第 10 卷第 3 期 2012 年 9 月
1672-6553/2012/10(3)/213-9
动力学与控制学报
JOURNAL OF DYNAMICS AND CONTROL
Vol.10 No.3
Sep. 2012

航空发动机叶片非线性动力学分析*

张伟† 冯志青 曹东兴
(北京工业大学机电学院,北京 100022)

摘要 论文研究了航空发动机叶片的非线性振动问题,将叶片简化为功能梯度材料薄壁悬臂梁,考虑几何大变形的影响,基于一阶活塞气动力理论,利用 Hamilton 原理建立了叶片的非线性偏微分运动方程. 综合运用 Galerkin 方法、多尺度方法和数值方法对叶片模型进行了非线性动力学分析,通过相图、波形图和频谱图分析了不同气流流速情况下旋转叶片的动态响应. 结果表明:随着气流流速的增加,系统呈现倍周期运动和混沌运动等多种复杂动力学行为.

关键词 旋转叶片, 非线性动力学, 动态响应, 混沌

引言

叶片是航空发动机中最重要的关键部件之一,它的可靠性直接影响发动机能否正常工作,甚至整架飞机的飞行安全. 叶片在高温高压燃气包围下工作不仅需要承受气动力、热应力、振动负荷以及高速旋转时自身的离心力,还受到燃气的严重腐蚀. 已有研究表明,叶片故障占航空发动机故障的 62% 以上. 因此,对叶片振动问题的研究具有非常重要的工程意义. 薄壁空心叶片具有增加发动机压气机喘振裕度、抗外物损伤、提高发动机推力、减少叶片数和减轻重量等优点,在航空发动机 I 级叶片(即风扇叶片)中得到广泛应用. 本文主要研究航空发动机压气机旋转叶片的非线性动力学问题

论文把旋转叶片简化为功能梯度材料的等截面旋转薄壁梁,国内外一些学者对旋转悬臂梁进行了动力学研究,如 Carnegie[1] 首次研究了旋转叶片的自振频率,并用 Rayleigh 法得到理论的势能表达式. Yokoyama[2] 用有限元法研究了 Timoshenko 旋转梁的自由振动特性. Yang[3] 等推导了一系列包括轴向、横向和扭转运动的耦合积分微分方程来描述旋转的欧拉梁,其中包含了离心力引起的刚化作用. Cort ́Inez 和 Piovan[4] 针对横截面开口或闭口的薄壁梁,发展了一个理论模型来进行振动和屈曲分析,随后对此模型受到剪切变形时进行了稳定性分析. Librescu 和 Oh[5-7] 提出了纤维复合材料旋转叶片的线性动力学理论,并且研究了功能梯度材料的旋转薄壁梁在高温环境下的的振动和稳定性问题. Fazelzadeh[8] 研究了功能梯度材料的旋转叶片在空气热弹性载荷下的动力学行为.

以上文献都是从线性理论来研究梁的振动特性,但对于高速旋转机械,非线性因素的影响对旋转机械的振动响应也越来越重要,Simo 和 Vu - Quoc[9] 的研究显示适当的考虑非线性应变位移关系,对正确模拟柔性梁的几何硬化有非常重要的作用. Chen 和 Peng[10] 用几何非线性研究了旋转叶片的动力学稳定性. Chandiramani 和 Librescu[11] 用高阶剪切变形理论研究了复合材料旋转梁的自由振动,通过几何非线性结合各向异性、横向剪切和翘曲等因素对旋转梁振动的影响进行了分析. Yao 和 Chen[12] 研究了预弯曲薄壁变转速叶片的非线性振动问题.

本文主要研究航空发动机压气机旋转叶片的非线性动力学问题. 首先考虑几何大变形和气动力的影响,利用 Hamilton 原理建立柔性叶片的非线性偏微分运动方程,综合运用 Galerkin 方法、高阶多尺度方法,并通过数值模拟研究旋转叶片的非线性动力学行为.

1 叶片非线性动力学模型

本文将航空发动机压气机 I 级叶片简化为如图 1 所示的功能梯度材料的等截面旋转薄壁梁,考虑长度为 L 的薄壁叶片固定在半径为 R_0 的轮毂

2011-11-25 收到第 1 稿,2011-12-08 收到修改稿.
* 国家自然科学基金重点资助项目(10732020);国家自然科学基金资助项目(11072008,10802001)
† 通讯作者 E-mail:sandyzhang0@yahoo.com

上，以常速 Ω 旋转，其中 γ 是安装角，横截面的尺寸如图2所示. 模型受到 Y 方向的气动荷载.

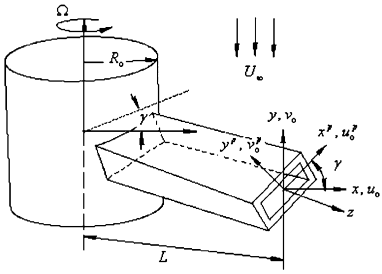

图1　旋转叶片的悬臂梁模型

Fig. 1　Geometry of the thin－walled beam

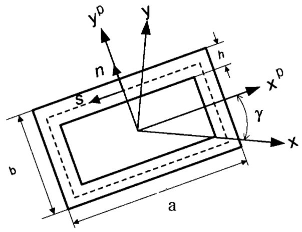

图2　梁的横截面

Fig. 2　Blade cross－section

首先建立两个主要的坐标系，其中 (X,Y,Z) 为惯性坐标系，(x,y,z) 为旋转坐标系，z 为顺翼展方向，x,y 为横截面上任一点的横向坐标. 由于叶片有安装角，我们建立了局部坐标 (x^p,y^p,z^p)，其中 x^p 和 y^p 是横截面上任意一点的坐标.

旋转坐标 (x,y,z) 与局部坐标 (x^p,y^p,z^p) 之间的关系为

$$x=x^p\cos\gamma-y^p\sin\gamma,\ y=x^p\sin\gamma+y^p\cos\gamma,\ z=z^p \tag{1}$$

在旋转坐标系中，环面坐标 $(x(s),y(s),z(z))$ 为中面任一点的坐标. 其中 s 和 n 分别为中面环长和横截面厚度，令 $-\frac{1}{2}h\leqslant n\leqslant\frac{1}{2}h$，$\vec{t}$ 为中面的切线方向，$\vec{n}$ 为法线方向. 如图2所示.

在建立旋转叶片的运动控制方程前，假设：

1）横截面的形状保持不变.

2）模型采用限制扭转，即扭转率 ϕ' 与顺翼展方向的 z 有关，表示为 $\phi'=\phi'(z)$

3）沿 s 方向的力与其它力 N_{ss} 相比很小，我们忽略不计.

梁的位移场表示为

$$u(x,y,z,t)=u_0(z,t)-y\phi(z,t) \tag{2}$$

$$v(x,y,z,t)=v_0(z,t)+x\phi(z,t) \tag{3}$$

$$w(x,y,z,t)=\theta_x(z,t)\left[y(s)-n\frac{dx}{ds}\right]+\theta_y(z,t)\left[x(s)+n\frac{dy}{ds}\right] \tag{4}$$

其中 u_0、v_0 是叶片沿着 x、y 轴的刚体位移，ϕ 和 θ_x、θ_y 是相对于 z 轴的扭转角和 x、y 的弯曲角.

不考虑剪切，则 θ_x、θ_y 表示为

$$\theta_x=-v'_0,\quad \theta_y=-u'_0 \tag{5}$$

叶片变形后任一点 $M(x,y,z)$ 的位置矢量表示为

$$\{R(x,y,z)\}=(x+u)\vec{i}+(y+v)\vec{j}+(z+w+R_0)\vec{k} \tag{6}$$

则速度矢量及加速度矢量可表示为

$$\{\dot{R}\}=v_x\vec{i}+v_y\vec{j}+v_z\vec{k},\ \{\ddot{R}\}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k} \tag{7}$$

其中

$$v_x=\dot{u}+(R_0+z+w)\Omega,\ v_y=\dot{v},\ v_z=\dot{w}-(x+u)\Omega \tag{8}$$

$$a_x=\ddot{u}+2\dot{w}\Omega-(x+u)\Omega^2,\ a_y=\ddot{v},\quad a_z=\ddot{w}-2\dot{u}\Omega-(R_0+z+w)\Omega^2 \tag{9}$$

不考虑剪切，应变与位移几何关系为

$$\varepsilon_{zz}=\frac{\partial w}{\partial z}+\frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right],\ \gamma_{xz}=\gamma_{yz}=0 \tag{10}$$

假设横截面的形状不变，则有

$$\varepsilon_{xx}=\varepsilon_{yy}=\gamma_{xy}=0 \tag{11}$$

从而有

$$\varepsilon_{nn}=\varepsilon_{ss}=\varepsilon_{sn}=0 \tag{12}$$

切线方向的剪切为

$$\varepsilon_{sz}=2\frac{A_c}{\beta}\phi' \tag{13}$$

其中 A_c 和 β 分别代表横截面的面积和 s 的长度.

