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旋转粘弹性夹层梁非线性自由振动特性研究*

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摘要　对旋转粘弹性夹层梁的非线性自由振动特性进行了分析. 基于 Kelvin-Voigt 粘弹性本构关系和大挠度理论，建立了旋转粘弹性夹层梁的非线性自由振动方程，并使用 Galerkin 法将偏微分形式振动方程化为常微分振动方程. 采用多重尺度法对非线性常微分振动方程进行求解，通过小参数同次幂系数相等获得微分方程组，并通过求解方程组及消除久期项来获得旋转粘弹性夹层梁非线性自由振动的一次近似解. 用数值方法讨论了粘弹性夹层厚度、转速和轮毂半径对梁固有频率的影响. 结果表明：固有频率随转速增大而增大，随夹层厚度增大而减小，随轮毂半径的增大而增大.

关键词　旋转粘弹性夹层梁，　Kelvin-Voigt，　非线性振动，　多重尺度法，　近似解，　固有频率

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引言

粘弹性夹层梁结构通常是由刚度较大的上下约束层和中间夹心层构成. Younesian[1] 等研究了粘弹性旋转梁的非线性振动；应祖光[2]、吴强[3] 等研究了粘弹性夹层梁的线性与非线性的振动特性和响应；李中华[4-5] 等研究了轴向运动粘弹性夹层板的振动分析以及轴向运动粘弹性夹层板的多模态耦合横向振动；吕海炜[6] 等对轴向变速运动粘弹性夹层梁的横向振动分析作了研究；Valverde[7] 等分析了旋转梁的附属结构的稳定性；Mahmood[8] 等对 Kelvin-Voigt 粘弹性梁的非线性自由振动作了研究；Nayfeh[9] 等研究了线性和非线性结构力学；Abolghasemi[10] 等研究了旋转粘弹性梁的吸引子；Nayfeh[11] 等对非线性波动作了研究. 目前，对旋转梁的研究尚为少见，而工程中常会遇到旋转梁类问题. 本文基于 Kelvin-Voigt 粘弹性本构关系和几何大变形理论，建立旋转粘弹性夹层梁自由振动方程，采用 Galerkin 法和多重尺度法求解非线性振动方程，给出了振动方程的一次近似解.

1　旋转粘弹性夹层梁控制方程

本文基于如下基本假设：

（1）不考虑转动惯量和剪切变形影响；（2）只考虑横向位移；（3）截面变形满足平面假设；（4）层与层之间没有相对滑移；（5）层与层之间横向位移连续.

1.1　旋转粘弹性夹层梁模型

图 1 为旋转粘弹性夹层梁模型，上下两层为对称约束层，厚度均为 $h/2$，中间为夹心层，厚度为 H，轮毂半径为 R 并以转速 Ω 绕转轴转动. 上下层弹性模量 E，密度 ρ，夹心层为粘弹性材料，弹性模量 E_0，密度 ρ_0，阻尼系数 η_0，等效线密度为：$\rho_{eq}=(\rho h+\rho_0 H)/(h+H)$.

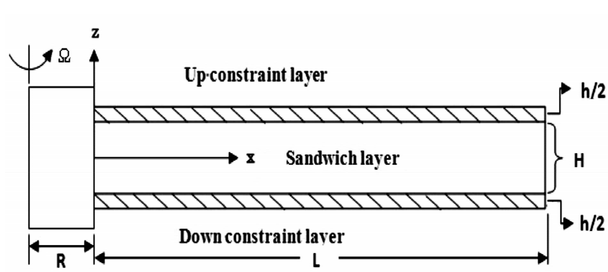

图 1　旋转粘弹性夹层梁模型
Fig. 1　Model of rotating sandwich beam

旋转粘弹性夹层梁的平衡方程：

$$M_{,xx}+(Nw_{,x})_{,x}-(\rho_0 H+\rho h)bw_{,tt}=0 \qquad (1)$$

式中，w 为 z 方向上的挠度，N 为轴力，b 为梁的宽

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度，M 为弯矩，$w_{,x}$，$M_{,xx}$ 分别表示 w 和 M 对 x 的一阶、二阶偏导.

