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功能梯度材料椭圆板的非线性热振动及屈曲*

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摘要 采用弹性理论建立了功能梯度材料板的静力平衡方程,利用静力平衡方程确定了功能梯度材料板的中性面位置,在此基础上推导出了功能梯度材料板在均匀温度场中的非线性振动及屈曲微分方程组,求得了功能梯度材料椭圆板的非线性振动及屈曲的近似解,讨论分析了中性面位置、梯度指数、温度等因素对功能梯度材料椭圆板非线性振动及屈曲的影响. 把该方法计算结果与有限元计算结果进行了比较,验证了该方法的计算结果是可靠的. 算例分析表明,中性面位置对均匀温度场中功能梯度材料椭圆板的非线性振动及屈曲有一定影响.

关键词 功能梯度, 材料, 椭圆板, 非线性, 振动, 屈曲

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引言

功能梯度材料是基于一种全新的材料设计概念合成的新型复合材料[1-6],日本科学家于 1984 年提出了功能梯度材料的概念,即根据具体的要求,选择使用两种不同性能材料,通过连续平滑地改变两种材料的组织和结构,使其结合部位的界面消失,从而得到功能相应于组织变化而变化的均质材料,最终减小或消除结合部位的性能不匹配因素. 对于陶瓷和金属混合而成的功能梯度材料,由于陶瓷具有低传热系数而用于抵抗高温,金属则由于其良好的延展性而防止了短时间内温度剧变产生的应力而导致断裂破坏,因此被广泛地应用在航空航天等实际工程中. 所以,功能梯度材料板的力学性能引起了工程设计人员的高度关注[7-11]. 但是,有关研究功能梯度材料板壳的文献都没有确定功能梯度材料板壳中性面的真实位置,而是假设功能梯度材料板壳相对于中性面具有几何和弹性对称,然后建立功能梯度材料板壳的振动及屈曲的微分方程,然而一般功能梯度材料板壳中性面与板壳中面是不重合的,这种研究方法显然是具有局限性的. 基于上述原因,本文首先确定了功能梯度材料椭圆板的中性面位置,建立了功能梯度材料椭圆板在均匀温度场中的非线性振动及屈曲微分方程组,讨论分析了中性面位置、温度、梯度指数等因素对椭圆板非线性振动及屈曲的影响.

1 振动及屈曲微分方程

假设均匀温度场中功能梯度材料板下侧为金属材料,上侧为陶瓷材料,中间为两种材料组成的混合物,由于金属材料与陶瓷材料的泊松比相近,可令它们的泊松比均为 μ. 设金属材料的弹性模量、热膨胀系数、密度分别为 E_m、α_m、ρ_m,陶瓷材料的弹性模量、热膨胀系数、密度分别为 E_c、α_c、ρ_c,则板内任一点的弹性模量、热膨胀系数、密度分别为

$$E(z)=E_1V_m+E_c,\ \alpha(z)=\alpha_1V_m+\alpha_c,\ \rho(z)=\rho_1V_m+\rho_c \tag{1}$$

式中,$E_1=E_m-E_c$,$\alpha_1=\alpha_m-\alpha_c$,$\rho_1=\rho_m-\rho_c$,$V_m$ 为金属材料组分的体积比例系数.

可设功能梯度材料板中金属材料组分的体积比例系数为板厚方向坐标 z 的幂函数,即

$$V_m=\left(\frac{z-z_0}{h}+\frac{1}{2}\right)^k \tag{2}$$

其中,坐标原点在板中性面,k 为梯度指数,z_0 为中性面与板中面之间的距离.

根据弹性理论,功能梯度材料板在均匀温度场

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中的物理方程为

$$\begin{cases}\sigma_x = -\dfrac{E(z)z}{1-\mu^2}(\dfrac{\partial^2 w}{\partial x^2}+\mu\dfrac{\partial^2 w}{\partial y^2}) - \dfrac{E(z)\alpha(z)}{1-\mu}\Delta T \\ \sigma_y = -\dfrac{E(z)z}{1-\mu^2}(\dfrac{\partial^2 w}{\partial y^2}+\mu\dfrac{\partial^2 w}{\partial x^2}) - \dfrac{E(z)\alpha(z)}{1-\mu}\Delta T \\ \tau_{xy} = -\dfrac{E(z)z}{1+\mu}\dfrac{\partial^2 w}{\partial x\partial y}\end{cases} \tag{3}$$

式中，ΔT 为温度增量，w 为板的平面外位移.