应力与应变的关系为

$$\begin{bmatrix}\sigma_{ss}\\ \sigma_{zz}\\ \sigma_{zn}\\ \sigma_{ns}\\ \sigma_{sz}\end{bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\ Q_{12}&Q_{22}&0&0&0\\ 0&0&Q_{44}&0&0\\ 0&0&0&Q_{55}&0\\ 0&0&0&0&Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{ss}\\ \varepsilon_{zz}\\ \varepsilon_{zn}\\ \varepsilon_{ns}\\ \varepsilon_{sz}\end{bmatrix}-\begin{bmatrix}\hat{\alpha}\nabla T\\ \hat{\alpha}\nabla T\\ 0\\ 0\\ 0\end{bmatrix} \tag{14}$$

其中

$$Q_{11}=Q_{22}=\frac{E}{1-\upsilon^2},\ Q_{12}=\frac{E\upsilon}{1-\upsilon^2},\ Q_{66}=\frac{E}{2(1+\upsilon)},$$

$$Q_{44}=Q_{55}=k_s^2\frac{E}{2(1+\upsilon)},\ \hat{\alpha}=\frac{E}{1-\upsilon^2}\alpha.$$

这里 E 和 υ 分别是杨氏模量和泊松比,k_s^2 是横向剪切修正系数,α 是热膨胀系数.

则系统的动能和势能表示如下

$$\delta K = -\int_{\tau}\rho\ddot{R}\delta R\mathrm{d}\tau = -\int_{\tau}\rho\{[\ddot{u}+2\dot{w}\Omega-(x+u)\Omega^2]\delta u+\ddot{v}\delta v+[\ddot{w}-2\dot{u}\Omega-(R_0+Z+w)\Omega^2]\delta w\}\mathrm{d}\tau \tag{15}$$

$$\delta U = \delta U_1 + \delta U_2 \tag{16}$$

其中 δU_1 和 δU_2 的表达式如下

$$\delta U_1 = \int_{\tau}\sigma_{ij}\delta\varepsilon_{ij}\mathrm{d}\tau = \int_{\tau}(\sigma_{zz}\delta\varepsilon_{zz}+\sigma_{sz}\delta\varepsilon_{sz})\mathrm{d}\tau \tag{17a}$$

$$\delta U_2 = \int_{l0}F(\frac{\partial u_0}{\partial z}-y\frac{\partial\phi}{\partial z})\delta(\frac{\partial u_0}{\partial z}-y\frac{\partial\phi}{\partial z})\mathrm{d}z + \int_{l0}F(\frac{\partial v_0}{\partial z}+x\frac{\partial\phi}{\partial z})\delta(\frac{\partial v_0}{\partial z}+x\frac{\partial\phi}{\partial z})\mathrm{d}z \tag{17b}$$

势能由叶片变形产生的 U_1 和旋转产生的 U_2 组成,F 为离心力,表示为

$$F = \int_{l0}b_1\Omega^2(z+R_0)\mathrm{d}z = b_1\Omega^2[R_0(l-z)+\frac{1}{2}(l^2-z^2)] = b_1\Omega^2R(z) \tag{17c}$$

外力所作的虚功的变分表示为

$$\delta V = \int_{l0}(p_x\delta u_0+p_y\delta v_0)\mathrm{d}z \tag{18}$$

根据一阶活塞理论,气动力在横截面上单位长度的分量为

$$\Delta P_{x^p} = C_{\infty}\rho_{\infty}(\frac{\partial u^p}{\partial t}+U_{x^p}^t\frac{\partial u^p}{\partial z}),$$

$$\Delta P_{y^p} = C_{\infty}\rho_{\infty}(\frac{\partial v^p}{\partial t}+U_{y^p}^t\frac{\partial v^p}{\partial z}) \tag{19}$$

其中

$$u^p = u\cos\gamma+v\sin\gamma,\quad v^p = -u\sin\gamma+v\cos\gamma,$$

$$U_{x^p}^t = U_{\infty}\cos\gamma,\quad U_{y^p}^t = U_{\infty}\sin\gamma \tag{20a}$$

则 x 轴方向和 y 轴方向的的气动力的分量为

$$P_x = a\Delta P_{y^p}\sin\gamma - b\Delta P_{x^p}\cos\gamma,$$

$$P_y = -a\Delta P_{y^p}\cos\gamma - b\Delta P_{x^p}\sin\gamma \tag{20b}$$

根据 Hamilton 原理,建立系统广义位移形式的偏微分动力学方程为

$$\frac{3}{2}A_{11}(\frac{\partial u_0}{\partial z})^2\frac{\partial^2u_0}{\partial z^2}+(B_{11}+3\underline{B_{11}})\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial z^2}\frac{\partial u_0}{\partial z}+\frac{1}{2}(B_{11}+3\underline{B_{11}})(\frac{\partial\phi}{\partial z})^2\frac{\partial^2u_0}{\partial z^2}+A_{11}\frac{\partial v_0}{\partial z}\frac{\partial^2v_0}{\partial z^2}\frac{\partial u_0}{\partial z}+\frac{1}{2}A_{11}(\frac{\partial v_0}{\partial z})^2\frac{\partial^2u_0}{\partial z^2}+(\underline{B_{11}}+B_{11})\frac{\partial^3v_0}{\partial z^3}\frac{\partial\phi}{\partial z}-B_{11}\frac{\partial^4u_0}{\partial z^4}+(B_{11}+2B_{11})\frac{\partial^2v_0}{\partial z^2}\frac{\partial^2\phi}{\partial z^2}-\frac{3}{2}(\underline{D_{11}}+H_{22})(\frac{\partial\phi}{\partial z})^2\frac{\partial^2\phi}{\partial z^2}+(D_{11}+H_{12})(\frac{\partial^2\phi}{\partial z^2})^2+(H_{12}+D_{11})\frac{\partial\phi}{\partial z}\frac{\partial^3\phi}{\partial z^3}+B_{11}\frac{\partial v_0}{\partial z}\frac{\partial^3\phi}{\partial z^3}-A_{\alpha11}\Delta T\frac{\partial^2u_0}{\partial z^2}-I_0\Omega^2(R+z)\frac{\partial u_0}{\partial z}=I_0\ddot{u}_0-I_0u_0\Omega^2-I_{xx}\frac{\partial^4u_0}{\partial z^2\partial t^2}+I_{xx}\Omega^2\frac{\partial^2u_0}{\partial z^2} \tag{21a}$$

$$\frac{1}{2}A_{11}(\frac{\partial u_0}{\partial z})^2\frac{\partial^2v_0}{\partial z^2}+A_{11}\frac{\partial u_0}{\partial z}\frac{\partial^2u_0}{\partial z^2}\frac{\partial v_0}{\partial z}+\frac{1}{2}(\underline{B_{11}}+3B_{11})(\frac{\partial\phi}{\partial z})^2\frac{\partial^2v_0}{\partial z^2}+(\underline{B_{11}}+3B_{11})\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial z^2}\frac{\partial v_0}{\partial z}+\frac{3}{2}A_{11}(\frac{\partial v_0}{\partial z})^2\frac{\partial^2v_0}{\partial z^2}-(B_{11}+\underline{B_{11}})\frac{\partial^3u_0}{\partial z^3}\frac{\partial\phi}{\partial z}-(B_{11}+2\underline{B_{11}})\frac{\partial^2u_0}{\partial z^2}\frac{\partial^2\phi}{\partial z^2}+\frac{3}{2}(H_{12}+D_{11})(\frac{\partial\phi}{\partial z})^2\frac{\partial^2\phi}{\partial z^2}-\underline{B_{11}}\frac{\partial^4v_0}{\partial z^4}+(H_{21}+\underline{D_{11}})(\frac{\partial^2\phi}{\partial z^2})^2+(H_{21}+\underline{D_{11}})\frac{\partial\phi}{\partial z}\frac{\partial^3\phi}{\partial z^3}-\underline{B_{11}}\frac{\partial u_0}{\partial z}\frac{\partial^3\phi}{\partial z^3}-A_{\alpha11}\Delta T\frac{\partial^2v_0}{\partial z^2}-I_0\Omega^2(R+z)\frac{\partial v_0}{\partial z}+I_0\Omega^2R(z)\frac{\partial^2v_0}{\partial z^2}+P_y=I_0\ddot{v}_0-I_{yy}\frac{\partial^4v_0}{\partial z^2\partial t^2}+2\Omega I_{yy}\frac{\partial^2\phi}{\partial z\partial t}+I_{yy}\Omega^2\frac{\partial^2v_0}{\partial z^2} \tag{21b}$$