1.2 旋转粘弹性夹层梁控制方程

使用几何大变形理论，夹层梁轴向应变为

$$\varepsilon_x=\frac{1}{2}w_{,x}^2-zw_{,xx} \tag{2}$$

式中，ε_x 为轴向应变，上、下约束层本构关系为

$$\sigma^c=E\varepsilon_x \tag{3}$$

夹心层为 Kelvin 粘弹性材料，其本构关系为

$$\sigma^j=E_0\varepsilon_x+\eta_0\dot{\varepsilon}_x \tag{4}$$

σ^c 和 σ^j 分别为约束层和夹层在 x 方向的正应力，其截面弯矩为

$$M=\int_{-\frac{H+h}{2}}^{-\frac{H}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\sigma^c zdydz+\int_{-\frac{H}{2}}^{\frac{H}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\sigma^j zdydz+\int_{\frac{H}{2}}^{\frac{H+h}{2}}\int_{\frac{b}{2}}^{\frac{b}{2}}\sigma^c zdydz=-\frac{Eb}{12}[(H+h)^3-H^3]w_{,xx}-\frac{E_0bH^3}{12}w_{,xx}-\frac{\eta_0bH^3}{12}w_{,xxt} \tag{5}$$

梁的总质量 $m=(\rho h+\rho_0H)bl$，则距固定端 x 处截面上的离心力为

$$P=\int_x^l\frac{m}{l}(R+x)\Omega^2dx=\frac{mR\Omega^2(l-x)}{l}+\frac{m\Omega^2}{2l}(l^2-x^2) \tag{6}$$

其中 Ω 为旋转角速度，轴力 N 可表示为

$$N=\int_{-\frac{H+h}{2}}^{-\frac{H}{2}}\sigma^c bdz+\int_{-\frac{H}{2}}^{\frac{H}{2}}\sigma^j bdz+\int_{\frac{H}{2}}^{\frac{H+h}{2}}\sigma^c bdz+P=\frac{(Ebh+E_0bH)}{2}w_{,x}^2+\eta_0bHw_{,x}w_{,xt}+\frac{mR\Omega^2}{l}(l-x)+\frac{m\Omega^2}{2l}(l^2-x^2) \tag{7}$$

将(5)、(7)式代入(1)式中，得

$$\frac{\eta_0H^3}{12}w_{,xxxxt}+\{\frac{E}{12}[(H+h)^3-H^3]+\frac{E_0H^3}{12}\}w_{,xxxx}-\frac{3(Eh+E_0H)}{2}w_{,x}^2w_{,xx}-2\eta_0Hw_{,x}w_{,xt}w_{,xx}-\eta_0Hw_{,x}^2w_{,xxt}-(\rho h+\rho_0H)R(l-x)\Omega^2w_{,xx}-\frac{(\rho h+\rho_0H)(l^2-x^2)\Omega^2}{2}w_{,xx}+[\rho h+\rho_0H)R\Omega^2+(\rho h+\rho_0H)x\Omega^2]w_{,x}+(\rho h+\rho_0H)w_{,tt}=0 \tag{8}$$

取 $w(x,t)=\varphi(x)q(t)$，其中 $\varphi(x)$ 为满足边界条件的模态函数，$q(t)$ 为广义模态坐标. 代入(8)式，两端同乘 $\varphi(x)$ 后在 $[0,l]$ 对 x 积分，整理得

$$\rho_{eq}(h+H)a_{00}\ddot{q}(t)+\frac{\eta_0H^3}{12}a_{04}\dot{q}(t)-3\eta_0Hd_{012}q^2(t)\dot{q}(t)-\rho_{eq}(h+H)R\Omega^2la_{02}q(t)+\{\frac{E[(H+h)^3-H^3]}{12}+\frac{E_0H^3}{12}\}a_{04}q(t)+\rho_{eq}(h+H)R\Omega^2b_{02}q(t)-\frac{\rho_{eq}(h+H)l^2\Omega^2}{2}a_{02}q(t)+\frac{\rho_{eq}(h+H)\Omega^2}{2}c_{02}q(t)+\rho_{eq}(h+H)R\Omega^2a_{01}q(t)+\rho_{eq}(h+H)\Omega^2b_{01}q(t)-\frac{3(Eh+E_0H)}{2}d_{012}q^3(t)=0 \tag{9}$$

