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微重力环境下刚液耦合系统液体晃动混沌现象研究*

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摘要 微重力下,柱形贮箱内液体晃动的速度势模态和表面位移模态难以解析表达,为揭示液-刚耦合运动的非线性特性,不失一般性地用常重力下液体晃动的速度势模态和表面位移模态近似表示微重力下液体晃动的速度势模态和表面位移模态. 用泰勒级数展开法分析了微重力下柱形贮箱内的液体晃动,运用Lagrange方法导出了微重力下贮箱内液体与结构耦合系统的无量纲动力学方程组,并用 Matlab 软件对该方程组进行数值计算,发现当系统稳定时,面内、外模态分别具有同类的稳态动力学行为,包括静止、周期运动、准周期运动和混沌运动;在不同的外激励参数下,面内、外模态的稳态动力学行为发生变化.

关键词 微重力, 非线性晃动, 耦合动力学, 稳态运动

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引言

从 Navier-Stokes 方程出发,对液体晃动进行时域的数值仿真,通常称为 CFD(计算流体动力学)方法,但这种方法需要大量的计算并且具有跨学科的特点,采用计算流体动力学方法研究晃动液体和刚体耦合系统动力学方面的研究成果也很少. 很多文献[1-11]在对充液航天器建模时,常假设液体晃动幅度远小于容器的特征尺寸,将液体晃动动力学行为线性化. 对于液体非线性晃动,和线性晃动一样,对液体作不可压缩、无黏、无旋的势流假设,由于自由液面上的运动学和动力学边界条件下不能再作线性化处理,同时液体晃动的各阶频率不再是常数,阻尼也是非线性的,各阶晃动会产生内共振,无法解耦处理,也无法利用线性叠加原理. 通常的处理方法是将未知的自由液面波高和液体速度势分别展成液体晃动模态坐标的级数形式,利用泛函变分导出速度势函数广义坐标与波高函数广义坐标之间的关系,运用 Lagrange 方程或 Hamilton 方程建立起波高函数广义坐标的动力学方程,再采用多尺度方法或 J. W. Mile[12]提出的平均 Lagrange 函数法进行分析求解. Peterson 等[13-14]指出,液体-航天器耦合系统在外界激励幅值较大时本质上是一个非线性耦合动力学系统,液体-航天器的动力学行为不能通过将未耦合的液体非线性晃动叠加到航天器的线性振动上来描述,如果试图这样做,就会导致液体-航天器耦合动力学特性错误的分析结果,他们把航天器模型化为一个弹簧-阻尼-质量系统,对受水平激励和竖直激励贮箱中液体考虑了前五阶晃动模态,研究揭示了十分复杂的动力学现象,如丰富的次生晃动模态、密集的内共振运动等. 苟兴宇[15]对邦德数远大于 1 的情况下考虑窄长方形贮箱中液体强迫晃动,给出了激励中含有恒定分量时的几种液面波高修正方法,并对它们作了对比. 尹立中[16]对贮箱内液体带有自由液面的充液耦合系统动力学的研究方法进行简要介绍,概要地综述了这些问题的研究进展. 王照林等[17]分析了失重状态下球腔内液体的静液面形状,构造特征函数计算得到了液体晃动参数. HE Yuan-jun[18]用变分原理建立了微重力环境下液体晃动的压力体积分形式的 Lagrange 函数;并将速度势函数在自由液面处作波高函数的级数展开,从而导出自由液面运动学和动力学边界条件非线性方程组,并用多尺度法对其进行了解析研究,分析了系统的幅频响应特性随 Bond 数的变化规律、跳跃和滞后等非线性晃动现象. Faltinsen[19-23]等提出多维模态方法,用来

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分析矩形贮箱中的液体非线性晃动问题,它们的研究成果接连发表在流体力学的顶级杂志 JFM 上,理论分析与实验观测结果取得了一致,是一种用来分析液体非线性晃动的有效的模态解析方法.其基本思想是根据问题的具体情况选取主导模态和与其关系密切的次生模态,然后采用 Narimanov-Moiseev 三阶渐进假设,完成无穷维模态系统的降阶.对于实验中出现的液体飞溅现象所导致的能量耗散,它们通过假设飞溅液体的动能和势能消失来加以考虑. 余延生[24]将多维模态方法应用于圆柱贮箱液体非线性晃动问题的研究中,选取了两阶主导模态和三阶次生模态,推导出了描述圆柱贮箱液体自由晃动的五阶渐进模态系统,最后通过数值仿真揭示了一些典型的非线性现象. Abramson[25]等对推进剂晃动的早期研究进行了全面、系统综述,并附录了大量的参考文献. 岳宝增[26]阐述了储液罐动力学与控制的工程应用背景,从储液罐类液体晃动动力学、液体晃动等效力学模型和储液罐多体系统动力学与控制等三个方面回顾了储液罐动力学与控制的研究进展,并附录了大量的参考文献.

本文研究航天器的水平二维平动和液体晃动的液刚耦合动力学.因为不考虑接触角迟滞时,晃动液体微重力时弯曲的静液面和常重力时水平的静液面的特征模态没有显著的区别[27],本文的微重力下的液体晃动模态取前常重力时的前五个模态.通过推导系统总的拉格朗日函数,由拉格朗日方程得到系统的动力学方程组.然后对无量纲的动力学方程组进行数值计算,发现结构的 x 方向位移、波高面内主次模态及轴对称二阶模态(本文将此四者简称为面内模态)表现出同类的稳态动力学行为;结构的 y 方向位移、波高面外主次模态(本文将此三者简称为面外模态)表现出同类的稳态动力学行为,面内、外模态在一定幅值不同频率或一定频率不同幅值的外激励下随外激励频率或幅值变化具有不同类型的稳态动力学行为,包括静止、周期运动、准周期运动和混沌运动.