$$-(H_{21}+\underline{D_{11}})\frac{\partial^3v_0}{\partial z^3}\frac{\partial\phi}{\partial z}-(H_{21}+\underline{D_{11}})\frac{\partial^2v_0}{\partial z^3}\frac{\partial^2\phi}{\partial z^2}-(D_{11}+H_{12})\frac{\partial^3u_0}{\partial z^3}\frac{\partial\phi}{\partial z}-(D_{11}+H_{12})\frac{\partial^2u_0}{\partial z^2}\frac{\partial^2\phi}{\partial z^2}+(B_{11}+\underline{B_{11}})\frac{\partial u_0}{\partial z}\frac{\partial^2u_0}{\partial z^2}\frac{\partial\phi}{\partial z}+\frac{1}{2}(B_{11}+\underline{B_{11}})(\frac{\partial u_0}{\partial z})^2\frac{\partial^2\phi}{\partial z^2}+\frac{3}{2}(E_{11}+\underline{E_{11}}+2H_{22})\frac{\partial^2\phi}{\partial z^2}(\frac{\partial\phi}{\partial z})^2-B_{11}\frac{\partial^3u_0}{\partial z^3}\frac{\partial v_0}{\partial z}-B_{11}\frac{\partial^2u_0}{\partial z^2}\frac{\partial^2v_0}{\partial z^2}-(H_{21}+\underline{D_{11}})(\frac{\partial\phi}{\partial z})^2\frac{\partial^2u_0}{\partial z^2}-2(H_{21}+D_{11})\frac{\partial^2\phi}{\partial z^2}\frac{\partial\phi}{\partial z}\frac{\partial u_0}{\partial z}+(B_{11}+\underline{B_{11}})\frac{\partial v_0}{\partial z}\frac{\partial^2v_0}{\partial z^2}\frac{\partial\phi}{\partial z}+\frac{1}{2}(B_{11}+\underline{B_{11}})(\frac{\partial v_0}{\partial z})^2\frac{\partial^2\phi}{\partial z^2}+2(D_{11}+H_{21})\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial z^2}\frac{\partial v_0}{\partial z}+(D_{11}+H_{12})(\frac{\partial\phi}{\partial z})^2\frac{\partial^2v_0}{\partial z^2}+\underline{B_{11}}\frac{\partial^3v_0}{\partial z^3}\frac{\partial u_0}{\partial z}+\underline{B_{11}}\frac{\partial^2v_0}{\partial z^2}\frac{\partial^2u_0}{\partial z^2}-\frac{1}{2}(\underline{D_{11}}+H_{21})(\frac{\partial\phi}{\partial z})^2\frac{\partial^2u_0}{\partial z^2}-(\underline{D_{11}}+H_{21})\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial z^2}\frac{\partial u_0}{\partial z}+2\underline{B_{11}}\frac{\partial u_0}{\partial z}\frac{\partial^2u_0}{\partial z^2}\frac{\partial\phi}{\partial z}+\underline{B_{11}}(\frac{\partial u_0}{\partial z})^2\frac{\partial^2\phi}{\partial z^2}+(H_{12}+$$

$$D_{11})\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial z^2}\frac{\partial v_0}{\partial z}+B_{11}(\frac{\partial v_0}{\partial z})^2\frac{\partial^2\phi}{\partial z^2}+\frac{1}{2}(H_{12}+D_{11})(\frac{\partial\phi}{\partial z})^2\frac{\partial^2 v_0}{\partial z^2}+2B_{11}\frac{\partial v_0}{\partial z}\frac{\partial^2 v_0}{\partial z^2}\frac{\partial\phi}{\partial z}-A_{\alpha11}\nabla T\frac{\partial^2\phi}{\partial z^2}-(I_{xx}+I_{yy})\Omega^2(R+z)\frac{\partial\phi}{\partial z}+(I_{xx}+I_{yy})\Omega^2R(z)\frac{\partial^2\phi}{\partial z^2}=I_{yy}\ddot{\phi}+I_{xx}\ddot{\phi}-2\Omega I_{yy}\frac{\partial^2 v_0}{\partial z\partial t}-I_{yy}\phi\Omega^2 \qquad (21c)$$

边界条件:

$z=0$ 时,

$$u_0=v_0=\theta_x=\theta_y=\phi=\phi'=0 \qquad (22a)$$

$z=l$ 时,

$$\delta u_0:-\frac{1}{2}A_{11}(\frac{\partial u_0}{\partial z})^3-\frac{1}{2}(B_{11}+\underline{B}_{11})(\frac{\partial\phi}{\partial z})^2\frac{\partial u_0}{\partial z}-\frac{1}{2}A_{11}(\frac{\partial v_0}{\partial z})^2\frac{\partial u_0}{\partial z}+\underline{B}_{11}\frac{\partial\theta_x}{\partial z}\frac{\partial\phi}{\partial z}-\underline{B}_{11}\frac{\partial u_0}{\partial z}(\frac{\partial\phi}{\partial z})^2+\frac{1}{2}(\underline{D}_{11}+H_{21})(\frac{\partial\phi}{\partial z})^3+B_{11}\frac{\partial^3 u_0}{\partial z^3}-(D_{11}+H_{12})\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial z^2}-B_{11}\frac{\partial^2 v_0}{\partial z^2}\frac{\partial\phi}{\partial z}-B_{11}\frac{\partial v_0}{\partial z}\frac{\partial^2\phi}{\partial z^2}+A_{\alpha11}\nabla T\frac{\partial u_0}{\partial z}-I_0\Omega^2R(z)\frac{\partial u_0}{\partial z}=I_{xx}\frac{\partial^3 u_0}{\partial z\partial t^2}-I_{xx}\Omega^2\frac{\partial u_0}{\partial z} \qquad (22b)$$

$$\delta v_0:-\frac{1}{2}A_{11}(\frac{\partial v_0}{\partial z})^3-\frac{1}{2}(B_{11}+\underline{B}_{11})(\frac{\partial\phi}{\partial z})^2\frac{\partial v_0}{\partial z}-\frac{1}{2}A_{11}(\frac{\partial u_0}{\partial z})^2\frac{\partial v_0}{\partial z}-B_{11}\frac{\partial\theta_y}{\partial z}\frac{\partial\phi}{\partial z}-B_{11}\frac{\partial v_0}{\partial z}(\frac{\partial\phi}{\partial z})^2+\frac{1}{2}(D_{11}+H_{12})(\frac{\partial\phi}{\partial z})^3+\underline{B}_{11}\frac{\partial^3 v_0}{\partial z^3}-(H_{21}+\underline{D}_{11})\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial z^2}+\underline{B}_{11}\frac{\partial^2 u_0}{\partial z^2}\frac{\partial\phi}{\partial z}+\underline{B}_{11}\frac{\partial u_0}{\partial z}\frac{\partial^2\phi}{\partial z^2}+A_{\alpha11}\nabla T\frac{\partial v_0}{\partial z}-I_0\Omega^2R(z)\frac{\partial v_0}{\partial z}=I_{yy}\frac{\partial^3 v_0}{\partial z\partial t^2}-2I_{yy}\Omega\frac{\partial\phi}{\partial t}-I_{yy}\Omega^2\frac{\partial v_0}{\partial z} \qquad (22c)$$