其中：

$$a_{00}=\int_0^l\varphi^2dx,\ a_{01}=\int_0^l\varphi'(x)\varphi(x)dx,$$

$$a_{02}=\int_0^l\varphi''(x)\varphi(x)dx,$$

$$a_{04}=\int_0^l\varphi''''(x)\varphi(x)dx,$$

$$b_{01}=\int_0^lx\varphi'(x)\varphi(x)dx,\ b_{02}=\int_0^lx\varphi''(x)\varphi(x)dx,$$

$$c_{02}=\int_0^lx^2\varphi''(x)\varphi(x)dx,$$

$$d_{012}=\int_0^l\varphi(x)[\varphi'(x)]^2\varphi''(x)dx.$$

(9)式可化为

$$M\ddot{q}(t)+C\dot{q}(t)+Dq^2(t)\dot{q}(t)+Kq(t)+Gq^3(t)=0 \tag{10}$$

其中，

$$M=\rho_{eq}(h+H)a_{00},\ C=\frac{\eta_0H^3}{12}a_{04},\ D=-3\eta_0Hd_{012},$$

$$K=\{\frac{E[(H+h)^3-H^3]}{12}+\frac{E_0H^3}{12}\}a_{04}+\rho_{eq}(h+H)\Omega^2(-Rla_{02}+Rb_{02}-\frac{1}{2}l^2a_{02}+\frac{1}{2}c_{02}+Ra_{01}+b_{01}),$$

$$G=-\frac{3}{2}(Eh+E_0H)d_{012}$$

(10)式可进一步整理为：

$$\ddot{q}(t)+2\zeta\omega_0\dot{q}(t)+\omega_0^2q(t)+\alpha q^2(t)\dot{q}(t)+\beta q^3(t)=0 \tag{11}$$

其中，

$$\omega_0^2=\Omega^2[\frac{b_{01}}{a_{00}}+\frac{1}{2}\frac{c_{02}}{a_{00}}-\frac{1}{2}\frac{a_{02}}{a_{00}}l^2+$$

$$R(\frac{a_{01}}{a_{00}}+\frac{b_{02}}{a_{00}}-\frac{a_{02}}{a_{00}}l)]$$

$$\alpha=\frac{-3\eta_0 H}{(\rho h+\rho_0 H)}\frac{d_{012}}{a_{00}}$$

$$\beta=\frac{-3(Eh+E_0 H)}{(\rho h+\rho_0 H)}\frac{d_{012}}{a_{00}}$$

令 $q(t)=\varepsilon\hat{q}(t)$,$2\zeta\omega_0=\varepsilon\bar{\zeta}$,$\varepsilon$ 为小参数. 将 $q(t)$ 和 $2\zeta\omega_0$ 代入(11)式中,得

$$\ddot{\hat{q}}(t)+\omega_0^2\hat{q}(t)=-\varepsilon\bar{\zeta}\dot{\hat{q}}(t)-\varepsilon^2\alpha\hat{q}^2(t)\dot{\hat{q}}(t)-\varepsilon^2\beta\hat{q}^3(t)\quad(12)$$

(12)式即为旋转粘弹性夹层梁的自由振动方程.

2　旋转粘弹性夹层梁振动方程的求解

2.1　求解方法及过程

采用多重尺度法,令

$$\hat{q}(T_0,T_1,T_2)=\hat{q}_0(T_0,T_1,T_2)+\varepsilon\hat{q}_1(T_0,T_1,T_2)+\varepsilon^2\hat{q}_2(T_0,T_1,T_2)\quad(13)$$

其中,$T_n=\varepsilon^n\tau$,将(13)式代入到(12)式中,比较 ε 的同次幂系数

$$D_0^2\hat{q}_0+\omega_0^2\hat{q}_0=0\quad(14)$$

$$D_0^2\hat{q}_1+\omega_0^2\hat{q}_1=-2D_0D_1\hat{q}_0-\bar{\zeta}D_0\hat{q}_0\quad(15)$$

$$D_0^2\hat{q}_2+\omega_0^2\hat{q}_2=-2D_0D_2\hat{q}_0-D_1^2\hat{q}_0-2D_0D_1\hat{q}_1-[\bar{\zeta}(D_1\hat{q}_0+D_0\hat{q}_1)+\alpha\hat{q}_0{}^2D_0\hat{q}_0+\beta\hat{q}_0{}^3]\quad(16)$$