1　充液系统的液-刚耦合动力学模型

带柱形贮箱的充液航天器液-刚耦合动力学系统模型如图 1 所示,假设航天器和晃动液体的耦合作用等效成航天器运动时受到弹簧-质量系统的约束,航天器在 x 方向外激励作用下作 x 和 y 方向二维平动.假设液体无粘、无旋、不可压.

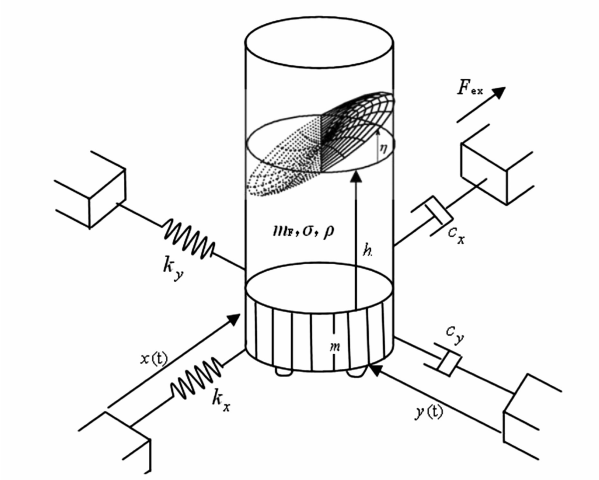

图 1　液-刚耦合动力学模型固支

Fig. 1　The model of fluid-rigid coupling dynamics Clamped

常重力下,静液面水平,液体速度势可表示为:

$$\phi(r,\theta,z,t) = \sum_{i=1}^{N}\frac{\cosh(k_i(z+h))}{\cosh(k_i h)}\psi_i(r,\theta)\phi_i(t) \tag{1}$$

其中 $\psi_i(r,\theta)$ 的表达式见附件 A.

在不考虑接触角迟滞的情况下,液体表面位移模态形状也可表示为:

$$\xi_i = A_{mn}J_m(k_{mn}r)\begin{cases}\cos(m\theta)\\ \sin(m\theta)\end{cases} \tag{2}$$

微重力下的特征模态形状可以用(1)、(2)近似.晃动液体的动能为:

$$T_F = \frac{1}{2}m_F\dot{\boldsymbol{R}}\cdot\dot{\boldsymbol{R}} + \frac{1}{2}\rho\iint_{S_F}\phi\frac{\partial\phi}{\partial\boldsymbol{n}}\mathrm{d}S_F + \rho\iiint_V\dot{\boldsymbol{R}}\nabla\phi\mathrm{d}V \tag{3}$$

$\dot{\boldsymbol{R}}$ 用势函数表示为 $\dot{\boldsymbol{R}} = \nabla(x\dot{x} + y\dot{y})$,对式(3)的第三项应用格林第一定理,将式(3)的第二项表示成液体晃动模态坐标的泰勒级数展开的形式[13],略去四阶以上的高阶项,晃动液体的动能可以写成:

$$\begin{aligned}T_F =& \frac{1}{2}m_F(\dot{x}^2+\dot{y}^2) + \\ &\frac{1}{2}\rho\Big(\sum_{m=1}^{5}\sum_{n=1}^{5}a_{mn}^{0}\dot{q}_m\dot{q}_n + \sum_{m=1}^{5}\sum_{n=1}^{5}\sum_{r=1}^{5}a_{mnr}^{(1)}q_r\dot{q}_m\dot{q}_n + \\ &\sum_{m=1}^{5}\sum_{n=1}^{5}\sum_{r=1}^{5}\sum_{s=1}^{5}a_{mnrs}^{(2)}q_rq_s\dot{q}_m\dot{q}_n\Big) + \\ &\rho\dot{x}\Big(\sum_{n=1}^{5}\dot{q}_n\iint_{S_B}\xi_n r\cos(\theta)dS_B\Big) + \\ &\rho\dot{y}\Big(\sum_{n=1}^{5}\dot{q}_n\iint_{S_B}\xi_n r\sin(\theta)dS_B\Big)\end{aligned} \tag{4}$$

具体推导这里略去，$a_{mn}^{(0)}$，$a_{mnr}^{(1)}$，$a_{mnrs}^{(2)}$ 的具体表达式是文献[13]中的 $a_{mn}^{(0)}$，$a_{mnr}^{(1)}$，$a_{mnrs}^{(2)}$ 与 S_B 之积.