$$\delta\phi:(H_{21}+\underline{D}_{11})\frac{\partial^2 v_0}{\partial z^2}\frac{\partial\phi}{\partial z}+(D_{11}+H_{12})\frac{\partial^2 u_0}{\partial z^2}\frac{\partial\phi}{\partial z}-\frac{1}{2}(B_{11}+3\underline{B}_{11})(\frac{\partial u_0}{\partial z})^2\frac{\partial\phi}{\partial z}-\frac{1}{2}(E_{11}+\underline{E}_{11}+2H_{22})(\frac{\partial\phi}{\partial z})^3+B_{11}\frac{\partial^2 u_0}{\partial z^2}\frac{\partial v_0}{\partial z}+\frac{3}{2}(H_{21}+\underline{D}_{11})(\frac{\partial\phi}{\partial z})^2\frac{\partial u_0}{\partial z}-\frac{1}{2}(3B_{11}+\underline{B}_{11})(\frac{\partial v_0}{\partial z})^2\frac{\partial\phi}{\partial z}-\underline{B}_{11}\frac{\partial^2 v_0}{\partial z^2}\frac{\partial u_0}{\partial z}-\frac{3}{2}(D_{11}+H_{12})(\frac{\partial\phi}{\partial z})^2\frac{\partial v_0}{\partial z}-(A_{66}+A_{\alpha12}+A_{\alpha21})\nabla T\frac{\partial\phi}{\partial z}-(I_{xx}+I_{yy})\Omega^2R(z)\frac{\partial\phi}{\partial z} \qquad (22d)$$

$$\delta\theta_y:-B_{11}\frac{\partial^2 u_0}{\partial z^2}+\frac{1}{2}(D_{11}+H_{12})(\frac{\partial\phi}{\partial z})^2+B_{11}\frac{\partial v_0}{\partial z}\frac{\partial\phi}{\partial z}=0 \qquad (22e)$$

$$\delta\theta_x:-\underline{B}_{11}\frac{\partial^2 v_0}{\partial z^2}+\frac{1}{2}(\underline{D}_{11}+H_{21})(\frac{\partial\phi}{\partial z})^2+\underline{B}_{11}\frac{\partial u_0}{\partial z}\frac{\partial\phi}{\partial z}=0 \qquad (22f)$$

为便于分析,引入如下无量纲变换

$$\bar{z}=\frac{z}{l},\overline{u_0}=\frac{u_0}{l},\overline{v_0}=\frac{v_0}{l},\bar{R}=\frac{R}{l},\bar{a}=\frac{a}{l},\bar{b}=\frac{b}{l},$$

$$\bar{t}=\frac{t\sqrt{E}}{\sqrt{\rho l^2}},\underline{B}_{11}^*=\frac{\underline{B}_{11}}{E\bar{A}l^2},B_{11}^*=\frac{B_{11}}{E\bar{A}l^2},A_{11}^*=\frac{A_{11}}{E\bar{A}},$$

$$D_{11}^*=\frac{D_{11}}{E\bar{A}l^3},\underline{D}_{11}^*=\frac{\underline{D}_{11}}{E\bar{A}l^3},E_{11}^*=\frac{E_{11}}{E\bar{A}l^4},\underline{E}_{11}^*=\frac{\underline{E}_{11}}{E\bar{A}l^4}$$

$$\overline{\rho_\infty}=\frac{\rho_\infty l^2}{\rho\bar{A}},H_{22}^*=\frac{H_{22}}{E\bar{A}l^3},H_{21}^*=\frac{H_{21}}{E\bar{A}l^3},H_{12}^*=\frac{H_{12}}{E\bar{A}l^3},$$

$$I_0^*=\frac{I_0}{\rho\bar{A}}=1,I_{xx}^*=\frac{I_{xx}}{\rho\bar{A}l^2},I_{yy}^*=\frac{I_{yy}}{\rho\bar{A}l^2},\bar{\Omega}=\Omega\sqrt{\frac{\rho l^2}{E}},$$

$$C_\infty^*=\frac{C_\infty\sqrt{\rho}}{\sqrt{E}},U_\infty^*=\frac{U_\infty\sqrt{\rho}}{\sqrt{E}},A_{\alpha11}^*=\frac{A_{\alpha11}}{E\bar{A}} \qquad (23)$$

将方程(23)代入方程(21a)、(21b)和(21c)中,并取位移的一阶模态

$$u_0(z,t)=u_i(t)G_j(z),\quad v_0(z,t)=v_i(t)G_j(z),$$
$$\phi(z,t)=\phi_i(t)S_j(z) \qquad (24)$$

其中

$$G(z)=\cos kz-chkz-\frac{\cos kl+chkl}{\sin kl+shkl}(\sin kz-shkz) \qquad (25a)$$

$$S(z)=\sin kz-shkz+\frac{\sin kl+shkl}{\cos kl-chkl}(\cos kz-chkz) \qquad (25b)$$

利用 Galerkin 法进行离散,得到3自由度的常微分方程

$$\ddot{u}-\mu_1\dot{u}+w_1^2u=e_{01}u^3+e_{01}uv^2+e_{02}u\phi+e_{03}u\phi^2+e_{04}v\phi+e_{05}v+e_{06}\phi^3+e_{07}\phi^2+e_{08}\phi+e_{09}\dot{v}+e_{10}\dot{\phi} \qquad (26a)$$

$$\ddot{v}-\mu_2\dot{v}+w_2^2v=e_{11}v^3+e_{11}vu^2+e_{12}v\phi+e_{13}v\phi^2+e_{14}u\phi+e_{15}u+e_{16}\phi^3+e_{17}\phi^2+e_{18}\phi+e_{19}\dot{u}+e_{20}\dot{\phi} \qquad (26b)$$

$$\ddot{\phi}+w_3^2\phi=e_{21}\phi^3+e_{22}\phi^2u+e_{23}\phi^2v+e_{24}u^2\phi+e_{25}v^2\phi+e_{26}u\phi+e_{27}v\phi+e_{28}uv+e_{29}\dot{v} \qquad (26c)$$

2 摄动分析

为便于摄动分析,引入小参数 ε,方程(26)变

换为:

$$\begin{aligned}&\ddot{u}-\varepsilon\mu_1\dot{u}+w_1^2u-\varepsilon e_{01}u^3-\varepsilon e_{01}uv^2-\varepsilon e_{02}u\phi-\\&\quad\varepsilon e_{03}u\phi^2-\varepsilon e_{04}v\phi-\varepsilon e_{05}v-\varepsilon e_{06}\phi^3-\varepsilon e_{07}\phi^2-\\&\quad\varepsilon e_{08}\phi-\varepsilon e_{09}\dot{v}-\varepsilon e_{10}\dot{\phi}=0\end{aligned}\tag{27a}$$

$$\begin{aligned}&\ddot{v}-\varepsilon\mu_1\dot{u}+w_2^2v-\varepsilon e_{11}v^3-\varepsilon e_{11}vu^2-\varepsilon e_{12}v\phi-\\&\quad\varepsilon e_{13}v\phi^2-\varepsilon e_{14}u\phi-\varepsilon e_{15}u-\varepsilon e_{16}\phi^3-\varepsilon e_{17}\phi^2-\\&\quad\varepsilon e_{18}\phi-\varepsilon e_{19}\dot{u}-\varepsilon e_{20}\dot{\phi}=0\end{aligned}\tag{27b}$$

$$\begin{aligned}&\ddot{\phi}+w_3^2\phi-\varepsilon e_{21}\phi^3-\varepsilon e_{22}\phi^2u-\varepsilon e_{23}\phi^2v-\\&\quad\varepsilon e_{24}u^2\phi-\varepsilon e_{25}v^2\phi-\varepsilon e_{26}u\phi-\varepsilon e_{27}v\phi-\\&\quad\varepsilon e_{28}uv-\varepsilon e_{29}\dot{v}=0\end{aligned}\tag{27c}$$

利用高阶多尺度法对功能梯度材料旋转叶片运动方程(27)进行摄动分析,方程一致渐进解表示为

$$\begin{aligned}x(t,\varepsilon)=&x_0(T_0,T_1,T_2)+\varepsilon x_1(T_0,T_1,T_2)+\\&\varepsilon^2x_2(T_0,T_1,T_2)\cdots\end{aligned}\tag{28a}$$

$$\begin{aligned}y(t,\varepsilon)=&y_0(T_0,T_1,T_2)+\varepsilon y_1(T_0,T_1,T_2)+\\&\varepsilon^2y_2(T_0,T_1,T_2)\cdots\end{aligned}\tag{28b}$$

$$\begin{aligned}z(t,\varepsilon)=&z_0(T_0,T_1,T_2)+\varepsilon z_1(T_0,T_1,T_2)+\\&\varepsilon^2z_2(T_0,T_1,T_2)\cdots\end{aligned}\tag{28c}$$

式中 $T_0=t,T_1=\varepsilon t,T_2=\varepsilon^2t.$

考虑 1:1:2 内共振关系

$$\begin{aligned}&\omega_1^2=\omega^2+\varepsilon\sigma_1,\quad\omega_2^2=\omega^2+\varepsilon\sigma_2,\\&\omega_3^2=4\omega^2+\varepsilon\sigma_3,\quad\omega=\Omega=1\end{aligned}\tag{29}$$

其中 σ_1,σ_2 和 σ_3 是调谐参数.