由(14)式得

$$\hat{q}_0=H(T_1,T_2)e^{iw_0T_0}+\bar{H}(T_1,T_2)e^{-iw_0T_0}\quad(17)$$

将(17)代入(15)式,得

$$D_0^2\hat{q}_1+\omega_0^2\hat{q}_1=-2i\omega_0D_1He^{i\omega_0T_0}-\bar{\zeta}i\omega_0He^{i\omega_0T_0}+cc\quad(18)$$

(18)式中,cc 表示共轭项,消除(18)式的久期项,可得

$$D_1H=-\frac{1}{2}\bar{\zeta}H\quad(19)$$

这样,由(18)式可以解得

$$\hat{q}_1=e^{i\omega_0T_0}+e^{-i\omega_0T_0}\quad(20)$$

将(17)、(20)式代入(16)式,得

$$D_0^2\hat{q}_2+\omega_0^2\hat{q}_2=-2i\omega_0e^{i\omega_0T_0}-e^{i\omega_0T_0}D_1^2H-\bar{\zeta}e^{i\omega_0T_0}D_1H-i\omega_0\bar{\zeta}e^{i\omega_0T_0}-\alpha H^3i\omega_0e^{3i\omega_0T_0}-2\alpha H^2\bar{H}i\omega_0e^{i\omega_0T_0}+\alpha H^2\bar{H}i\omega_0e^{i\omega_0T_0}-\beta H^3e^{3i\omega_0T_0}-3\beta H^2\bar{H}e^{i\omega_0T_0}+cc\quad(21)$$

消除(21)式中的久期项,得

$$-2i\omega_0D_2H-D_1^2H-\bar{\zeta}D_1H-i\omega_0\bar{\zeta}-2\alpha H^2\bar{H}i\omega_0+\alpha H^2\bar{H}i\omega_0-3\beta H^2\bar{H}=0\quad(22)$$

经整理后,(22)式化为

$$D_2H-\frac{i}{2\omega_0}D_1^2H-\frac{i}{2\omega_0}\bar{\zeta}D_1H+\frac{1}{2}\bar{\zeta}+(\frac{1}{2}\alpha-\frac{3i}{2\omega_0}\beta)H^2\bar{H}=0\quad(23)$$

将(19)式代入(23)式中,经整理后得

$$D_2H=-\frac{i}{8\omega_0}\bar{\zeta}^2H-(\frac{1}{2}\alpha-\frac{3i}{2\omega_0}\beta)H^2\bar{H}-\frac{1}{2}\bar{\zeta}\quad(24)$$

H 对时间求导

$$\frac{dH}{dt}=D_0H+\varepsilon D_1H+\varepsilon^2D_2H\quad(25)$$

因为 $H=H(T_1,T_2)$,所以 $D_0H=0$,即

$$\frac{dH}{dt}=\varepsilon D_1H+\varepsilon^2D_2H\quad(26)$$

将(19)、(24)式代入(26)式中,得

$$\dot{H}=\varepsilon^2\{-\frac{i}{8\omega_0}\bar{\zeta}^2H-(\frac{1}{2}\alpha-\frac{3i}{\omega_0}\beta)H^2\bar{H}\}-\frac{1}{2}\varepsilon^2\bar{\zeta}-\frac{1}{2}\varepsilon\bar{\zeta}H\quad(27)$$

为便于解出 H,将它写成复数形式,即设

$$H=\frac{1}{2}ae^{i\gamma}\quad(28)$$

其中 a 和 γ 是时间的实函数,则 H 对时间求导得

$$\dot{H}=\frac{1}{2}\dot{a}\cos\gamma-\frac{1}{2}a\dot{\gamma}\sin\gamma+i(\frac{1}{2}\dot{a}\sin\gamma+\frac{1}{2}a\dot{\gamma}\cos\gamma)\quad(29)$$

将(28)式代入(27)式中,经整理得

$$\dot{H}=-\frac{a\varepsilon}{4}\bar{\zeta}\cos\gamma+\frac{\varepsilon^2a}{16\omega_0}\bar{\zeta}^2\sin\gamma-\frac{\varepsilon^2a^4}{32}\alpha\cos\gamma-\frac{3\varepsilon^2a^4}{16\omega_0}\beta\sin\gamma-\frac{\varepsilon^2}{2}\bar{\zeta}+i(-\frac{a\varepsilon}{4}\bar{\zeta}\sin\gamma-\frac{\varepsilon^2a}{16\omega_0}\bar{\zeta}^2\cos\gamma-\frac{\varepsilon^2a^4}{32}\alpha\sin\gamma+\frac{3\varepsilon^2a^4}{16\omega_0}\beta\cos\gamma)\quad(30)$$