贮箱内晃动液体的势能由重力势能 U_G 和表面张力势能 U_σ 组成，其中表面张力势能 U_σ 也写成晃动模态坐标的泰勒级数展开的形式[13]，如下所示：

$$U_G = \frac{\rho g S_B}{2}\sum_{i=1}^{5} q_i^2 \tag{5}$$

$$\begin{aligned} U_\sigma = {} & \sigma\sum_{i=1}^{5} S_{ii}^{(2)} q_i^2 + \sigma\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=3}^{5}(2S_{ijk}^{(3)} + \\ & S_{kji}^{(3)})q_i q_j q_k + \sigma S_{1111}^{(4)}(q_1^4 + q_2^4) + \\ & \sigma(S_{1122}^{(4)} + S_{2211}^{(4)} + 4S_{1212}^{(4)})q_1^2 q_2^2 \end{aligned} \tag{6}$$

$S_{ii}^{(2)}$，$S_{ijk}^{(3)}$，$S_{1111}^{(4)}$，$S_{1122}^{(4)}$，$S_{2211}^{(4)}$，$S_{1212}^{(4)}$ 的表达式见文献[13].

记 $m_{xq_1} = \rho\iint_{S_B}\psi_1 r\cos(\theta)\mathrm{d}S_B$，$m_{yq_2} = \rho\iint_{S_B}\psi_2 r\sin(\theta)\mathrm{d}S_B$，晃动液体和航天器总的 Lagrange 函数为：

$$\begin{aligned} L = {} & \frac{1}{2}\rho(\sum_{m=1}^{5} a_{mn}^{0}\dot{q}_m^2 + \sum_{m=1}^{2}\sum_{n=1}^{2}\sum_{r=3}^{5}(a_{mnr}^{(1)} q_r\dot{q}_m\dot{q}_n + \\ & 2a_{rmn}^{(1)} q_n\dot{q}_r\dot{q}_m) + \sum_{m=1}^{2}\sum_{n=1}^{2}\sum_{r=1}^{2}\sum_{s=1}^{2}(a_{mnrs}^{(2)} q_r q_s\dot{q}_m\dot{q}_n) + \\ & m_{xq_1}\dot{x}\dot{q}_1 + m_{yq_2}\dot{y}\dot{q}_2 + \frac{1}{2}m_F(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}m\dot{x}^2 + \\ & \frac{1}{2}m\dot{y}^2 - \frac{1}{2}k_x x^2 - \frac{1}{2}k_y y^2 - \frac{\rho g S_B}{2}\sum_{i=1}^{5} q_i^2 - \\ & \sigma\sum_{i=1}^{5} S_{ii}^{(1)} q_i^2 - \sigma\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=3}^{5}(2S_{ijk}^{(3)} + S_{kji}^{(3)})q_i q_j q_k - \\ & \sigma S_{1111}^{(4)} q_1^4 - \sigma S_{2222}^{(4)} q_2^4 - \sigma(S_{1122}^{(4)} + S_{2211}^{(4)} + \\ & 4S_{1212}^{(4)})q_1^2 q_2^2 \end{aligned} \tag{7}$$