当 $\Delta T=0$ 时，功能梯度材料板纯弯曲的横截面内力应满足以下关系

$$\int_A \sigma_x \mathrm{d}A = 0,\ \int_A \sigma_y \mathrm{d}A = 0 \tag{4}$$

把式(3)代入式(4)中可以得到

$$\int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}} E(z)z\mathrm{d}z = 0 \tag{5}$$

把式(1)、式(2)代入式(5)中可以求得

$$z_0 = \frac{(E_c - E_m)kh}{2(k+2)(E_m + kE_c)} \tag{6}$$

利用式(3)可以得到功能梯度材料板弯矩、扭矩表达式为

$$\begin{cases} M_x = -D(\dfrac{\partial^2 w}{\partial x^2}+\mu\dfrac{\partial^2 w}{\partial y^2}) - M_T \\ M_y = -D(\dfrac{\partial^2 w}{\partial y^2}+\mu\dfrac{\partial^2 w}{\partial x^2}) - M_T \\ M_{xy} = -(1-\mu)D\dfrac{\partial^2 w}{\partial x\partial y}\end{cases} \tag{7}$$

式中，

$$D = \frac{E_1}{1-\mu^2}[\frac{h}{k+1}(z_0+\frac{h}{2})^2 - \frac{2h^2}{(k+1)(k+2)}(z_0+\frac{h}{2}) + \frac{2h^3}{(k+1)(k+2)(k+3)}] + \frac{E_c}{3(1-\mu^2)}[(z_0+\frac{h}{2})^3-(z_0-\frac{h}{2})^3]$$

$$M_T = \frac{E_c\alpha_c\Delta T}{2(1-\mu)}[(z_0+\frac{h}{2})^2-(z_0-\frac{h}{2})^2] + \frac{E_1\alpha_1 h\Delta T}{(2k+1)(1-\mu)}(z_0+\frac{kh}{2k+2}) + \frac{(E_c\alpha_1+E_1\alpha_c)h\Delta T}{(k+1)(1-\mu)}(z_0+\frac{kh}{2k+4})$$

由弹性理论可知，功能梯度材料板在外扰力作用下的内力满足以下关系式

$$\begin{cases}\dfrac{\partial M_x}{\partial x}+\dfrac{\partial M_{xy}}{\partial y} = Q_x \\ \dfrac{\partial M_{xy}}{\partial x}+\dfrac{\partial M_y}{\partial y} = Q_y\end{cases} \tag{8}$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+N_x\frac{\partial^2\varphi}{\partial x^2}+N_y\frac{\partial^2\varphi}{\partial y^2}+2N_{xy}\frac{\partial^2\varphi}{\partial x\partial y}-\rho h\frac{\partial^2 w}{\partial t^2}+q(x,y,t)=0 \tag{9}$$

式中，$\rho h = \dfrac{(\rho_m + k\rho_c)h}{k+1}$，$N_x$、$N_y$、$N_{xy}$ 为中面拉力及剪力，$q(x,y,t)$ 为外扰力.

由弹性理论可得板中面内点的应变表达式为

$$\begin{cases}\varepsilon_x = \dfrac{\partial u}{\partial x}+\dfrac{1}{2}(\dfrac{\partial w}{\partial x})^2 \\ \varepsilon_y = \dfrac{\partial v}{\partial y}+\dfrac{1}{2}(\dfrac{\partial w}{\partial y})^2 \\ \gamma_{xy} = \dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y}\end{cases} \tag{10}$$

式中，u（或 v）为中面内点沿 x（或 y）方向的位移.

由式(10)可得相容方程为：

$$\frac{\partial^2\varepsilon_x}{\partial y^2}+\frac{\partial^2\varepsilon_y}{\partial x^2}-\frac{\partial^2\gamma_{xy}}{\partial x\partial y} = (\frac{\partial^2 w}{\partial x\partial y})^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{11}$$

把板中面上应力可用板中面内点的应变表示为

$$\begin{cases}\sigma_x^0 = \dfrac{E(z)}{1-\mu^2}[\varepsilon_y+\mu\varepsilon_y-(1+\mu)\alpha(z)\Delta T] \\ \sigma_y^0 = \dfrac{E(z)}{1-\mu^2}[\varepsilon_y+\mu\varepsilon_x-(1+\mu)\alpha(z)\Delta T] \\ \tau_{xy}^0 = \dfrac{E(z)}{2(1+\mu)}\gamma_{xy}\end{cases} \tag{12}$$

