$$

那么，由剪滞效应产生的动正应力和剪应为：

$$\sigma_{sx}=E[w_{s0}-zw_{sy}]u_2'(x,t) \quad (18)$$

$$\tau_{sx}=G\frac{\partial\eta}{\partial y}=-G\frac{\partial w_{sy}}{\partial y}zu_2(x,t) \quad (19)$$

式中：w_{s0}为常量，且w_{s0}由$\int_A\sigma_{sx}dA=0$确定为$w_{s0}=(4h_1tb)/(3A)$.

式(19)满足截面平衡条件$\int_A\tau_{sx}dA=0$，那么剪滞效应产生的竖向动弯矩M_{sy}可表示为：

$$M_{sy}=\int_A\sigma_{sx}zdA=-EI_{sy}u_2'(x,t) \quad (20)$$

$$I_{sy}=-\int_A z[w_{s0}-zw_{sy}]dA \quad (21)$$

总的竖向动弯矩及对应的动正应力分别为：

$$M_y=-EI_y\theta_2'(x,t)-EI_{sy}u_2'(x,t) \quad (22)$$

$$\sigma_{sa}=-Ez\theta_2'+E(w_{s0}-zw_{sy})u_2'(x,t) \quad (23)$$

那么，垂直弯矩产生的动应变能为：

$$v_z=\frac{1}{2}\iint(\frac{\sigma_{sa}^2}{E}+\frac{\tau_{sx}^2}{G})dAdx$$
$$=\frac{1}{2}\int_0^L\{E[I_s(u_2')^2+2I_{sy}\theta_2'u_2'+I_y(\theta_2')^2]+Gk_{sy}u_2^2\}dx \quad (24)$$

式中：$I_s=\int_A(w_{s0}-zw_{sy})^2dA$；$I_y=\int_A z^2dA$；

$$k_{sy}=\int_A(\frac{\partial w_{sy}}{\partial y})^2z^2dA.$$

铁木辛柯剪切应变能为：

$$V_1=\frac{1}{2}\int_0^L kGA(\theta_2-\frac{\partial w_2}{\partial x})^2dx \quad (25)$$

结构总动能为[1,11]：

$$T=\frac{1}{2}\int_0^l(\frac{\partial w_2}{\partial t})^2\rho Adx+\frac{1}{2}\rho I_s\int_0^l(\dot{u}_2)^2dx+\frac{1}{2}\rho I\int_0^l(\dot{\theta}_2)^2dx \quad (26)$$

T形梁稳态振动时，考虑剪滞翘曲应力自平衡条件，其各项势能

T形截面梁的荷载势能V_q为：

$$V_q=-\int_0^L q(x,t)w_2(x,t)dx-[Q(x,t)w_2(x,t)]\Big|_0^L+[M_{1x}(x,t)u_2(x,t)+M_{xA}(x,t)\theta_2(x,t)]\Big|_0^L \quad (27)$$

系统总势能为：

$$V_B=V_z+V_I+V_q \quad (28)$$

式中：$\theta_z(x,t)$为T形截面竖向动转角；$u_2(x,t)$为振动时剪力滞效应引起翼板的纵向动位移差函数；且I_y；I_{sy}；k_{sy}；I_s的含义如上表示，其它符号意义与不计剪滞翘曲应力自平衡条件时相同.

1.2.2 T形梁强迫振动微分方程及自然边界条件

由哈密顿原理$\delta\int_0^L(T_B-V_B)dt=0$，可推导出T形梁动力反应控制微分方程及自然边界条件为[12]：

$$-\rho A\ddot{w}_2-kGA(\theta_2'-w_2'')+q(x,t)=0 \quad (29)$$

$$-\rho I_s\ddot{u}_2+EI_{sy}\theta_2''+EI_su_2''-Gk_{sy}u_2=0 \quad (30)$$

$$-\rho I_y\ddot{\theta}_2+EI_{sy}u_2''+EI_y\theta_2''-kGA(\theta_2-w_2')=0 \quad (31)$$

$$[-EI_su_2'-EI_y\theta_2'+M_{1x}(x,t)]\delta u_2\Big|_0^L=0 \quad (32)$$

$$[-EI_{sy}u_2'-EI_y\theta_2'+M_{xA}]\delta\theta_2\Big|_0^L=0 \quad (33)$$

$$[-kGA(\theta_2-w_2')-Q(x,t)]\delta w_2\Big|_0^L=0 \quad (34)$$

同样，式(18)～(34)中，符号“.”和“′”分别表示对时间t和对坐标x求偏导数.