由拉格朗日方程得系统的非线性动力学方程组：

$$\begin{aligned} & (m + m_F)\ddot{x} + m_{xq_1}\ddot{q}_1 + c_x\dot{x} + k_x x = F_{ex} \\ & (m + m_F)\ddot{y} + m_{yq_2}\ddot{q}_2 + c_y\dot{y} + k_y y = 0 \\ & m_{xq_1}\ddot{x} + \rho a_{11}^{(0)}\ddot{q}_1 + c_{q_1}\dot{q}_1 + (\rho g S_B + 2\sigma S_{11}^{(2)})q_1 + \\ & \quad \rho a_{113}^{(1)}\ddot{q}_1 q_3 + \rho a_{311}^{(1)}\ddot{q}_3 q_1 + \rho a_{114}^{(1)}\ddot{q}_1 q_4 + \rho a_{411}^{(1)}\ddot{q}_4 q_1 + \\ & \quad \rho a_{125}^{(1)}\ddot{q}_2 q_5 + \rho a_{512}^{(1)}\ddot{q}_5 q_2 + \rho a_{113}^{(1)}\dot{q}_1\dot{q}_3 + \rho a_{114}^{(1)}\dot{q}_1\dot{q}_4 + \\ & \quad \rho(a_{125}^{(1)} + a_{512}^{(1)} - a_{521}^{(1)})\dot{q}_2\dot{q}_5 + \rho(a_{2121}^{(2)} + \\ & \quad a_{2112}^{(2)})\ddot{q}_2 q_1 q_2 + 2\rho a_{2211}^{(2)}\dot{q}_1 q_2\dot{q}_2 + \rho(a_{2121}^{(2)} + \\ & \quad a_{2112}^{(2)} - a_{2211}^{(2)})q_1\dot{q}_2^2 + \rho a_{2211}^{(2)}\ddot{q}_1 q_2^2 + \rho a_{1111}^{(2)}\ddot{q}_1 q_1^2 + \\ & \quad \rho a_{1111}^{(2)}\dot{q}_1^2 q_1 + \sigma(2S_{311}^{(3)} + 4S_{113}^{(3)})q_1 q_3 + \\ & \quad \sigma(2S_{411}^{(3)} + 4S_{114}^{(3)})q_1 q_4 + \sigma(2S_{521}^{(3)} + 4S_{125}^{(3)})q_2 q_5 + \\ & \quad \sigma(4S_{2211}^{(4)} + 8S_{2121}^{(4)})q_1 q_2^2 + 4\sigma S_{1111}^{(4)})q_1^3 = 0 \\ & m_{yq_2}\ddot{y} + \rho a_{22}^{(0)}\ddot{q}_2 + c_{q_2}\dot{q}_2 + (\rho g S_B + 2\sigma S_{22}^{(2)})q_2 + \\ & \quad \rho a_{223}^{(1)}\ddot{q}_2 q_3 + \rho a_{322}^{(1)}\ddot{q}_3 q_2 + \rho a_{224}^{(1)}\ddot{q}_2 q_4 + \rho a_{422}^{(1)}\ddot{q}_4 q_2 + \\ & \quad \rho a_{125}^{(1)}\ddot{q}_1 q_5 + \rho a_{521}^{(1)}\ddot{q}_5 q_1 + \rho a_{223}^{(1)}\dot{q}_2\dot{q}_3 + \rho a_{224}^{(1)}\dot{q}_2\dot{q}_4 + \\ & \quad \rho(a_{125}^{(1)} + a_{521}^{(1)} - a_{512}^{(1)})\dot{q}_1\dot{q}_5 + \rho(a_{2121}^{(2)} + \\ & \quad a_{2112}^{(2)})\ddot{q}_1 q_1 q_2 + 2\rho a_{2211}^{(2)}\dot{q}_1 q_1\dot{q}_2 + \rho(a_{2121}^{(2)} + a_{2112}^{(2)} - \\ & \quad a_{2211}^{(2)})\dot{q}_1^2 q_2 + \rho a_{2211}^{(2)} q_1^2\ddot{q}_2 + \rho a_{2222}^{(2)}\ddot{q}_2 q_2^2 + \rho a_{2222}^{(2)}\dot{q}_2^2 q_2 + \\ & \quad \sigma(2S_{322}^{(3)} + 4S_{223}^{(3)})q_2 q_3 + \sigma(2S_{422}^{(3)} + 4S_{224}^{(3)})q_2 q_4 + \\ & \quad \sigma(2S_{521}^{(3)} + 4S_{125}^{(3)})q_1 q_5 + \sigma(4S_{2211}^{(4)} + 8S_{2121}^{(4)})q_1^2 q_2 + \\ & \quad 4\sigma S_{2222}^{(4)})q_2^3 = 0 \\ & \rho a_{33}^{(0)}\ddot{q}_3 + c_{q_3}\dot{q}_3 + (\rho g S_B + 2\sigma S_{33}^{(2)})q_3 + \rho a_{311}^{(1)}\ddot{q}_1 q_1 + \\ & \quad \rho(a_{311}^{(1)} - \frac{1}{2}a_{113}^{(1)}\dot{q}_1^2 + \rho a_{322}^{(1)}\ddot{q}_2 q_2 + \rho(a_{322}^{(1)} - \frac{1}{2}a_{223}^{(1)}\dot{q}_2^2 + \\ & \quad \sigma(2S_{113}^{(3)} + S_{311}^{(3)})q_1^2 + \sigma(2S_{223}^{(3)} + S_{322}^{(3)})q_2^2 = 0 \\ & \rho a_{44}^{(0)}\ddot{q}_4 + c_{q_4}\dot{q}_4 + (\rho g S_B + 2\sigma S_{44}^{(2)})q_4 + \\ & \quad \rho a_{411}^{(1)}\ddot{q}_1 q_1 + \rho(a_{411}^{(1)} - \frac{1}{2}a_{114}^{(1)}\dot{q}_1^2 + \rho a_{422}^{(1)}\ddot{q}_2 q_2 + \\ & \quad \rho(a_{422}^{(1)} - \frac{1}{2}a_{224}^{(1)}\dot{q}_2^2 + \sigma(2S_{114}^{(3)} + S_{411}^{(3)})q_1^2 + \\ & \quad \sigma(2S_{224}^{(3)} + S_{422}^{(3)})q_2^2 = 0 \\ & \rho a_{55}^{(0)}\ddot{q}_5 + c_{q_5}\dot{q}_5 + (\rho g S_B + 2\sigma S_{55}^{(2)})q_5 + \\ & \quad \rho a_{512}^{(1)}\ddot{q}_1 q_2 + \rho a_{521}^{(1)} q_1\ddot{q}_2 + \rho(a_{521}^{(1)} + a_{512}^{(1)} - \\ & \quad a_{125}^{(1)})\dot{q}_1\dot{q}_2 + \sigma(4S_{125}^{(3)} + 2S_{521}^{(3)})q_1 q_2 = 0 \end{aligned} \tag{8}$$

其中，$F_{ncx} = -c_x\dot{x} + F_{ex}$，$F_{ncy} = -c_y\dot{y}$，$F_{ncq_i} = -c_{q_i}\dot{q}_i$ 是系统各自由度的非保守外力，c_x，c_y，c_{q_i} 是各自由度的阻尼系数，F_{ex} 是外激励.