由式(12)可得功能梯度材料板的中面拉力为

$$\begin{cases} N_x = \dfrac{Eh}{1-\mu^2}(\varepsilon_x+\mu\varepsilon_y) - N_T \\ N_y = \dfrac{Eh}{1-\mu^2}(\varepsilon_y+\mu\varepsilon_x) - N_T \\ N_{xy} = \dfrac{Eh}{2(1+\mu)}\gamma_{xy}\end{cases} \tag{13}$$

式中，$E = E_c + \dfrac{E_1}{k+1}$，$N_T = \dfrac{E_c\alpha_c h\Delta T}{1-\mu} + \dfrac{E_1\alpha_1 h\Delta T}{(2k+1)(1-\mu)} + \dfrac{(E_1\alpha_c+E_c\alpha_1)h\Delta T}{(k+1)(1-\mu)}$.

由式(13)还可以得到板中面点应变的另一种表达式为

$$\begin{cases}\varepsilon_x = \dfrac{1}{Eh}(N_x-\mu N_y)+\dfrac{(1-\mu)N_T}{Eh} \\ \varepsilon_y = \dfrac{1}{Eh}(N_y-\mu N_x)+\dfrac{(1-\mu)N_T}{Eh} \\ \gamma_{xy} = \dfrac{2(1+\mu)}{Eh}N_{xy}\end{cases} \tag{14}$$

再令

$$N_x=\frac{\partial^2\varphi}{\partial y^2},N_y=\frac{\partial^2\varphi}{\partial x^2},N_{xy}=-\frac{\partial^2\varphi}{\partial x\partial y} \tag{15}$$

把式(7)、式(8)、式(15)代入式(9)中,把式(14)、式(15)代入式(11)中,可以得到功能梯度材料板的非线性热振动及屈曲微分方程组为

$$\begin{cases} D\nabla^4 w+\nabla^2 M_T+\rho h\dfrac{\partial^2 w}{\partial t^2}= \\ (\dfrac{\partial^2\varphi}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2}+\dfrac{\partial^2\varphi}{\partial y^2}\dfrac{\partial^2 w}{\partial x^2}-2\dfrac{\partial^2\varphi}{\partial x\partial y}\dfrac{\partial^2 w}{\partial x\partial y})+q(x,y,t) \\ \nabla^4\varphi+(1-\mu)\nabla^2 N_T=Eh[(\dfrac{\partial^2 w}{\partial x\partial y})^2-\dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2}] \end{cases} \tag{16}$$

2　非线性热振动及屈曲近似解

对于图 1 所示周边固支功能梯度材料椭圆板,在椭圆板中心中性面建立坐标原点,则沿其周边的边界条件为

$$w=\frac{\partial w}{\partial x}=\frac{\partial w}{\partial y}=0 \tag{17}$$

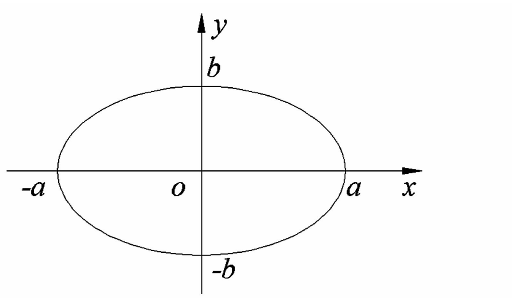

图 1　周边固支椭圆板

Fig. 1　the fixed elliptical plate

设功能梯度材料椭圆板的横振位移为

$$w(x,y)=T(t)(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1)^2 \tag{18}$$

把式(18)代入式(16)第二分式中可得

$$\nabla^4\varphi=-EhT^2(\frac{48x^4}{a^6b^2}+\frac{48y^4}{a^2b^6}-\frac{64x^2}{a^4b^2}-\frac{64y^2}{a^2b^4}+\frac{96x^2y^2}{a^4b^4}+\frac{16}{a^2b^2}) \tag{19}$$

对式(19)进行积分后可以得到下式

$$\varphi(x,y)=\frac{EhT^2}{210a^4b^4}(x^8+y^8)-\frac{EhT^2}{35}(\frac{x^8}{a^6b^2}+\frac{y^8}{a^2b^6})+\frac{8EhT^2}{45}(\frac{x^6}{a^4b^2}+\frac{y^6}{a^2b^4})-\frac{EhT^2}{3a^4b^4}x^4y^4-\frac{EhT^2}{3a^4b^4}(x^4+y^4)+\frac{1}{2}P_xy^2+\frac{1}{2}P_yx^2 \tag{20}$$