通过多尺度摄动分析,得到直角坐标形式的平均方程为

$$\begin{aligned}\dot{x}_1=&b_{08}x_1-j_{05}x_2+j_{08}x_3-j_{09}x_4+4b_{14}x_2x_3x_4x_5x_6+\\&2x_1x_2(-j_{02}x_3+c_{01}x_4+j_{06}(x_3x_5-x_4x_6)-j_{01}(x_3x_5+\\&x_4x_6))+2x_3x_4(-c_{08}x_1+\frac{1}{2}a_{06}x_2+c_{06}(x_1x_5-\\&x_2x_6)-c_{02}(x_1x_5+x_2x_6))+j_{14}(x_4x_5-x_3x_6)+\\&j_{12}(x_2x_5-x_1x_6)+j_{13}(x_3x_5+x_4x_6)+b_{15}(x_1x_5+\\&x_2x_6)+b_{14}x_2(x_3^2-x_4^2)(x_5^2-x_6^2)+(b_{02}x_6+2b_{01}x_3x_4-\\&2b_{13}x_5x_6)(x_1^3-3x_1x_2^2)+b_{02}x_5(-3x_1^2x_2+x_2^3)+\\&c_{07}x_6(x_3^3-3x_3x_4^2)+c_{07}x_5(-3x_3^2x_4+x_4^3)+(c_{01}x_3+\\&j_{02}x_4+j_{01}(x_3x_6-x_4x_5)-j_{06}(x_3x_6+x_4x_5))(x_1^2-\\&x_2^2)+(\frac{1}{2}a_{06}x_1+c_{08}x_2+c_{02}(x_1x_6-x_2x_5)-\\&c_{06}(x_1x_6+x_2x_5)-2b_{14}x_1x_5x_6+b_{01}(-3x_1^2x_2+\\&x_2^3))(x_3^2-x_4^2)+(2b_{14}x_1x_3x_4-b_{13}(-3x_1^2x_2+\\&x_2^3))(x_5^2-x_6^2)+(a_{02}x_1-a_{03}x_2+c_{03}x_3-c_{04}x_4-\\&2b_{05}x_1x_3x_4+j_{03}(x_4x_5-x_3x_6)+b_{07}(x_2x_5-x_1x_6)-\\&b_{03}x_2(x_3^2+x_4^2)+b_{05}x_2(x_3^2-x_4^2)-b_{04}x_2(x_5^2+\\&x_6^2))(x_1^2+x_2^2)+(a_{06}x_1-c_{05}x_2+b_{11}x_3-b_{10}x_4-\\&2a_{07}x_1x_3x_4+b_{12}(x_2x_5-x_1x_6)+c_{09}(x_4x_5-x_3x_6)+\\&a_{07}x_2(x_3^2-x_4^2)-a_{05}x_2(x_5^2+x_6^2))(x_3^2+x_4^2)+\\&(-j_{04}x_2+c_{10}x_3-j_{07}x_4-2b_{09}x_1x_3x_4+j_{10}(x_2x_5-\\&x_1x_6)+j_{11}(x_4x_5-x_3x_6)+b_{09}x_2(x_3^2-x_4^2))(x_5^2+\\&x_6^2)-x_2(a_{01}(x_1^2+x_2^2)^2+a_{04}(x_3^2+x_4^2)^2)-(b_{06}x_2+\\&a_{08}x_4)(x_5^2+x_6^2)^2\end{aligned}\tag{30a}$$

$$\begin{aligned}\dot{x}_2=&j_{05}x_1+b_{08}x_2+j_{09}x_3+j_{08}x_4+4b_{14}x_1x_3x_4x_5x_6+\\&2x_1x_2(c_{01}x_3+j_{02}x_4+j_{01}(x_3x_6-x_4x_5)+j_{06}(x_3x_6+\\&x_4x_5))+2x_3x_4(\frac{1}{2}a_{06}x_1+c_{08}x_2+c_{02}(x_1x_6-\\&x_2x_5)+c_{06}(x_1x_6+x_2x_5))-j_{13}(x_4x_5-x_3x_6)-\\&b_{15}(x_2x_5-x_1x_6)+j_{14}(x_3x_5+x_4x_6)+j_{12}(x_1x_5+\\&x_2x_6)+b_{14}x_1(x_3^2-x_4^2)(x_5^2-x_6^2)-(b_{02}x_6+\\&2b_{01}x_3x_4+2b_{13}x_5x_6)(-3x_1^2x_2+x_2^3)+b_{02}x_5(x_1^3-\\&3x_1x_2^2)+c_{07}x_5(x_3^3-3x_3x_4^2)-c_{07}x_6(-3x_3^2x_4+x_4^3)+\\&(j_{02}x_3-c_{01}x_4+j_{06}(x_3x_5-x_4x_6)+j_{01}(x_3x_5+\\&x_4x_6))(x_1^2-x_2^2)+(c_{08}x_1-\frac{1}{2}a_{06}x_2+c_{02}(x_1x_5+\\&x_2x_6)+c_{06}(x_1x_5-x_2x_6)+2b_{14}x_2x_5x_6+b_{01}(x_1^3-\\&3x_1x_2^2))(x_3^2-x_4^2)+(-2b_{14}x_2x_3x_4+b_{13}(x_1^3-\\&3x_1x_2^2))(x_5^2-x_6^2)+(a_{03}x_1+a_{02}x_2+c_{04}x_3+c_{03}x_4+\\&2b_{05}x_2x_3x_4+j_{03}(x_3x_5+x_4x_6)+b_{07}(x_1x_5+x_2x_6)+\\&b_{03}x_1(x_3^2+x_4^2)+b_{05}x_1(x_3^2-x_4^2)+b_{04}x_1(x_5^2+\\&x_6^2))(x_1^2+x_2^2)+(c_{05}x_1+a_{06}x_2+b_{10}x_3+b_{11}x_4+\\&2a_{07}x_2x_3x_4+b_{12}(x_1x_5+x_2x_6)+c_{09}(x_3x_5+x_4x_6)+\\&a_{07}x_1(x_3^2-x_4^2)+a_{05}x_1(x_5^2+x_6^2))(x_3^2+x_4^2)+\\&(j_{04}x_1+j_{07}x_3+c_{10}x_4+2b_{09}x_2x_3x_4+j_{10}(x_1x_5+\\&x_2x_6)+j_{11}(x_3x_5+x_4x_6)+b_{09}x_1(x_3^2-x_4^2))(x_5^2+\\&x_6^2)+x_1(a_{01}(x_1^2+x_2^2)^2+a_{04}(x_3^2+x_4^2)^2)+(b_{06}x_1+\\&a_{08}x_3)(x_5^2+x_6^2)^2\end{aligned}\tag{30b}$$

$$\begin{aligned}\dot{x}_3=&h_{13}x_1-h_{14}x_2+g_{05}x_3-h_{05}x_4+4g_{06}x_1x_2x_4x_5x_6+\\&2x_1x_2(-f_{13}x_3+\frac{1}{2}d_{07}x_4+g_{02}(x_3x_5-x_4x_6)-\\&g_{10}(x_3x_5+x_4x_6))+x_3x_4(-h_{02}x_1+g_{01}x_2+\\&h_{08}(x_1x_5-x_2x_6)-h_{01}(x_1x_5+x_2x_6))+h_{12}(x_2x_5-\\&x_1x_6)+h_{10}(x_4x_5-x_3x_6)+g_{06}x_4(x_1^2-x_2^2)(x_5^2-\\&x_6^2)+h_{11}(x_1x_5+x_2x_6)+g_{08}(x_3x_5+x_4x_6)+\\&f_{01}x_5(x_4^3-3x_3^2x_4)+g_{09}x_5(-3x_1^2x_2+x_2^3)+(f_{01}x_6+\\&2f_{02}x_1x_2-2f_{09}x_5x_6)(-3x_3x_4^2+x_3^3)+g_{09}x_6(-3x_1x_2^2+\end{aligned}$$