将方程组中的 x，$\dot{x}$，$\ddot{x}$，q_n，$\dot{q}_n$，$\ddot{q}_n$ 用 dx，$\omega d\dot{x}$，$\omega^2 d\ddot{x}$，dq_n，$\omega d\dot{q}_n$，$\omega^2 d\ddot{q}_n$ 代替（这里设 $k_x = k_y = k$，$\omega = \sqrt{k/m}$ 是航天器结构模态的自然频率，d 是贮箱内的液高），得到系统的无量纲的非线性动力学方程组：

$$\begin{aligned} & (1 + \mu)\ddot{x} + \mu_{xq_1}\ddot{q}_1 + 2\zeta_x\dot{x} + x = \varXi_{ex} \\ & (1 + \mu)\ddot{y} + \mu_{xq_2}\ddot{q}_2 + 2\zeta_y\dot{y} + y = 0 \\ & \lambda_{xq_1}\frac{\mu}{\mu_{xq_1}}\ddot{x} + \ddot{q}_1 + 2\zeta_{q_1} v\dot{q}_1 + v^2 q_1 + \\ & \quad \alpha_{113}\ddot{q}_1 q_3 + \alpha_{311}\ddot{q}_3 q_1 + \alpha_{114}\ddot{q}_1 q_4 + \alpha_{411}\ddot{q}_4 q_1 + \\ & \quad \alpha_{125}\ddot{q}_2 q_5 + \alpha_{512}\ddot{q}_5 q_2 + \alpha_{113}\dot{q}_1\dot{q}_3 + \alpha_{114}\dot{q}_1\dot{q}_4 + \\ & \quad (\alpha_{125} + \alpha_{512} - \alpha_{521})\dot{q}_2\dot{q}_5 + (\alpha_{2112} + \alpha_{2112} \\ & \quad - \alpha_{2211})\ddot{q}_2 q_1 q_2 + 2\alpha_{2211}\dot{q}_1 q_2\dot{q}_2 + (\alpha_{2121} + \alpha_{2112} - \\ & \quad \alpha_{2211})q_1\dot{q}_2^2 + \alpha_{2211}\ddot{q}_1 q_2^2 + \alpha_{1111}\ddot{q}_1 q_1^2 + \\ & \quad \alpha_{1111}\dot{q}_1^2 q_1 + v^2\beta_{113} q_1 q_3 + v^2\beta_{114} q_1 q_4 + \\ & \quad v^2\beta_{125} q_2 q_5 + v^2\beta_{2121} q_1 q_2^2 + v^2\beta_{1111} q_1^3 = 0 \end{aligned}$$

$$
\begin{aligned}
&\lambda_{xq_2}\frac{\mu}{\mu_{xq_2}}\ddot{y}+\mu_{22}\ddot{q}_2+2\mu_{22}\zeta_{q_2}v_2\dot{q}_2+\mu_{22}v_2^2q_2+\\
&\quad\alpha_{223}\ddot{q}_2q_3+\alpha_{322}\ddot{q}_3q_2+\alpha_{224}\ddot{q}_2q_4+\alpha_{422}\ddot{q}_4q_2+\\
&\quad\alpha_{125}\ddot{q}_1q_5+\alpha_{521}\ddot{q}_5q_1+\alpha_{223}\dot{q}_2\dot{q}_3+\alpha_{224}\dot{q}_2\dot{q}_4+\\
&\quad(\alpha_{125}+\alpha_{521}-\alpha_{512})\dot{q}_1\dot{q}_5+(\alpha_{2121}+\\
&\quad\alpha_{2112})\ddot{q}_1q_1q_2+2\alpha_{2211}\dot{q}_1q_1\dot{q}_2+(\alpha_{2121}+\alpha_{2112}-\\
&\quad\alpha_{2211})\dot{q}_1^2q_2+\alpha_{2211}q_1^2\ddot{q}_2+\alpha_{2222}\ddot{q}_2q_2^2+\alpha_{2222}\dot{q}_2^2q_2+\\
&\quad v^2\beta_{223}q_2q_3+v^2\beta_{224}q_2q_4+v^2\beta_{125}q_1q_5+\\
&\quad v^2\beta_{2121}q_1^2q_2+v^2\beta_{2222}q_2^3=0\\
&\mu_{33}(\ddot{q}_3+2\zeta_{q_3}v_3\dot{q}_3+v_3^2q_3)+\alpha_{311}\ddot{q}_1q_1+\\
&\quad(\alpha_{311}-\frac{1}{2}\alpha_{113})\dot{q}_1^2+\alpha_{322}\ddot{q}_2q_2+(\alpha_{322}-\\
&\quad\frac{1}{2}\alpha_{223})\dot{q}_2^2+\frac{1}{2}v^2\beta_{113}q_1^2+\frac{1}{2}v^2\beta_{223}q_2^2=0\\
&\mu_{44}(\ddot{q}_4+2\zeta_{q_4}v_4\dot{q}_4+v_4^2q_4)+\alpha_{411}\ddot{q}_1q_1+\\
&\quad(\alpha_{411}-\frac{1}{2}\alpha_{114})\dot{q}_1^2+\alpha_{422}\ddot{q}_2q_2+(\alpha_{422}-\\
&\quad\frac{1}{2}\alpha_{224})\dot{q}_2^2+\frac{1}{2}v^2\beta_{114}q_1^2+\frac{1}{2}v^2\beta_{224}q_2^2=0\\
&\mu_{55}(\ddot{q}_5+2\zeta_{q_5}v_5\dot{q}_5+v_5^2q_5)+\alpha_{512}\ddot{q}_1q_2+\\
&\quad+\alpha_{521}q_1\ddot{q}_2+(\alpha_{521}+\alpha_{512}-\alpha_{125})\dot{q}_1\dot{q}_2+\\
&\quad v^2\beta_{125}q_1q_2=0
\end{aligned}
\tag{9}
$$

其中所含参数的具体表达式见附件 B.