周边固支功能梯度材料椭圆板的伸长为

$$\begin{cases} \Delta x=\dfrac{1}{b}\displaystyle\int_{-a}^{a}\int_{-b\sqrt{1-\frac{x^2}{a^2}}}^{b\sqrt{1-\frac{x^2}{a^2}}}\Big[\dfrac{1}{Eh}(\dfrac{\partial^2\varphi}{\partial y^2}-\mu\dfrac{\partial^2\varphi}{\partial x^2})-\dfrac{1}{2}(\dfrac{\partial w}{\partial x})^2+\dfrac{1-\mu}{Eh}N_T\Big]dxdy=0 \\ \Delta y=\dfrac{1}{a}\displaystyle\int_{-a}^{a}\int_{-b\sqrt{1-\frac{x^2}{a^2}}}^{b\sqrt{1-\frac{x^2}{a^2}}}\Big[\dfrac{1}{Eh}(\dfrac{\partial^2\varphi}{\partial x^2}-\mu\dfrac{\partial^2 w}{\partial y^2})-\dfrac{1}{2}(\dfrac{\partial w}{\partial y})^2+\dfrac{1-\mu}{Eh}N_T\Big]dxdy=0 \end{cases} \tag{21}$$

利用式(17)、式(20)、式(21)可以求得

$$\begin{cases} P_x=\dfrac{Eh(3a^4-26a^2b^2-b^4+48)T^2}{48a^4b^2}+\dfrac{Eh(\mu a^2+b^2)T^2}{3a^2b^2(1-\mu^2)}-N_T \\ P_y=\dfrac{Eh(3b^4-26a^2b^2-a^4+48)T^2}{48a^2b^4}+\dfrac{Eh(a^2+\mu b^2)T^2}{3a^2b^2(1-\mu^2)}-N_T \end{cases} \tag{22}$$

利用伽辽金原理可以把式(16)第一分式化为如下形式

$$\begin{cases} \displaystyle\int_{-a}^{a}\int_{-b\sqrt{1-\frac{x^2}{a^2}}}^{b\sqrt{1-\frac{x^2}{a^2}}}\Big[D\nabla^4 w+\nabla^2 M_T+\rho h\dfrac{\partial^2 w}{\partial t^2}- \\ \dfrac{\partial^2\varphi}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2}-\dfrac{\partial^2\varphi}{\partial y^2}\dfrac{\partial^2 w}{\partial x^2}+2\dfrac{\partial^2\varphi}{\partial x\partial y}\dfrac{\partial^2 w}{\partial x\partial y}- \\ q(x,y,t)\Big](\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-1)^2dxdy=0 \end{cases} \tag{23}$$

把式(18)、式(20)、式(22)代入式(16)中且令 $q(x,y,t)=0$,可得功能梯度材料椭圆板的非线性固有振动微分方程为

$$\frac{d^2T}{dt^2}+\omega_0{}^2T+\beta T^3=0 \tag{24}$$

式中,

$$\omega_0{}^2=\frac{D}{\rho h}(\frac{40}{a^4}+\frac{40}{b^4}+\frac{80}{3a^2b^2})-\frac{10}{3\rho h}(\frac{1}{a^2}+\frac{1}{b^2})N_T$$

$$\beta=\frac{E(117a^4-710a^2b^2-31b^4+1008)}{504\rho a^6b^2}+\frac{E(117b^4-710a^2b^2-31a^4+1008)}{504\rho a^2b^6}-\frac{2E}{\rho a^2b^6}+\frac{10E(a^2+\mu b^2)}{9\rho(1-\mu^2)a^2b^4}+\frac{10E(\mu a^2+b^2)}{9\rho(1-\mu^2)a^4b^2}$$

在式(24)中引入“人工摄动参数”且令 $\tau=\omega t$ 可以得到

$$\omega^2\frac{d^2T}{d\tau^2}+{\omega_0}^2T+\varepsilon\beta T^3=0 \tag{25}$$

令式(25)的初始条件为

$$\tau=0,\ T(0)=B,\ \frac{dT(0)}{d\tau}=0 \tag{26}$$

令

$$\begin{cases}T(\tau)=T_0(\tau)+\varepsilon T_1(\tau)+\varepsilon^2T_2(\tau)+\Lambda\\ \omega=\omega_0+\varepsilon\omega_1+\varepsilon^2\omega_2+\Lambda\end{cases} \tag{27}$$