(29)式和(30)式实部、虚部对应相等,得

$$\begin{cases}\frac{1}{2}\dot{a}\cos\gamma-\frac{1}{2}a\dot{\gamma}\sin\gamma=-\frac{a\varepsilon}{4}\bar{\zeta}\cos\gamma+\frac{\varepsilon^2a}{16\omega_0}\bar{\zeta}^2\sin\gamma-\frac{\varepsilon^2a^4}{32}\alpha\cos\gamma-\frac{3\varepsilon^2a^4}{16\omega_0}\beta\sin\gamma-\frac{\varepsilon^2}{2}\bar{\zeta} & (31)\\ \frac{1}{2}\dot{a}\sin\gamma+\frac{1}{2}a\dot{\gamma}\cos\gamma=-\frac{a\varepsilon}{4}\bar{\zeta}\sin\gamma-\frac{\varepsilon^2a}{16\omega_0}\bar{\zeta}^2\cos\gamma-\frac{\varepsilon^2a^4}{32}\alpha\sin\gamma+\frac{3\varepsilon^2a^4}{16\omega_0}\beta\cos\gamma & (32)\end{cases}$$

由(31)、(32)式得

$$\frac{1}{2}\dot{a}=-\frac{a\varepsilon}{4}\bar{\zeta}-\frac{\varepsilon^2a^4}{32}\alpha-\frac{\varepsilon^2}{2}\bar{\zeta}\cos\gamma \quad (33)$$

$$\frac{1}{2}a\dot{\gamma}=-\frac{a\varepsilon^2}{16\omega_0}\bar{\zeta}^2+\frac{3\varepsilon^2a^4}{16\omega_0}\beta+\frac{\varepsilon^2}{2}\bar{\zeta}\sin\gamma \quad (34)$$

将 $\varepsilon\bar{\zeta}$ 还原成 $2\zeta\omega_0$，(33)、(34)式变为

$$\begin{cases}\dot{a}=-\frac{a}{2}(2\zeta\omega_0)-\frac{\varepsilon^2a^4}{16}\alpha-2\varepsilon\zeta\omega_0\cos\gamma & (35)\\ a\dot{\gamma}=-\frac{a}{8\omega_0}(2\zeta\omega_0)^2+\frac{3\varepsilon^2a^4}{8\omega_0}\beta+\varepsilon(2\zeta\omega_0)\sin\gamma & (36)\end{cases}$$

(35)、(36)式的稳态解对应系统的不动点，由 $\dot{a}=0,\dot{\gamma}=0$ 可得

$$\begin{cases}-\frac{a}{2}(2\zeta\omega_0)-\frac{\varepsilon^2a^4}{16}\alpha-\varepsilon(2\zeta\omega_0)\cos\gamma=0 & (37)\\ -\frac{a}{8\omega_0}(2\zeta\omega_0)^2+\frac{3\varepsilon^2a^4}{8\omega_0}\beta+\varepsilon(2\zeta\omega_0)\sin\gamma=0 & (38)\end{cases}$$

$$\left[\frac{\varepsilon^2}{256}\left(\frac{\alpha}{2\zeta\omega_0}\right)^2-\frac{9\varepsilon^2}{64\omega_0^2}\left(\frac{\beta}{2\zeta\omega_0}\right)^2\right]a^8+\left(\frac{\alpha}{32\zeta\omega_0}-\frac{\beta}{32\omega_0^2}\right)a^5+\left[\frac{1}{4\varepsilon^2}+\frac{1}{64\omega_0^2\varepsilon^2}(2\zeta\omega_0)^2\right]a^2=1 \quad (39)$$

2.2 一次近似解

(39)式中，α、β、ζ，ω_0 均为常数. 这样给定一个满足 $\cos\gamma$ 和 $\sin\gamma$ 均在[-1,1]这个区间内的 ε 值，就可以通过(40)式求出 a，结合(37)式，可以求出 γ. 由(17)式，可得系统的一次近似解为

$$\hat{q}=\frac{1}{2}ae^{i\beta}e^{i\omega_0T_0}+\frac{1}{2}ae^{-i\beta}e^{-i\omega_0T_0}=a\cos(\omega_0t+\beta) \quad (40)$$

由(40)式可见，a 为振幅，γ 为相位差.