2　数值结果

对式(9)这样复杂的非线性动力学方程组,要通过解析的方法很难获得深入的认识,因此这里采用数值方法求解. 液刚耦合项均以加速度形式出现,表明航天器结构与贮箱内液体是在惯性力意义下实现耦合的. 而波高模态在广义位移、广义速度及广义加速度意义上均存在耦合项,模态之间互相带动,互相牵制. 加速度项系数随广义位移变化反映了晃动过程中质量不断地迁移的特征.

仿真所用参数参考文献[13]:贮箱半径 $a=0.0155$ m,液体高度 $d=2a$,晃动液体的质量与航天器结构的质量(包括未晃动的液体的质量)之比 $\mu=0.16$,液体晃动频率比 $v=0.9$,阻尼比取为 $\zeta_x=\zeta_y=0.05$,$\zeta_i=0.0348(i=1..5)$[27]. 假设晃动液体是水,密度 $\rho=998$ kg/m^3,表面张力系数 $\sigma=0.07275$ N/m. Bond 数取为 $B_o=10$. $F_{ex}=Q_s\cos(\omega_{ex}t)$,$Q_s$ 和 ω_{ex} 分别是无量纲外激励振幅和频率.

给波高面外二阶模态 q_5 一个小的初始扰动,仿真时间为 200 个无量纲外激励周期,发现系统稳定时,面内模态表现出同类的稳态动力学行为,面外模态表现出同类的稳态动力学行为. 这里讨论 Q_s 取定值、ω_{ex} 变化时系统各自由度的运动. 当 Q_s 为 0.01、ω_{ex} 为 0.7 时,各自由度和波高的时程图如图 2 所示,从图中可看出面内模态趋于周期运动,面外模态趋于静止;Q_s 为 0.01,ω_{ex} 为 0.97 时,系统稳态运动的 Poincare 映射如图 3 所示,图中每个自由度的 Poincare 映射都只有 15 个点,因此系统趋于 15 倍外激励周期运动;Q_s 为 0.01,ω_{ex} 为 0.98 时,系统稳态运动的 Poincare 映射如图 4 所示,各自由度的 Poincare 映射都是一闭圈,因此系统作准周期运动;Q_s 为 0.01,ω_{ex} 为 0.995 时,系统稳态运动功率谱如图5所示,各自由度的功率谱呈连续

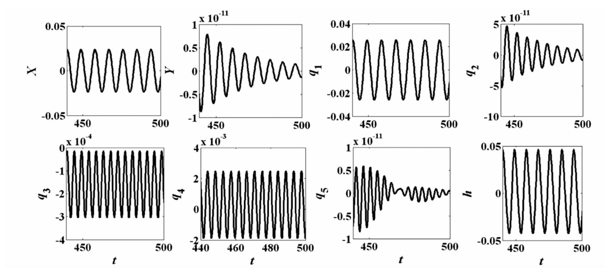

图 2　$Q_s=0.01,\omega_{ex}=0.7$ 时系统的时程图($440\leqslant t\leqslant500$)

Fig. 2　Time change history of the system when $Q_s=0.01,\omega_{ex}=0.7(440\leqslant t\leqslant500)$

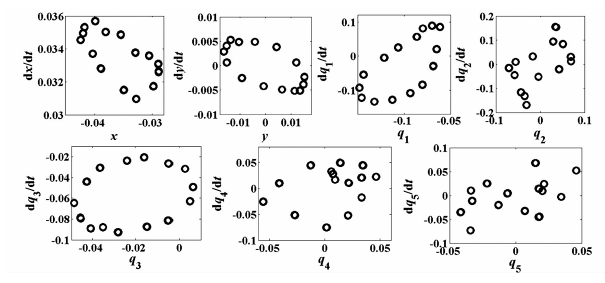

图 3　$Q_s=0.01,\omega_{ex}=0.97$ 时系统稳态运动的 Poincare 映射

Fig. 3　Poincare mapping of the system at stable state when $Q_s=0.01,\omega_{ex}=0.97$

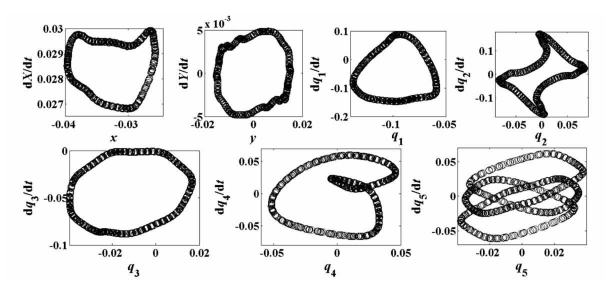

图 4　$Q_s=0.01,\omega_{ex}=0.98$ 时系统稳态运动的 Poincare 映射

Fig. 4　Poincare mapping of this system at stable state when $Q_s=0.01,\omega_{ex}=0.98$

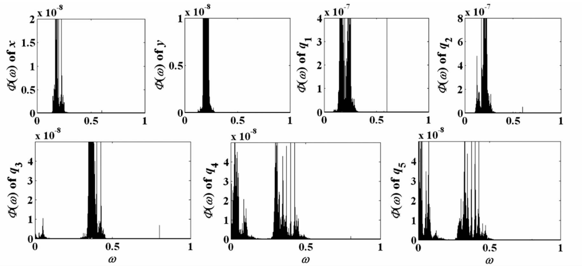

图 5 $Q_s=0.01$，$\omega_{ex}=0.995$ 时系统稳态运动的功率谱

Fig. 5 Power spectrum density of this system at stable state when $Q_s=0.01$，$\omega_{ex}=0.995$

状，因此系统作混沌运动.