$$
x_1^3) + (\frac{1}{2}d_{07}x_3 + f_{13}x_4 - g_{02}(x_4x_5 + x_3x_6) +
g_{10}(x_3x_6 - x_4x_5) - 2g_{06}x_3x_5x_6 + f_{02}(-3x_3^2x_4 +
x_4^3))(x_1^2 - x_2^2) + (g_{01}x_1 + h_{02}x_2 - h_{08}(x_1x_6 +
x_2x_5) + h_{01}(x_1x_6 - x_2x_5))(x_3^2 - x_4^2) + (2g_{06}x_1x_2x_3 -
f_{09}(-3x_3^2x_4 + x_4^3))(x_5^2 - x_6^2) + (f_{14}x_1 - f_{15}x_2 + d_{07}x_3 -
f_{08}x_4 - 2d_{09}x_1x_2x_3 + f_{11}(x_4x_5 - x_3x_6) + g_{07}(x_2x_5 -
x_1x_6) - f_{04}x_4(x_3^2 + x_4^2) + d_{09}x_4(x_1^2 - x_2^2) - d_{04}x_4(x_5^2 +
x_6^2))(x_1^2 + x_2^2) + (g_{03}x_1 - g_{04}x_2 + d_{02}x_3 - d_{03}x_4 -
2f_{07}x_1x_2x_3 + h_{03}(x_2x_5 - x_1x_6) + f_{06}(x_4x_5 - x_3x_6) +
f_{07}x_4(x_1^2 - x_2^2) - f_{03}x_4(x_5^2 + x_6^2))(x_3^2 + x_4^2) + (f_{12}x_1 -
h_{09}x_2 + d_{05}x_3 - h_{04}x_4 - 2f_{10}x_1x_2x_3 + h_{07}(x_2x_5 - x_1x_6) +
h_{06}(x_4x_5 - x_3x_6) + f_{10}x_4(x_1^2 - x_2^2))(x_5^2 + x_6^2) -
x_4(d_{06}(x_1^2 + x_2^2)^2 + d_{01}(x_3^2 + x_4^2)^2) - (d_{08}x_2 +
f_{05}x_4)(x_5^2 + x_6^2)^2 \quad (30c)
$$

$$
\dot{x}_4 = h_{14}x_1 + h_{13}x_2 + h_{05}x_3 + g_{05}x_4 + 4g_{06}x_1x_2x_3x_5x_6 +
2x_1x_2(\frac{1}{2}d_{07}x_3 + f_{13}x_4 + g_{02}(x_4x_5 + x_3x_6) +
g_{10}(x_3x_6 - x_4x_5)) + 2x_3x_4(g_{01}x_1 + h_{02}x_2 +
h_{08}(x_1x_6 + x_2x_5) + h_{01}(x_1x_6 - x_2x_5)) + h_{12}(x_1x_5 +
x_2x_6) + h_{10}(x_3x_5 + x_4x_6) + g_{08}(x_3x_6 - x_4x_5) +
h_{11}(x_1x_6 - x_2x_5) + g_{06}x_3(x_1^2 - x_2^2)(x_5^2 - x_6^2) +
f_{01}x_5(x_3^3 - 3x_3x_4^2) + g_{09}x_5(x_1^3 - 3x_1x_2^2) +
(-g_{09}x_6(-3x_1^2x_2 + x_2^3) - (f_{01}x_6 + 2f_{02}x_1x_2 +
2f_{09}x_5x_6)(-3x_3^2x_4 + x_4^3) + (f_{13}x_3 - \frac{1}{2}d_{07}x_4 +
g_{02}(x_3x_5 - x_4x_6) + g_{10}(x_3x_5 + x_4x_6) + 2g_{06}x_4x_5x_6 +
f_{02}(x_3^3 - 3x_3x_4^2))(x_1^2 - x_2^2) + (h_{02}x_1 - g_{01}x_2 +
h_{08}(x_1x_5 - x_2x_6) + h_{01}(x_1x_5 + x_2x_6))(x_3^2 - x_4^2) +
(-2g_{06}x_1x_2x_4 + f_{09}(x_3^3 - 3x_3x_4^2))(x_5^2 - x_6^2) +
(f_{15}x_1 + f_{14}x_2 + f_{08}x_3 + d_{07}x_4 + 2d_{09}x_1x_2x_4 + f_{11}(x_3x_5 +
x_4x_6) + g_{07}(x_1x_5 + x_2x_6) + f_{04}x_3(x_3^2 + x_4^2) +
d_{09}x_3(x_1^2 - x_2^2) + d_{04}x_3(x_5^2 + x_6^2))(x_1^2 + x_2^2) +
(g_{04}x_1 + g_{03}x_2 + d_{03}x_3 + d_{02}x_4 + 2f_{07}x_1x_2x_4 +
h_{03}(x_1x_5 + x_2x_6) + f_{06}(x_3x_5 + x_4x_6) + f_{07}x_3(x_1^2 -
x_2^2) + f_{03}x_3(x_5^2 + x_6^2))(x_3^2 + x_4^2) + (h_{09}x_1 + f_{12}x_2 +
h_{04}x_3 + d_{05}x_4 + 2f_{10}x_1x_2x_4 + h_{07}(x_1x_5 + x_2x_6) +
h_{06}(x_3x_5 + x_4x_6) + f_{10}x_3(x_1^2 - x_2^2))(x_5^2 + x_6^2) +
x_3(d_{06}(x_1^2 + x_2^2)^2 + d_{01}(x_3^2 + x_4^2)^2) + (d_{08}x_1 +
f_{05}x_3)(x_5^2 + x_6^2)^2 \quad (30d)
$$

$$
\dot{x}_5 = k_{07}x_5 - l_{08}x_6 - 2k_{08}x_1x_2 - 2k_{10}x_3x_4 + m_{11}(x_1x_3 -
x_2x_4) + m_{12}(x_1x_4 + x_2x_3) - 4(m_{06} + m_{04} +
m_{08})x_1x_2x_3x_4x_6 - 2x_5x_6(n_{02}x_1x_3 - n_{02}x_2x_4) +
(x_3x_5 - x_4x_6)(n_{05}x_1 + n_{06}x_2) + (x_4x_5 + x_3x_6)(n_{05}x_2 -
n_{06}x_1) + (x_1x_5 - x_2x_6)(m_{05}x_3 + n_{04}x_4) + (x_2x_5 +
x_1x_6)(m_{05}x_4 - n_{04}x_3) + k_{09}(x_1^2 - x_2^2) + l_{11}(x_3^2 -
x_4^2) + (m_{08} - m_{06} - m_{04})x_6(x_1^2 - x_2^2) + m_{07}x_4(x_1^3 -
3x_1x_2^2) + m_{07}x_3(x_2^3 - 3x_1^2x_2) + m_{13}x_2(x_3^3 - 3x_3x_4^2) +
m_{13}x_1(x_4^3 - 3x_3^2x_4) + (-2l_{02}x_5x_6 + 2(m_{04} - m_{08} -
m_{06})x_3x_4x_5 - 4l_{09}x_1x_2x_5)(x_1^2 - x_2^2) + (-2l_{03}x_5x_6 +
2(m_{06} - m_{08} - m_{04})x_1x_2x_5 - 4l_{10}x_3x_4x_5)(x_3^2 - x_4^2) +
(2l_{02}x_1x_2 + 2l_{03}x_3x_4 + n_{02}(x_1x_4 + x_2x_3))(x_5^2 -
x_6^2) + l_{10}x_6((x_3^2 - x_4^2)^2 - 4x_3^2x_4^2) + l_{09}x_6(x_1^2 - x_2^2)^2 -
4x_1^2x_2^2) + (k_{04}x_5 - l_{05}x_6 - m_{09}(x_1x_4 + x_2x_3) -
k_{03}x_6(x_3^2 + x_4^2) - m_{02}x_6(x_5^2 + x_6^2))(x_1^2 + x_2^2) +
(k_{06}x_5 - l_{07}x_6 - m_{10}(x_1x_4 + x_2x_3) - m_{03}x_6(x_5^2 +
x_6^2))(x_3^2 + x_4^2) + (l_{01}x_5 - n_{01}x_6 - 2l_{04}x_1x_2 -
2l_{06}x_3x_4 - n_{03}(x_1x_4 + x_2x_3) + k_{01}x_1(-x_4x_5 -
x_3x_6) + k_{01}x_2(x_3x_5 - x_4x_6) + k_{01}x_3(-x_2x_5 -
x_1x_6) + k_{01}x_4(x_1x_5 - x_2x_6))(x_5^2 + x_6^2) - x_6(k_{02}(x_1^2 +
x_2^2)^2 + k_{05}(x_3^2 + x_4^2)^2) - m_{01}x_6(x_5^2 + x_6^2)^2 \quad (30e)
$$