1.2.3 T形梁强迫振动微分方程的求解

若T形梁强迫振动频率为ω_0，那么

$q(x,t)=q_0(x)\sin(\omega_0t+\varphi)$，

可令：$w_2(x,t)=W_2(x)\sin(\omega_0t+\varphi)$；

$u_2(x,t)=U_2(x)\sin(\omega_0t+\varphi)$；

$\theta_2(x,t)=\psi_2(x)\sin(\omega_0t+\varphi)$，

由方程(31)得$u_2^{(3)}$、$u_2^{(5)}$的表达式，对方程(30)三次

求导,且将 $u_2^{(3)}$、$u_2^{(5)}$ 代入可得关于 w_2 和 θ_2 项的微分方程,将方程(29)代入,可得新微分方程为:

$$W_2^{(6)}+W_2^{(4)}\left[\frac{\rho A\omega_0^2E^2(I_{sy}^2-I_sI_y)-kGAEI_y(\rho I_s\omega_0^2+T)}{kGAE^2(I_{sy}^2-I_sI_y)}\right]+W_2^{(2)}\left[\frac{\rho\omega_0^2kGA(EI_sA-I_yT)-\rho A\omega_0^2EI_y(\rho I_s\omega_0^2+T)}{kGAE^2(I_{sy}^2-I_sI_y)}\right]+W_2\frac{\rho A\omega_0^2T(kGA-\rho I_y\omega_0^2)}{kGAE^2(I_{sy}^2-I_sI_y)}=\frac{-T(kGA-\rho I_y\omega_0^2)}{kGAE^2(I_{sy}^2-I_sI_y)}q_0 \tag{35}$$

式中:$T=\rho I_s\omega_0^2-Gk_{sy}$.

对方程(35)分析可知,其特征方程解可为:

$r_{1,2}=\pm(\alpha_1^s+\beta_1^si)$;$r_{3,4}=\pm(\alpha_2^s+\beta_2^si)$;

$r_{5,6}=\pm(\alpha_3^s+\beta_3^si)$

分析可知,方程(35)通解形式与方程(14)相同. 且 $\psi_2(x)$ 与 $\psi_1(x)$,$U_2(x)$ 与 $U_1(x)$ 解的表示形式亦相同,但此时

$T_2=-(\rho I_s\omega_0^2-Gk_{sy})/(EI_{sy})$; $T_1=-(I_s/I_{sy})$.

2　T 形梁动力反应的简支边界条件

分析可知,两种情况下 T 形梁的边界条件相同,那么将 $W_1(x)$,$W_2(x)$;$\psi_1(x)$,$\psi_2(x)$ 和 $U_1(x)$,$U_2(x)$ 统一以 $W(x)$,$\psi(x)$,$U(x)$ 的形式表示,则:

1) 简支 T 形梁的位移和力学边界条件

A、简谐分布力

$$W(x)\Big|_0^L=0;\ \psi'(x)\Big|_0^L=0;\ U'(x)\Big|_0^L=0 \tag{36}$$

B、简谐集中力

对于简支 T 形梁,若跨间所受力为一简谐集中力,且集中力 $P_k=P_0\sin(\omega_0t+\varphi)$ 左右相邻边界距离为 L_{k1} 和 L_{k2}. 如图 3 所示,则 k 点处还须引入下列连续边界条件为:

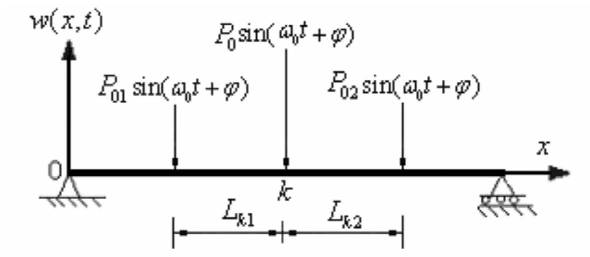

图 3　算例中坐标系的约定

Fig. 3　The fixed coordinate system in the calculation examples

$U_{k1}(L_{k1})=U_{k2}(0)$; $U'_{k1}(L_{k1})=U'_{k2}(0)$;

$\psi_{k1}(L_{k1})-\psi_{k2}(0)=\dfrac{-P_0}{kGA}$; $\psi'_{k1}(L_{k1})=\psi'_{k2}(0)$;

$$W_{k1}(L_{k1})=W_{k2}(0);\ W'_{k1}(L_{k1})=W'_{k2}(0) \tag{37}$$

将方程式 $W_1(x)$,$W_2(x)$;$\psi_1(x)$,$\psi_2(x)$;$U_1(x)$,$U_2(x)$ 或其求导式代入边界条件(36)~(37),然后应用 MATLAB 软件和相应剪滞系数公式便可得到 T 形梁翼板简支边界条件下动剪滞效应的变化规律.