把式(27)代入式(25)中可以得到下式

$$\begin{cases}\dfrac{d^2T_0}{d\tau^2}+T_0=0\\ \dfrac{d^2T_1}{d\tau^2}+T_1=\dfrac{2\omega_1}{\omega_0}\dfrac{d^2T_0}{d\tau^2}-\dfrac{\beta}{{\omega_0}^2}{T_0}^3\\ \dfrac{d^2T_2}{d\tau^2}+T_2=\dfrac{2\omega_1}{\omega_0}\dfrac{d^2T_1}{d\tau^2}-\dfrac{3\beta}{{\omega_0}^2}{T_0}^2T_1-\\ \quad\dfrac{({\omega_1}^2+2\omega_0\omega_2)}{{\omega_0}^2}\dfrac{d^2T_0}{d\tau^2}\end{cases} \tag{28}$$

把式(25)的解表示为系数待定的傅立叶级数

$$\begin{aligned}T(\tau)&=T_0(\tau)+\varepsilon T_1(\tau)+\varepsilon^2T_2(\tau)+\Lambda\\ &=B\cos\tau+\sum_{j=1}^{\infty}\varepsilon^j(c_j+b_j\cos\tau+\\ &\quad\sum_{i=2}^{\infty}a_{ij}\cos i\tau)+\Lambda\end{aligned} \tag{29}$$

为了使式(29)满足初始条件式(26)可补充条件

$$c_j+b_j+\sum_{i=2}^{\infty}a_{ij}=0 \tag{30}$$

把式(29)代入式(28)中利用系数待定法及式(30)可以求得

$$\begin{aligned}\omega&=\lim_{\varepsilon\to1}(\omega_0+\varepsilon\omega_1+\varepsilon^2\omega_2)\\ &=\omega_0(1+\frac{3\beta B^2}{8{\omega_0}^2}-\frac{15\beta^2B^4}{256{\omega_0}^4})\end{aligned} \tag{31}$$

$$\begin{aligned}T(t)&=\lim_{\varepsilon\to1}(T_0+\varepsilon T_1+\varepsilon^2T_2)\\ &=B\cos\omega t+(\frac{\beta B^3}{32{\omega_0}^2}\cos3\omega t-\frac{\beta B^3}{32{\omega_0}^2}\cos\omega t)\\ &\quad(\frac{20\beta^2B^5}{1024{\omega_0}^4}\cos\omega t-\frac{21\beta^2B^3}{1024{\omega_0}^4}\cos3\omega t+\\ &\quad\frac{\beta^2B^5}{1024{\omega_0}^4}\cos5\omega t)\end{aligned} \tag{32}$$

在式(18)中把时间函数 $T(t)$ 用椭圆板中心挠度 f 代替,且在式(24)中略去惯性项可得功能梯度材料椭圆板热屈曲关系式为

$$N_T=(p_0+q_0f^2)/R \tag{33}$$

式中,

$$p_0=D(\frac{40}{a^4}+\frac{40}{b^4}+\frac{80}{3a^2b^2}),\ R=\frac{10}{3}(\frac{1}{a^2}+\frac{1}{b^2})$$

$$\begin{aligned}q_0=&\frac{Eh(117a^4-710a^2b^2-31b^4+1008)}{504a^6b^2}+\\ &\frac{Eh(117b^4-710a^2b^2-31a^4+1008)}{504a^2b^6}-\frac{2Eh}{a^2b^2}+\\ &\frac{10Eh(a^2+\mu b^2)}{9(1-\mu^2)a^2b^4}+\frac{10Eh(\mu a^2+b^2)}{9(1-\mu^2)a^4b^2}\end{aligned}$$