进一步可以得到当 ω_{ex} 变化时面内、外模态的稳态运动类型的变化，Q_s 取 0.01，数值积分得到代表面内外模态的结构位移的稳态运动关于 ω_{ex} 的分岔如图 6、7 所示，两图中的空白区域对应使系统不稳定的 ω_{ex} 的范围. 从图中可看出不同的 ω_{ex} 系统有不同类型的稳态运动.

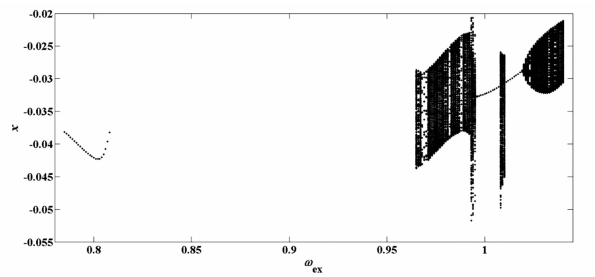

图 6 稳态运动时 x 关于 ω_{ex} 的分岔图（$Q_s=0.01$）

Fig. 6 Bifurcation diagrams of stable x on（$Q_s=0.01$

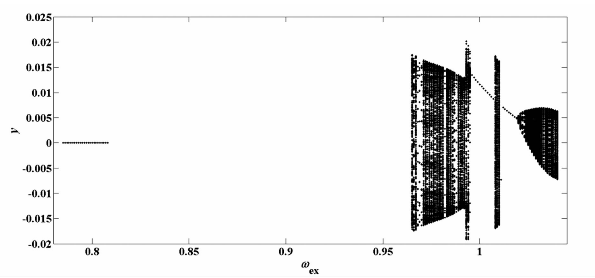

图 7 稳态运动时 y 关于 ω_{ex} 的分岔图（$\omega_{ex}=0.994$）

Fig. 7 Bifurcation diagrams of stable y on（$\omega_{ex}=0.994$）

稳态时关于 Q_s 的分岔如图 8、9 所示. 和 Q_s 固定、ω_{ex} 变化的情形类似，系统不稳定对应于图中空白间隔区域，不同的 Q_s 取值可能导致面内、外模态不同类型的稳态运动.

为了充分证明在某些参数情形下，系统的稳态运动是混沌的，分别计算[28-29]了前述两种情况下系统的最大的四个 Lyapunov 指数 如图 10、11 所示，从图中可以看出，在某些参数情况下，系统的最大 Lyapunov 指数大于零，系统此时的确作混沌运动.

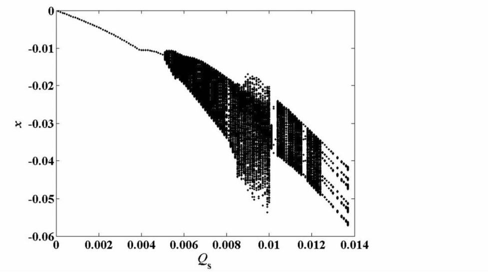

图 8 稳态运动时 x 关于 Q_s 的分岔图

Fig. 8 Bifurcation diagrams of stable x on Q_s

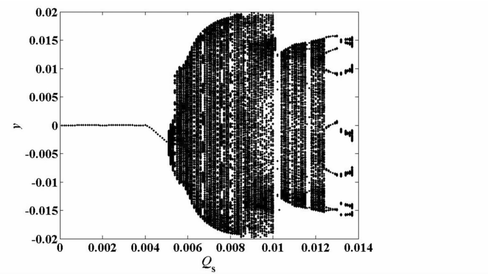

图 9 稳态运动时 y 关于 Q_s 的分岔图

Fig. 9 Bifurcation diagrams of stable y on Q_s

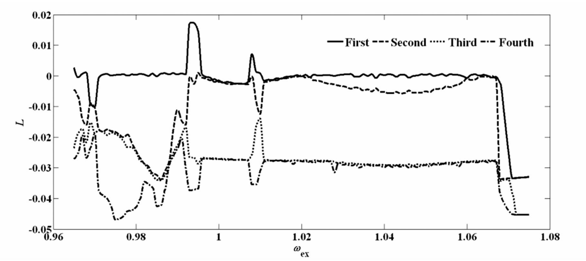

图 10 当 Q_s 为 0.01，ω_{ex} 变化时系统的前四个 Lyapunov 指数

Fig. 10 The first four Lyapunov exponents of the system when Q_s is 0.01 and ω_{ex} is variable

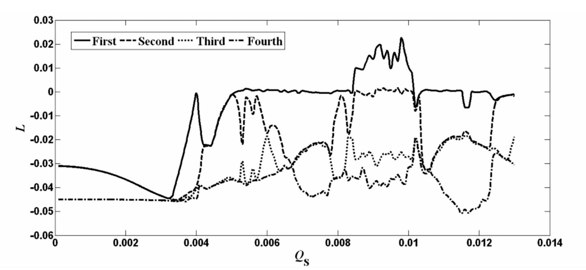

图 11 当 ω_{ex} 为 0.994，Q_s 变化时系统的前四个 Lyapunov 指数

Fig. 11 The first four Lyapunov exponents of the system when ω_{ex} is 0.994 and Q_s is variable

3 结论

本文对受单向外激励力的带柱形贮箱航天器

液-刚非线性耦合系统进行研究,不失一般性地用液体常重力下的特征模态近似微重力下的特征模态,用模态展开法分析微重力下柱形贮箱内的液体晃动,得到耦合系统的 Lagrange 函数,由 Lagrange 原理得到耦合系统的非线性动力学方程组并进行无量纲化和数值计算,发现系统的面内、外模态分别具有同类的稳态动力学行为;面内模态和面外模态在一定幅值不同频率或一定频率不同幅值的外激励下随外激励频率或幅值变化具有不同类型的稳态动力学行为,包括静止、周期运动、准周期运动和混沌运动.