3 数值仿真与讨论

计算所用材料参数如表 1，几何参数如表 2.

表 1 旋转粘弹性夹层梁材料参数

Table 1 Material parameters of rotating viscoelastic beam

	E/MPa	η	ρ/(kg·m^{-3})
Constraint layer	7.2×10^4	—	2700
Sandwich layer	10	-0.1	1300

表 2 粘弹性夹层梁几何参数

Table 2 Geometry parameters of viscoelastic sandwich beam

l/m	b/m	$(H+h)$/m
1	0.002	0.005

3.1 转速的影响

首先讨论转速对结构一阶固有频率和损耗因子的影响. $l=1$ m，$b=0.002$ m，$H+h=0.005$ m，$R=0.05$ m，转速从 0 到 50 rad/s 间变化. 三种夹层厚度比下一阶固有频率随转速变化如图 2.

可见，在夹心层厚度一定时，旋转粘弹性夹层梁的一阶固有频率随转速增大而增大；在转速相同时，夹心层越厚，一阶固有频率越小.

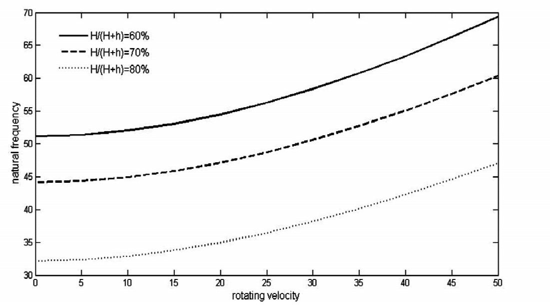

图 2 一阶固有频率随转速变化图

Fig. 2 First natural frequency versus rotating velocity

3.2 轮毂半径的影响

取 $l=1$ m，$b=0.002$ m，$H+h=0.005$ m，$\Omega=50$ rad/s，计算 $H/(H+h)=0.6,0.7,0.8$ 时，轮毂半径 R 的变化对系统一阶固有频率和损耗因子的影响，结果如图 3.

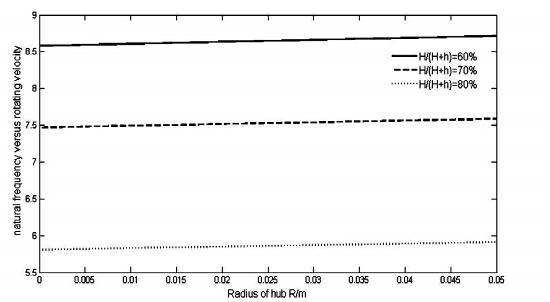

图 3 一阶固有频率与转速比随 R 的变化

Fig. 3 The ratio of first natural frequency to rotating velocity versus the variation of R

可见，R 的增大会使旋转粘弹性夹层梁的一阶固有频率略有增加，但不明显.

4 结论

本文对旋转粘弹性夹层梁的非线性自由振动特性作了研究，通过多尺度法求解非线性振动方程，并得到一次近似解. 此外还讨论了固有频率随转速及轮毂半径的变化，结论如下：

1）固有频率随转速增大而增大，随夹层厚度增大而减小；

2）固有频率随轮毂半径 R 的增大而增大.

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VIBRATION ANALYSIS OF ROTATING VISCOELASTIC SANDWICH BEAM*

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Abstract The nonlinear free vibration analysis of rotating viscoelastic sandwich beam is presented in this article. The control equation of the rotating viscoelastic sandwich beam was established based on Kelvin-Voigt constitutive equation and large deflection theory. Partial differential equation of vibration was transformed into an ordinary differential one using Galerkin method. The ordinary differential equation of nonlinear vibration was solved by multiple scale method. Systems of equations were obtained by comparing coefficient of power of the micro parameter. First approximate solution of the nonlinear free vibration of rotating viscoelastic sandwich beam could be acquired by solving the systems of equations as well as eliminating the secular terms. Numerical simulation was used to discuss the effect of thickness of the sandwich layer, variation of rotating velocity and radius of the hub on nature frequency. The results indicated that natural frequency of the rotating viscoelastic sandwich beam increased with the increase of rotating velocity and radius of the hub while decreased with the increase of thickness of the sandwich layer.

Key words rotating viscoelastic sandwich beam, Kelvin-Voigt, nonlinear vibration, multiple scale method, approximate solution, natural frequency

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