3　算例

对于 T 形截面梁,其材料和几何参数为:$\rho=2500\ \text{kg/m}^3$;$E=3.5\times10^4$ MPa;$G=1.5\times10^4$ MPa;$t_w=0.15$ m;$t=0.11$ m;$b=2.85$ m. 且梁高为 $h=1$ m,简谐集中力幅值为 $P_0=9800$ N. 则根据本文推导公式和其它方法可计算出 T 形梁的自振频率及 E、F 点动应力幅值.

在简支 T 形梁自振频率的求解过程中,令简谐分布力 $q(x,t)=0$,然后应用 MATLAB 软件和边界条件(36)便可得到表 1T 形梁的自振频率值.

表 1　简支 T 形梁的固有频率(单位:Hz)

Table 1　Natural frequency of simply-supported T-beam (unit: Hz)

Span width ratio($L/2b$)	Number of vibration modes	first order	second order	Third order	Fourth order	Fifth order
1.75	Timoshenko beam theory	12.408	45.354	90.531	141.471	194.624
1.75	Without considering self - equilibrium	12.107	43.659	87.250	137.039	189.455
1.75	Considering self - equilibrium	12.035	42.297	83.276	130.448	180.994
1.75	Finite element method	11.939	41.463	79.407	* * *	* * *
	Shear lag contribution (%)	3.01	6.74	8.01	7.792	7.00
2.63	Timoshenko beam theory	5.621	21.513	45.354	74.577	107.062
2.63	Without considering self - equilibrium	5.539	20.838	43.659	71.796	103.379
2.63	Considering self - equilibrium	5.534	20.550	42.300	68.908	98.472
2.63	Finite element method	5.428	19.929	41.438	65.231	* * *
	Shear lag contribution (%)	1.55	4.48	6.73	7.60	8.02

注:有限元法对于自振频率难以辨识的,本文以符号 * * * 作了标示. 本文理论为同时考虑剪滞翘曲应力自平衡条件和剪滞效应影响的计算理论;而传统理论则为仅考虑剪滞效应影响的计算理论,下同.

表 1 表明:

1) 本文 T 形梁自振频率计算值小于传统剪滞理论计算值,而传统剪滞理论计算值又小于铁木辛柯梁理论值,故由能量最低原理可以判断,在 T 形梁静、动力学分析中,本文计算理论优于传统剪滞理论,而传统剪滞理论又优于铁木辛柯梁理论;

2) 本文自振特性分析为文献[1]的静力学分析提供了理论依据,且尽管三种理论自振频率计算值相差很小,但动力反应分析中本文计算理论有无实际意义尚需进一步的动力学分析加以验证;

3) T 形梁自振特性受跨宽比的影响,跨宽比小自平衡条件和剪滞效应影响大,反之相反.

同样,简谐集中力作用下,在 T 形梁动应力幅值的求解过程中,应用 MATLAB 软件和边界条件(36)、(37)可得表 2、表 3 和表 4 T 形梁的动应力幅值为:

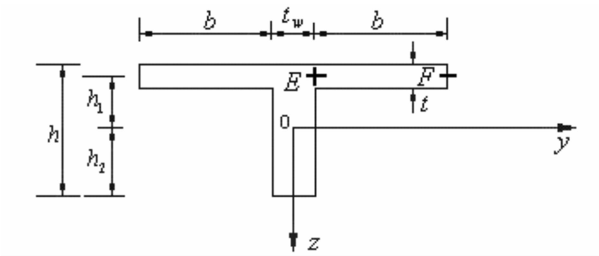

图 4 交点 E、F 为所求 T 形梁跨中动应力幅值位置

Fig. 4 Dynamic stress amplitude of middle-span of the T-beam located at the crossing points E and F

表 2 简支 T 形梁 E 点的动应力幅值 (单位: 10^4 Pa) (简谐集中力)