3 算例分析

为了验证本文计算方法正确性,分别用ANSYS和本文方法计算了温度载荷作用下周边固支椭圆板及四边不可移简支矩形板中点挠度 f 和板非线性振动频率 ω,并比较了直接考虑 $z_0=0$,即认为中面与中性面重合的情况. 椭圆板、矩形板均 $a=1000$ mm,板厚 $h=50$ mm,陶瓷材料的弹性模量和热膨胀系数、密度分别为 $\alpha_c=7.4\times10^{-6}$/℃,$E_c=380$ GPa,$\rho_c=2.5\times10^3$ kg/m^3, 金属材料的弹性模量和热膨胀系数、密度分别为 $\alpha_m=23\times10^{-6}$/℃,$E_m=70$ GPa,$\rho_m=2.7\times10^3$ kg/m^3,泊松比均为 $\mu=0.3$. k 分别取 0.25,0.5. 有限元建立模型求解,单元为 8 节点 SOLID46 实体层状单元,定义 50 层材料层来模拟功能梯度材料的材料性能的变化,顶层 $E_c=380$ GPa,$\alpha_c=7.4\times10^{-6}$/℃,$\rho_c=2.5\times10^3$ kg/m^3,$\mu=0.3$,底层 $E_m=70$ GPa,$\alpha_m=23\times10^{-6}$/℃,$\rho_m=2.7\times10^3$ kg/m^3,$\mu=0.3$,中间层按照式(1)、式(2)来确定 E、α 和 ρ. 采用 Large Displacement Static Analysis 进行求解. $k=0.25$,$\Delta T=500°$,$a=1.5b$ 和 $k=0.5$,$\Delta T=500°$,$a=1.5b$ 时椭圆板节点平面外位移如图 2 和图 3 所示; $k=0.25$,$\Delta T=500°$,$a=b$ 和 $k=0.5$,$\Delta T=500°$,$a=1.5b$ 时矩形板节点平面外位移如图 4 和图 5 所示. 本文计算结果与有限元结果比较可见一下各表. 表 1 ~ 表 4 为周边固支椭圆板的计算数据结果,表 5 ~ 表 8 为四边不可移简支矩形板的计算数据结果.

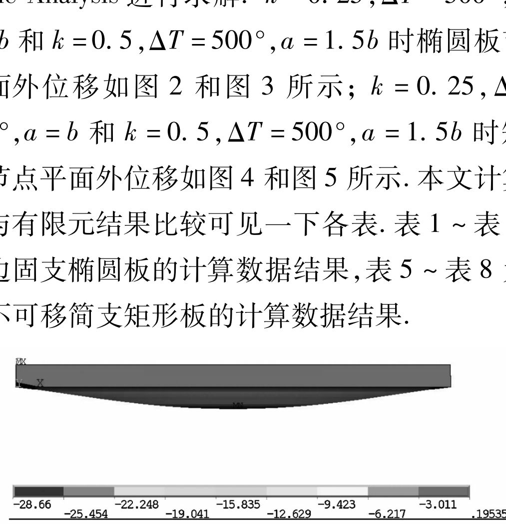

图 2 节点平面外位移($k=0.25$,$a=1.5b$)

Fig. 2 the nodes displacement out of plane

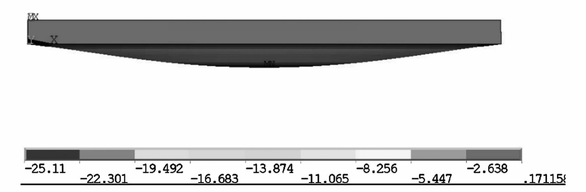

图 3　节点平面外位移($k = 0.5, a = 1.5b$)

Fig. 3　the nodes displacement out of plane

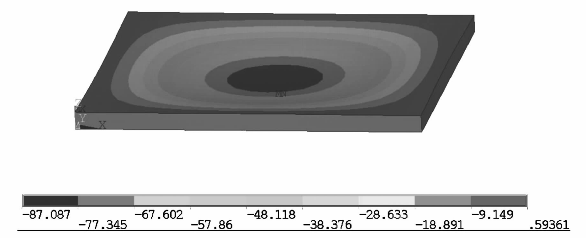

图 4　节点平面外位移($k = 0.25, a = b$)

Fig. 4　the nodes displacement out of plane

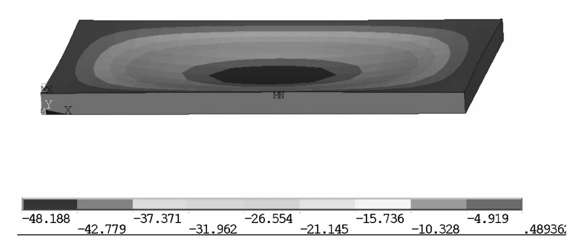

图 5　节点平面外位移($k = 0.5, a = 1.5b$)