$$
\dot{x}_6 = l_{08}x_5 + k_{07}x_6 + 2k_{09}x_1x_2 + 2l_{11}x_3x_4 + m_{11}(x_1x_4 +
x_2x_3) + m_{12}(x_1x_3 - x_2x_4) + 4(m_{06} + m_{04} -
m_{08})x_1x_2x_3x_4x_5 - 2x_5x_6(n_{02}(x_1x_3 - x_2x_4) + 2l_{02}x_1x_2 +
2l_{03}x_3x_4) + (x_4x_5 + x_3x_6)(n_{05}x_1 + n_{06}x_2) + (x_3x_5 -
x_4x_6)(n_{06}x_1 - n_{05}x_2) + (x_2x_5 + x_1x_6)(m_{05}x_3 +
n_{04}x_4) + (x_1x_5 - x_2x_6)(n_{04}x_3 - m_{05}x_4) + k_{08}(x_1^2 -
x_2^2) + k_{10}(x_3^2 - x_4^2) + (m_{08} + m_{06} + m_{04})x_5(x_1^2 -
x_2^2)(x_3^2 - x_4^2) + l_{02}(x_1^2 - x_2^2)(x_5^2 - x_6^2) + l_{03}(x_3^2 -
x_4^2)(x_5^2 - x_6^2) + l_{04}(x_1^2 - x_2^2)(x_5^2 + x_6^2) + l_{06}(x_3^2 -
x_4^2)(x_5^2 + x_6^2) + m_{07}x_3(x_1^3 - 3x_1x_2^2) - m_{07}x_4(x_2^3 -
3x_1^2x_2) + m_{13}x_1(x_3^3 - 3x_3x_4^2) - m_{13}x_2(x_4^3 - 3x_3^2x_4) +
(2(m_{04} + m_{08} - m_{06})x_3x_4x_6 + 4l_{09}x_1x_2x_6)(x_1^2 - x_2^2) +
(2(m_{06} + m_{08} - m_{04})x_1x_2x_6 + 4l_{10}x_3x_4x_6)(x_3^2 - x_4^2) +
n_{02}(x_1x_4 + x_2x_3)(x_5^2 - x_6^2) + l_{10}x_5((x_3^2 - x_4^2)^2 -
4x_3^2x_4^2) + l_{09}x_5(x_1^2 - x_2^2)^2 - 4x_1^2x_2^2) + (l_{05}x_5 +
k_{04}x_6 + m_{09}(x_1x_3 - x_2x_4) + k_{03}x_5(x_3^2 + x_4^2) +
m_{02}x_5(x_5^2 + x_6^2))(x_1^2 + x_2^2) + (l_{07}x_5 + k_{06}x_6 +
m_{10}(x_1x_3 - x_2x_4) + m_{03}x_5(x_5^2 + x_6^2))(x_3^2 + x_4^2) +
(n_{01}x_5 + l_{01}x_6 + n_{03}(x_1x_3 - x_2x_4) + k_{01}x_1(x_3x_5 -
x_4x_6) + k_{01}x_2(x_4x_5 + x_3x_6) + k_{01}x_3(x_1x_5 - x_2x_6) +
$$

$k_{01}x_4(x_1x_6+x_2x_5))(x_5^2+x_6^2)^2+x_5(k_{02}(x_1^2+x_2^2)^2+k_{05}(x_3^2+x_4^2)^2)+m_{01}x_5(x_5^2+x_6^2)^2$ (30f)

3　数值模拟

基于方程(30),应用 Runge - Kutta 数值方法对旋转叶片系统进行数值模拟. 选取初始值 $x_1=0.13$, $x_2=0.7$, $x_3=-1.46$, $x_4=0.97$, $x_5=0.42$, $x_6=-0.28$. 以下各图中,图(a)、(b)、(c)分别表示(x_1,x_2)、(x_3,x_4)、(x_5,x_6)平面中的相图,图(d)、(e)、(f)分别表示 x_1、x_3、x_5 的波形图,图(g)、(h)、(i)分别表示 x_2、x_3、x_6 的功率谱,图(j)、(k)分别表示(x_1,x_2,x_3)、(x_4,x_5,x_6)中的三维相图.