Table 2 Dynamic stress amplitude of simply-supported T-beam located at the crossing point E (unit: 10^4 Pa) (Harmonic concentrated load)

Span width ratio($L/2b$)	Frequency value of harmonic force (Hz)	1	2	3	4	5	6	7
1.75	Timoshenko beam theory	5.7072	5.7943	5.9473	6.1801	6.5166	6.9975	7.6958
1.75	Without considering self – equilibrium (E point)	6.0884	6.1852	6.3558	6.6166	6.9959	7.5433	8.3495
1.75	Dynamic shear lag coefficients	1.0668	1.0675	1.0687	1.0706	1.0736	1.0780	1.0849
1.75	Considering self – equilibrium (E point)	10.860	10.999	11.246	11.623	12.173	12.967	14.198
1.75	Dynamic shear lag coefficients	1.9029	1.8982	1.8909	1.8807	1.8680	1.8531	1.8449
1.75	Finite element value (E point)	9.8963	10.313	10.627	11.115	11.638	12.569	14.103
1.75	Frequency value of harmonic force (Hz)	8	9	10	11	12	12.4	
1.75	Timoshenko beam theory	8.7520	10.474	13.682	21.533	68.176	3626.3	
1.75	Without considering self – equilibrium(E point)	9.5941	11.693	15.851	27.630	260.26	91.628	
1.75	Dynamic shear lag coefficients	1.0962	1.1164	1.1585	1.2832	3.8175	0.0253	
1.75	Considering self – equilibrium (E point)	15.963	19.061	25.300	43.749	1094.5	101.33	
1.75	Dynamic shear lag coefficients	1.8239	1.8198	1.8491	2.0317	16.054	0.0279	
1.75	Finite element value (E point)	16.348	19.726	23.604	39.115	842.662	76.476	

(注:根据静剪滞系数定义,动剪滞系数 λ_D = T 形梁翼板剪滞理论动应力幅值 σ_{JD}/T 形梁翼板铁木辛柯梁理论动应力幅值 σ_{DT}. 且有限元法计算中,按图 2 T 形梁各交点坐标绘制 T 形梁断面,然后应用 ANSYS 有限元的 Extrude 功能形成体,划分单元网格,模拟简支边界条件在 T 形梁一端节点 x,y,z 三向施以约束,另一端在节点 z,y 方向施以约束. 下同)

表 3 简支 T 形梁 F 点的动应力幅值 (单位: 10^4 Pa) (简谐集中力)

Table 3 Dynamic stress amplitude of simply-supported T-beam located at the crossing points F (unit: 10^4 Pa) (Harmonic concentrated load)

Span width ratio($L/2b$)	Frequency value of harmonic force (Hz)	1	2	3	4	5	6	7
1.75	Timoshenko beam theory	5.7072	5.7943	5.9473	6.1801	6.5166	6.9975	7.6958
1.75	Without considering self – equilibrium (F point)	2.0788	2.1235	2.2024	2.3229	2.4983	2.7515	3.1243
1.75	Dynamic shear lag coefficients	0.3642	0.3665	0.3703	0.3770	0.3834	0.3932	0.4060
1.75	Considering self – equilibrium (F point)	2.5705	2.6352	2.7492	2.9238	3.1782	3.5463	4.1152
1.75	Dynamic shear lag coefficients	0.4504	0.4548	0.4623	0.4731	0.4877	0.5068	0.5347
1.75	Finite element value (F point)	2.3219	2.4206	2.5348	2.6983	3.0235	3.4812	4.1434
1.75	Frequency value of harmonic force (Hz)	8	9	10	11	12	12.4	
1.75	Timoshenko beam theory	8.7520	10.474	13.682	21.533	68.176	3626.3	
1.75	Without considering self – equilibrium (F point)	3.7004	4.6720	6.5973	12.053	119.81	43.190	
1.75	Dynamic shear lag coefficients	0.4228	0.4461	0.4822	0.5598	1.7574	0.0119	
1.75	Considering self – equilibrium (F point)	4.9356	6.3736	9.2708	17.841	506.08	49.576	
1.75	Dynamic shear lag coefficients	0.5639	0.6085	0.6776	0.8285	7.4231	0.0137	
1.75	Finite element value (F point)	5.2453	6.8117	9.9837	18.945	359.446	40.994	

表 4　简支 *T* 形梁 E 点的动应力幅值(单位: 10^4Pa)(简谐集中力)