Fig. 5　the nodes displacement out of plane

表 1　温度与挠度的非线性关系($a = 1.2b$)

Table 1　the nonlinear relation between temperature and frequency

ΔT			300°	350°	400°	450°	500°
$k = 0.25$	f	$z = 0.1h$	22.57	26.62	29.87	32.86	35.83
		$z = 0$	20.88	25.11	28.61	32.09	34.77
		ANSYS	22.13	26.09	29.16	32.11	35.23
$k = 0.5$	f	$z = 0.12h$	20.21	23.98	27.26	30.01	32.52
		$z = 0$	17.89	21.88	25.08	28.01	30.91
		ANSYS	19.93	23.28	26.86	29.77	32.01

表 2　温度与频率的非线性关系($a = 1.2b$)

Table 2　the nonlinear relation between temperature and frequency

ΔT			10°	20°	30°	40°	50°
$k = 0.25$	$b = 0.1h$	$z = 0.1h$	2410	2331	2238	2152	2083
		$z = 0$	2580	2506	2419	2335	2263
	$b = 0.2h$	$z = 0.1h$	2433	2342	2266	2186	2096
		$z = 0$	2598	2516	2437	2366	2286
$k = 0.5$	$b = 0.1h$	$z = 0.12h$	2685	2600	2516	2421	2335
		$z = 0$	2937	2866	2788	2696	2626
	$b = 0.2h$	$z = 0.12h$	2706	2625	2535	2451	2360
		$z = 0$	2960	2888	2811	2733	2650

表 3　温度与挠度的非线性关系($a = 1.5b$)

Table 3　the nonlinear relation between temperature and deflection

ΔT			300°	350°	400°	450°	500°
$k = 0.25$	f	$z = 0.1h$	11.01	16.23	19.91	23.11	26.21
		$z = 0$	19.11	26.55	31.22	37.21	45.11
		ANSYS	12.15	18.19	21.30	25.71	28.66
$k = 0.5$	f	$z = 0.12h$	6.91	13.12	16.99	20.11	23.01
		$z = 0$	13.01	20.01	25.31	31.58	40.37
		ANSYS	7.16	13.66	18.06	22.22	25.11

表 4　温度与频率的非线性关系($a = 1.5b$)

Table 4　the nonlinear relation between temperature and deflection

ΔT			10°	20°	30°	40°	50°
$k = 0.25$	$b = 0.1h$	$z = 0.1h$	3991	3918	3855	3766	3701
		$z = 0$	4233	4196	4130	4006	3922
	$b = 0.2h$	$z = 0.1h$	4020	3955	3906	3812	3776
		$z = 0$	4256	4221	4159	4029	3955
$k = 0.5$	$b = 0.1h$	$z = 0.12h$	4430	4361	4300	4231	4168
		$z = 0$	4833	4778	4720	4661	4600
	$b = 0.2h$	$z = 0.12h$	4458	4398	4322	4266	4200
		$z = 0$	4876	4811	4760	4695	4656

表 5　温度与挠度的非线性关系($a = b$)

Table 5　the nonlinear relation between temperature and deflection

ΔT			300°	350°	400°	450°	500°
$k = 0.25$	f	$z = 0.1h$	57.10	66.61	75.02	82.50	89.32
		$z = 0$	52.37	62.69	71.39	79.28	86.35
		ANSYS	56.82	66.15	74.31	81.03	87.09
$k = 0.5$	f	$z = 0.12h$	50.39	59.61	67.60	74.71	81.18
		$z = 0$	43.17	53.70	62.38	69.92	76.90
		ANSYS	50.01	58.78	66.05	73.13	79.65

表 6　温度与频率的非线性关系($a = b$)

Table 6　the nonlinear relation between temperature and frequency

ΔT			10°	20°	30°	40°	50°
$k = 0.25$	$b = 0.1h$	$z = 0.1h$	2007	1940	1871	1799	1724
		$z = 0$	2146	2084	2020	1953	1885
	$b = 0.2h$	$z = 0.1h$	2023	1957	1888	1816	1742
		$z = 0$	2162	2099	2035	1970	1901
$k = 0.5$	$b = 0.1h$	$z = 0.12h$	2235	2166	2096	2022	1947
		$z = 0$	2449	2387	2323	2256	2189
	$b = 0.2h$	$z = 0.12h$	2253	2186	2116	2043	1968
		$z = 0$	2466	2404	2341	2275	2207

表 7　温度与挠度的非线性关系($a = 1.5b$)