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NONLINEAR COUPLED DYNAMICS OF LIQUID-FILLED CYLINDRICAL CONTAINER IN MICROGRAVITY *

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Abstract It is difficult to express analytically potential mode shapes and free surface mode shapes of sloshing liquid in cylindrical container in microgravity. To discover nonlinear behaviors, generally, potential mode shapes and free surface mode shapes in normal gravity are taken to approximate those in microgravity. The sloshing of liquid in cylindrical container in microgravity is analyzed using mode expanding method. Dimensionless form of nonlinear dynamic equations of the fluid-structure coupling system are derived by Lagrange principle and numerically solved. It is found that this coupled system presents resonance in some ranges of parameters of the external excitation. If the system does not resonate, in-plane modes and out-plane modes behave respectively the same kind stable motion and their types of stable motion change when parameters of the external excitation are different. These types of stable motion contain stillness, periodic motion, quasi-periodic motion and chaotic motion.

Key words microgravity, nonlinear sloshing, coupling dynamics, stable motion

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附录 A:

$\psi_i(r,\theta)=A_{mn}J_m(k_{mn}r)\begin{cases}\cos(m\theta)\\ \sin(m\theta)\end{cases}$,其中 $J_m'(k_{mn}a)=0$,a 为贮箱半径. 由模态归一化前提条件 $\iint_{S_B}\psi_i^2 dS_B=S_B$ (S_B 为贮箱的底面积),得,

$$A_{mn}=\frac{\sqrt{2}}{\{1+\delta_{0m})[1-(\frac{m}{k_{mn}a})^2]\}^{\frac{1}{2}}J_m(k_{mn}a)},$$

其中,$\delta_{0m}=\begin{cases}1, m=0\\ 0, m\neq 0.\end{cases}$

附录 B:

$$\mu_{xq_1}=\frac{m_{xq_1}}{m},\ \mu_{yq_2}=\frac{m_{yq_2}}{m},\ \lambda_{xq_1}=(\frac{\mu_{xq_1}}{\mu})^2\frac{V}{a_{11}^{(0)}},$$

$$\lambda_{yq_2}=(\frac{\mu_{yq_2}}{\mu})^2\frac{V}{a_{11}^{(0)}},\ \mu=\frac{m_F}{m},\ \mu_{ii}=\frac{a_{ii}^{(0)}}{a_{11}^{(0)}}(i=2..5),$$

$$v^2=\frac{\omega_s^2}{\omega^2}=\frac{g(\pi Bo+2S_{11}^{(2)})}{\omega^2 a_{11}^{(0)}Bo}a^2,$$

$$\zeta_x=\frac{c_x}{2m\omega},\zeta_y=\frac{c_y}{2m\omega},\zeta_{q_1}=\frac{c_{q_1}}{2\rho a_{11}^{(0)}\omega v},$$

$$v_i^2=v^2\frac{a_{11}^{(0)}\pi Bo+2S_{ii}^{(2)}}{a_{ii}^{(0)}\pi Bo+2S_{11}^{(2)}}(i=2..5),$$

$$\zeta_{q_i}=\frac{c_{q_i}}{2\rho a_i^{(0)}\omega v_i}(i=2..5),$$

$$\alpha_{mnr}=d\frac{a_{mnr}^{(1)}}{a_{11}^{(0)}},\ \alpha_{mnrs}=d^2\frac{a_{mnrs}^{(2)}}{a_{11}^{(0)}},$$

$$\beta_{113}=d\frac{(2S_{311}^{(3)}+4S_{113}^{(3)})}{\pi Bo+2S_{11}^{(2)}},\ \beta_{114}=d\frac{(2S_{411}^{(3)}+4S_{114}^{(3)})}{\pi Bo+2S_{11}^{(2)}},$$

$$\beta_{223}=d\frac{(2S_{322}^{(3)}+4S_{223}^{(3)})}{\pi Bo+2S_{11}^{(2)}},\ \beta_{224}=d\frac{(2S_{422}^{(3)}+4S_{224}^{(3)})}{\pi Bo+2S_{11}^{(2)}},$$

$$\beta_{125}=d\frac{(2S_{521}^{(3)}+4S_{125}^{(3)})}{\pi Bo+2S_{11}^{(2)}},\ \beta_{1111}=d^2\frac{4S_{1111}^{(4)}}{\pi Bo+2S_{11}^{(2)}},$$

$$\beta_{2222}=d^2\frac{4S_{2222}^{(4)}}{\pi Bo+2S_{11}^{(2)}},\ \beta_{2121}=d^2\frac{(4S_{2211}^{(4)}+8S_{2121}^{(4)})}{\pi Bo+2S_{11}^{(2)}}.$$