Table 4　Dynamic stress amplitude of simply-supported T-beam located at the crossing point E (unit: 10^4Pa) (Harmonic concentrated load)

Span width ratio(*L*/2*b*)	Frequency value of harmonic force (Hz)	0.5	1	1.5	2	2.5	3
2.63	Timoshenko beam theory	8.5721	8.7381	9.0336	9.4923	10.175	11.194
2.63	Withoutconsidering self – equilibrium (E point)	8.9672	9.1438	9.4587	9.9490	10.683	11.786
2.63	Dynamic shear lag coefficients	1.0461	1.0464	1.0471	1.0481	1.0499	1.0529
2.63	Considering self – equilibrium (E point)	13.727	13.941	14.321	14.913	15.800	17.133
2.63	Dynamic shear lag coefficients	1.6014	1.5954	1.5853	1.5711	1.5528	1.5306
2.63	Finite element value (E point)	12.806	13.297	13.592	14.464	15.629	17.558
2.63	Frequency value of harmonic force (Hz)	3.5	4	4.5	5	5.5	5.6
2.63	Timoshenko beam theory	12.772	15.411	20.513	34.016	161.43	979.43
2.63	Withoutconsidering self – equilibrium (E point)	13.514	16.459	22.358	39.415	507.95	305.01
2.63	Dynamic shear lag coefficients	1.0581	1.0680	1.0899	1.1587	3.1466	0.3114
2.63	Considering self – equilibrium (E point)	19.223	22.786	29.939	50.734	699.17	337.90
2.63	Dynamic shear lag coefficients	1.5051	1.4786	1.4595	1.4915	4.3311	0.3450
2.63	Finite element value (E point)	19.746	23.505	31.524	46.028	546.136	281.926

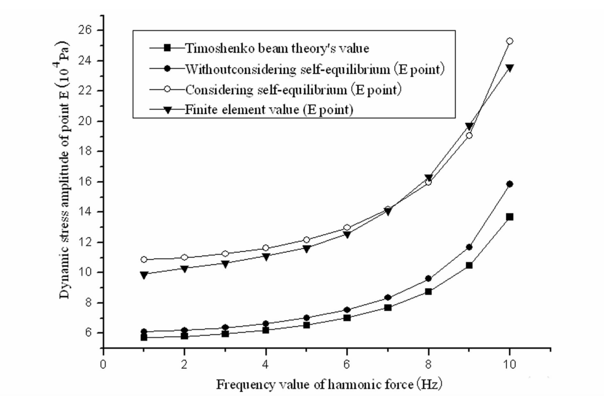

图 5　简支 T 形梁跨中 E 点动应力幅值比较图($L = 10$ m)(简谐集中力)

Fig. 5　The Comparison of dynamic stress amplitude of middle-span of simply-supported T-beam located at the crossing points E ($L = 10$ m) (Harmonic concentrated load)

4　结论

本文理论系统分析了薄壁宽翼 T 形梁桥的动力学特性,结果显示出剪滞翘曲应力自平衡条件影响下 T 形梁自振频率计算值减小,并且自平衡条件对翼板正应力幅值计算值的影响更大,与传统剪滞理论计算值相比较,翼板与腹板相交处本文理论正应力幅值计算值明显增大,而远离腹板处翼板的正应力幅值计算值增幅较小. 本文理论值与有限元数值解吻合较好,结果显示出剪滞翘曲应力自力平衡条件引入的必要性.

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ANALYSIS ON DYNAMIC THEORY AND CHARACTERISTICS OF THIN-WALLED T-BEAMS WITH WIDE FLANGES *

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Abstract In consideration of shear deformation and shear lag effects, A new warping displacement mode of T-beams is chosen to meet the axial self-equilibrium condition for corresponding stress, this paper proposes an approach of analyzing the dynamic characteristics of thin-walled T-beams with wide flanges generally used in engineering. three generalized displacement functions are employed in analyzing dynamic response of the thin-walled T-beams by calculus of variations, the differential equations and the corresponding natural boundary conditions of the T-beams are induced based on the minimum potential principle, and the dynamic characteristics of thin-walled T-beams are discussed. The calculation examples compare the finite solid element solutions with the analytical solutions, and the analytical solutions in consideration of the axial self-equilibrium condition is still more identical with the finite solid element solutions, the formulas obtained in this study strengthen the theoretical foundation for further research of dynamic characteristics of the structures.

Key words T-beam, shear lag effect, self-equilibrium condition, dynamic response, energy-variation principle

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