Table 7　the nonlinear relation between temperature and frequency

ΔT			300°	350°	400°	450°	500°
$k = 0.25$	f	$z = 0.1h$	22.80	34.91	43.72	51.10	57.52
		$z = 0$	44.72	64.11	89.55	109.22	125.90
		ANSYS	22.55	34.03	42.55	49.77	55.31
$k = 0.5$	f	$z = 0.12h$	13.90	28.11	37.30	44.62	50.81
		$z = 0$	5.03	12.01	27.31	36.58	44.37
		ANSYS	13.55	27.51	36.05	42.78	48.19

表 8 温度与频率的非线性关系($a=1.5b$)

Table 8 the nonlinear relation between temperature and deflection

ΔT			10°	20°	30°	40°	50°
$k=0.25$	$b=0.1h$	$z=0.1h$	3318	3268	3217	3165	3113
		$z=0$	3542	3495	3447	3399	3350
	$b=0.2h$	$z=0.1h$	3345	3295	3245	3193	3140
		$z=0$	3566	3519	3472	3424	3375
$k=0.5$	$b=0.1h$	$z=0.12h$	3690	3639	3586	3533	3480
		$z=0$	4032	3985	3937	3889	3841
	$b=0.2h$	$z=0.12h$	3721	3670	3618	3566	3512
		$z=0$	4060	4013	3967	3919	3870

由表 1 ~ 表 8 可以看出,随着温度升高均匀温度场中功能梯度材料椭圆板及矩形板的屈曲挠度将增大、非线性固有振动频率将变小,这主要是由于温度升高将降低功能梯度材料椭圆板及矩形板的弯曲刚度. 随着梯度指数 k 增大均匀温度场中功能梯度材料椭圆板及矩形板的屈曲挠度将变小、非线性固有振动频率将变大,主要是由于梯度指数 k 增大将增加功能梯度材料椭圆板的弯曲刚度,随着功能梯度材料椭圆板半长短轴比及功能梯度材料矩形板长宽比的增大均匀温度场中功能梯度材料椭圆板的屈曲挠度将减小、非线性固有振动频率将变大,主要是由于椭圆板半长短轴比及材料矩形板长宽比的增大将增加功能梯度材料椭圆板的弯曲刚度.

由表 1、表 3、表 5、表 7 还可知道,采用有限元方法计算的功能梯度材料椭圆板及矩形板屈曲挠度和本文方法计算的功能梯度材料椭圆板及矩形板屈曲挠度非常相近,两种方法的计算结果吻合的非常好,充分验证了本文方法的可靠性.

如按有关文献不确定功能梯度材料板壳中性面的真实位置,而是假设功能梯度材料板壳相对于中性面具有几何和弹性对称,来研究功能梯度材料椭圆板的非线性振动及屈曲. 算例分析表明,中性面位置对均匀温度场中功能梯度材料椭圆板的非线性振动及屈曲有较大的影响,这一点由表 1 ~ 表 4 就可以看出.

4 结论

由以上分析可以得到以下结论:

1) 采用有限元方法和本文方法计算的结果非常相近,两种方法的计算结果吻合的非常好,充分验证了本文方法的可靠性.

2) 温度升高将降低功能梯度材料椭圆板的弯曲刚度,梯度指数 k 增大、半长短轴比增大将增加功能梯度材料椭圆板的弯曲刚度.

3) 中性面位置对均匀温度场中功能梯度材料椭圆板的非线性振动及屈曲有较大影响.

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NONLINEAR THERMAL VIBRATION AND BUCKLING OF FUNCTIONALLY GRADED ELLIPTICAL PLATE *

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Abstract The static equilibrium equation of functionally graded circular plate was established by using elastic theory, and the neutral plane site of functionally graded elliptical plate was determined. On the basis, the nonlinear vibration and buckling differential equations for the functionally graded elliptical plate in uniform temperature field were derived, the approximate solution to nonlinear thermal vibration and buckling of functionally graded circular plate was obtained, and the effect of neutral plane site, gradient index and temperature on nonlinear thermal vibration and buckling of functionally graded elliptical plate was discussed and analyzed. The calculation results agreed well with the results, which verified the method. Analysis of examples indicates that the neutral plane site has certain influence on nonlinear thermal vibration and buckling of functionally graded elliptical plate in uniform temperature field.

Key words functionally graded, materials, elliptical plate, nonlinear, vibration, buckling

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