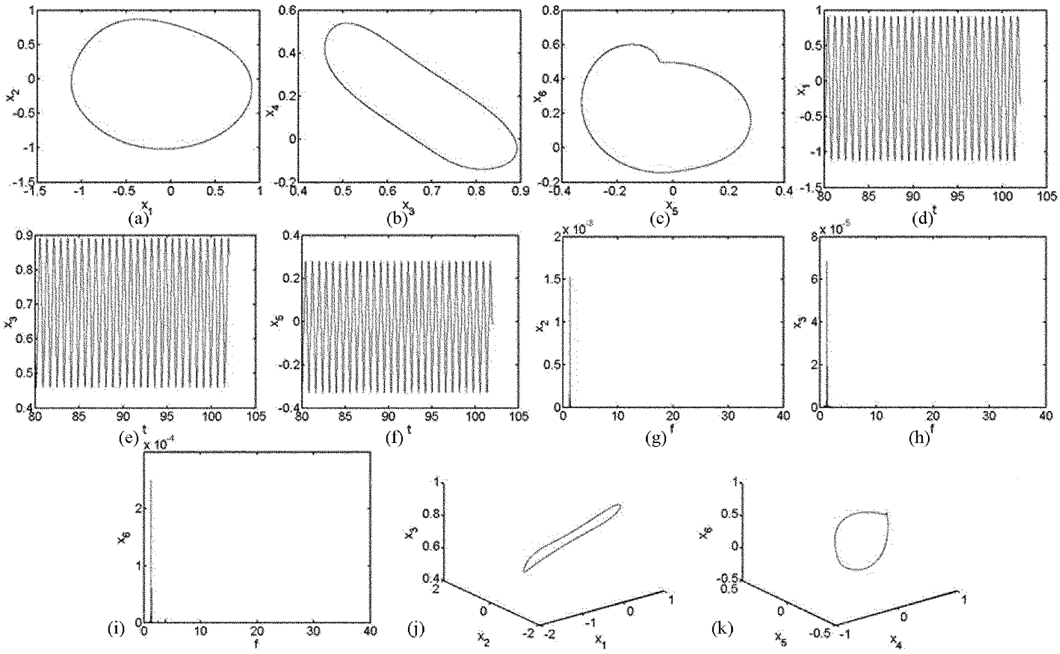

图 3　单倍周期运动

Fig. 3　period - 1 motion of the rotating blade

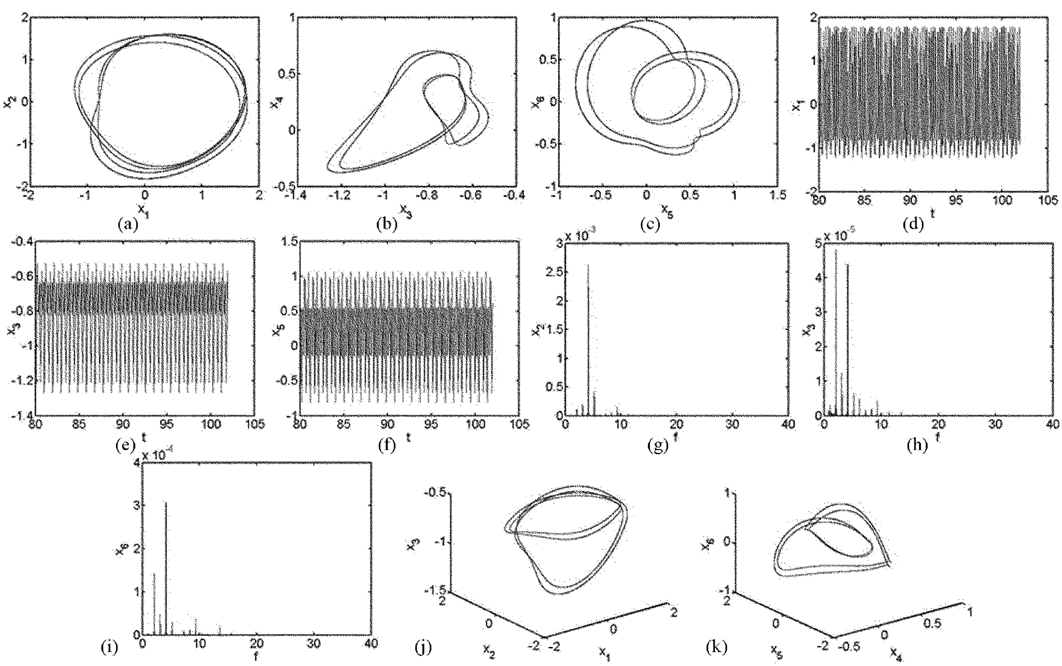

图 4　多倍周期运动

Fig. 4　multi - period motion of the rotating blade

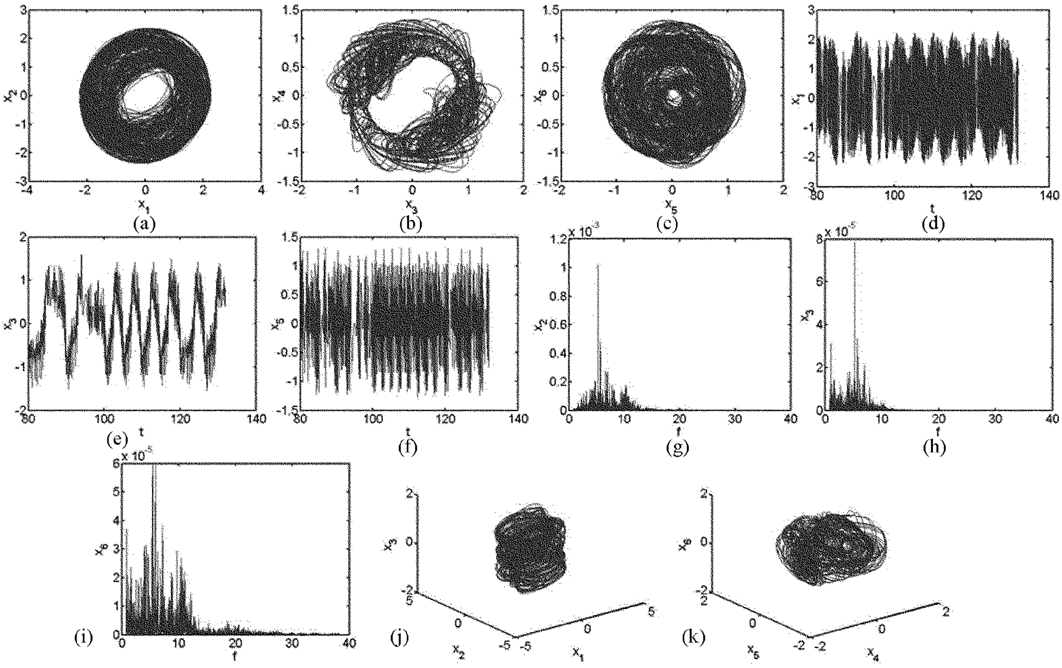

图5 混沌运动

Fig. 5 Chaotic motion of the rotating blade

为了研究气流流速对旋转叶片系统的影响，选定一组初始参数

$\mu_1=-0.61, \mu_2=-0.24, \sigma_1=1.1, \sigma_2=2.3,$
$\sigma_3=3.0,\ e_{01}=1.59,\ e_{02}=0.48,\ e_{03}=1.6,$
$e_{04}=1.05,\ e_{05}=f_1V=0.5V,\ e_{0.6}=-8.13,$
$e_{07}=1.38, e_{08}=f_2V=-0.42V, e_{09}=-1.41,$
$e_{10}=2.07, e_{11}=0.31, e_{12}=1.75, e_{13}=2.17,$
$e_{14}=-1.45,\ e_{15}=f_3V=0.24V,\ e_{16}=2.11,$
$e_{17}=1.9,\ e_{18}=f_4V=-0.3V,\ e_{19}=-1.15,$
$e_{20}=-3.89+0.1\Omega, E_{21}=-3.66, e_{22}=-0.22,$
$e_{23}=2.55,\ e_{24}=7.12,\ e_{25}=3.3,\ e_{26}=-2.4,$
$e_{27}=-2.73, e_{28}=1.73, e_{29}=-0.241\Omega, \Omega=10.$

以气流流速为控制参数，通过改变气流流速分析流速对旋转叶片振动响应的影响.

当气流流速 $V=1.6$ 时，系统发生单倍周期运动. 如图3所示.

当气流流速 $V=4.2$ 时，系统发生多倍周期运动，如图4所示.

当气流流速 $V=6.0$ 时，系统发生混沌运动，如图5所示.

4 结论

论文把航空发动机压气机叶片简化为功能梯度材料的悬臂薄壁梁，分析了气动力作用下叶片的非线性振动特性. 考虑几何大变形和气动力的影响，利用 Hamilton 原理建立了叶片的非线性动力学方程. 利用 Galerkin 方法进行一阶离散后得到了3自由度常微分方程，并用高阶多尺度法进行摄动分析得到了系统的六维平均方程. 采用数值方法研究了叶片系统在1:1:2内共振情况下的振动响应. 结果表明：气流流速的变化对系统动力学特性有重要影响，随着气流流速的增加，系统存在周期运动，多倍周期运动和混沌运动等多种复杂动力学行为.

参 考 文 献

1 Carnegie W. Vibrations of rotating cantilever blading: theoretical approaches to the frequency problem based on energy methods. *Journal of Mechanical Engineering Science*, 1959, 1: 235 ~ 240

2 Yokoyama T. Free vibration characteristics of rotating Timoshenko beam. *International Journal of Mechanical Sciences*, 1988, 30:743 ~ 755

3 Yang J B, Jiang L J, Chen D C. Dynamic modelling and control of a rotating Euler - Bernoulli beam. *Journal of Sound and Vibration*, 2004, 274:863 ~ 875

4 Cortìnez V H, Piovan M T. Vibration and buckling of composite thin-walled beams with shear deformability. *Journal of Sound and Vibration*, 2002, 258(4):70 ~ 723

5 Oh S Y, Librescu L. Thermoelastic modeling and vibration of functionally graded thin-walled rotating blades. *AIAA*

Journal, 2003, 41:2051 ~ 2060

6 Librescu L, Oh S Y, Song O. Spinning thin-walled beams made of functionally graded materials: modeling, vibration and instability. *European Journal of Mechanics A/Solids*, 2004, 23:499 ~ 515

7 Librescu L S, Oh Y, Song O. Thin-walled beams made of functionally graded materials and operating in a high temperature environment: vibration and instability. *Journal of Thermal Stresses*, 2005, 28:694 ~ 712

8 Fazelzadeh S A, Hosseini M. Aerothermoelastic behavior of supersonic rotating thin-walled beams made of functionally graded materials. Proceedings of the ASME Sixth International Symposium FSI, Aeroelasticity, FIV and Noise, 2006, ASME-PVP-ICPVT11-93624:1 ~ 12

9 Simo J, Vu-Quoc L. A three dimensional finite-strain rod model. *Part II*, *Computer Methods in Applied Mechanics and Engineering*, 1986, 58:79 ~ 116

10 Chen L W, Peng W K. Dynamic stability of rotating blades with geometric non-linearity. *Journal of Sound and Vibration*, 1995, 187 (3):421 ~ 433

11 Chandiramani N K, Librescu L. On the free-vibration of rotating composite beams using a higher-order shear formulation. *Aerospace Science and Technology*, 2002, 6:545 - 561

12 Yao M H, Chen Y P. Nonlinear vibrations of blade with varying rotating speed. *Nonlinear Dynamics*, 2012, 68: 487 ~ 504

ANALYSIS ON NONLINEAR DYNAMICS OF THE AERO-ENGINE BLADE *

Zhang Wei† Feng Zhiqing Cao Dongxing

(*College of Mechanical Engineering*, *Beijing University of Technology*, *Beijing* 100124, *China*)

Abstract The nonlinear vibration responses of rotating blades used in turbo-machinery under aerodynamic pressure loadings were investigated. Firstly, the rotating blade was settled as a rotating cantilever beam made by functional graded material. Then, the effect of geometric large deformation and aerodynamic force was taken into account in dynamic analysis. The nonlinear governing partial differential equations of the blade were established by using Hamiltonian Principle. The Galerkin method and the method of multiple scales were utilized to analyze the nonlinear dynamics of the blade. By using numerical simulation, the vibration responses of the blade under different air flow velocities were obtained. The results show that there exist complicated nonlinear behaviors in blade system such as periodic motions and chaotic motions.

Key words rotating blade, nonlinear dynamics, dynamic response, chaos

Received 25 November 2011, revised 8 December 2011.

* The project supported by Key Projects of National Science Foundation of China (10732020) and National Science Foundation of China (11072008, 10802001)

† Corresponding author E-mail: sandyzhang0@yahoo.com