第 9 卷第 4 期 2011 年 12 月
1672-6553/2011/09(4)/298-5
动 力 学 与 控 制 学 报
JOURNAL OF DYNAMICS AND CONTROL
Vol. 9 No. 4
Dec. 2011

轴向拉力对变速运动黏弹性梁参激振动稳定性的影响*

王波

(上海应用技术学院机械工程学院,上海 200235)

摘要 研究了变速轴向运动黏弹性梁参激振动受拉力扰动时在主参数共振和组合参数共振范围内的稳定性. 轴向运动梁的黏弹性本构关系引入了物质时间导数. 当参激频率接近某一阶固有频率 2 倍时将发生主参数共振;当参激频率接近某两阶固有频率之和时将发生组合参数共振. 运用多尺度法,直接求解轴向运动梁的控制方程,导出了稳定性边界方程. 最后,通过数值算例给出了变速轴向运动梁的黏阻尼和干扰拉力对失稳区域的影响结果.

关键词 轴向变速梁, 黏弹性, 拉力扰动, 参数共振, 稳定性

引 言

轴向运动梁可作为多种工程装置的力学模型[1],如动力传送带、磁带、纸带、带锯、空中缆车索道和高楼升降机缆绳等. 由于轴向速度变化或轴向拉力的影响,可能会出现大幅度的振动而受到众多学者的关注[1-14].

自 1972 年 Pansin[2] 的首次研究以来,轴向变速弹性梁的横向参数振动得到了广泛研究. Öz 等使用多尺度法研究了小弯曲刚度轴向变速弹性梁的动态稳定性[3]. Özkaya 和 Pakdemirli 使用多尺度法研究了混杂边界条件下轴向变速弹性梁的稳定边界[4]. 除了弹性梁,近年来变速轴向运动黏弹性梁也已经被广泛关注. Chen 和 Yang 首次研究了在两端带有弹簧铰支的混杂边界下轴向运动黏弹性梁的振动和稳定性[5]. Yang 和 Chen 提出了梁的积分形式黏弹性本构关系,并使用多尺度法分析了梁的振动和稳定性[6]. Ding 和 Chen 使用多尺度法分析了梁的稳定性并对其结果进行有限差分验证[7]. Chen 和 Wang 运用微分求积法研究了轴向变速黏弹性梁的运动稳定性并其验证了近似解析解[8]. Wang 和 Chen 使用 3 参数黏弹性本构关系描述梁的黏弹性材料并研究了基于此本构关系的运动稳定性问题[9]. 针对运动梁可能产生的内共振问题,冯志华和胡海岩运用多尺度方法研究了直线运动柔性梁的主参数共振与内共振联合激励[10]. 陈树辉和黄建亮运用 Galerkin 截断研究了简支边界下轴向运动体系横向非线性振动的内共振[11]. 张伟等利用 Galerkin 截断研究了前两阶固有频率之比为 1:3 时轴向运动带的内共振情况周期运动和混沌运动[12]. 张红星等用实验方法研究了粘弹性传送带的振动[13]. 不过这些研究极少涉及到轴向拉力变化对梁运动稳定性的影响. 本文研究了变速轴向运动黏弹性梁参激振动受拉力扰动时在参数共振范围内的稳定性.

1 运动方程

在无外部激励的情况下,考虑一均匀黏弹性梁,密度为 ρ,横截面积为 A,轴向拉力为 P,在支承两端间的长度为 l 上以轴向传输速度 $\gamma(t)$ 运动. 如果认为梁在平面内只有横向位移 $v(x,t)$ 的弯曲振动,t 为时间,x 为轴向坐标,由牛顿第二定律得[5]

$$\rho A(\nu_{,tt}+\dot{\gamma}\nu_{,x}+2\gamma\nu_{,xt}+\gamma^2\nu_{,xx})+M_{,xx}-P\nu_{,xx}=0 \tag{1}$$

其中,对于小挠度,梁的弯矩 $M(x,t)$ 可定义为

$$M(x,t)=EI\nu_{,xx}+\eta I(\nu_{,xxt}+\gamma\nu_{,xxx}) \tag{2}$$

这里,梁的黏弹性材料遵循取物质时间导数的 Kelvin 模型本构关系;其中 E 为弹性模量,I 为梁的横截面惯性矩,η 为黏性阻尼.

假设轴向拉力为 P 和轴向速度 γ 分别在初始拉力为 P_0 和恒定的轴向平均速度 γ_0 基础上受到微小扰动

2010-12-15 收到第 1 稿,2011-06-20 收到修改稿.
* 国家自然科学基金(10972143),上海市教育委员会重点学科建设项目资助(J51501),上海应用技术学院引进人才基金(YJ2011-26)

$$P(t) = P_0 + \varepsilon P_1 \sin\omega t \tag{3}$$

$$\gamma(t) = \gamma_0 + \varepsilon\gamma_1 \sin\omega t \tag{4}$$

其中,ω 为扰动频率;P_1 和 γ_1 分别为轴向拉力和速度的变化振幅;ε 为无量纲的小参数表示它们的变化振幅很小. 将方程(2)至(4)代入到(1)得

$$\begin{aligned}&\rho A(\nu_{,tt} + 2\gamma_0\nu_{,xt} + \gamma_0^2\nu_{,xx}) + EI\nu_{,xxxx} - P_0\nu_{,xx} = \\ &-\varepsilon\rho A\gamma_1[\omega\cos\omega t\nu_{,x} + 2\sin\omega t(\nu_{,xt} + 2\gamma_0\nu_{,xx})] - \\ &\varepsilon I\eta(\nu_{,xxxxt} + \gamma_0\nu_{,xxxx}) + \varepsilon P_1\sin\omega t\nu_{,xx} - \\ &\varepsilon^2(\rho A\gamma_1^2\sin\omega t\nu_{,xx} + I\eta\gamma_1\nu_{,xxxxx})\sin\omega t\end{aligned} \tag{5}$$

2　多尺度分析

引入无量纲变量和参数

$$\nu\leftrightarrow\frac{\nu}{l}, x\leftrightarrow\frac{x}{l}, t\leftrightarrow t\sqrt{\frac{P_0}{\rho Al^2}}, \nu_f^2 = \frac{EI}{P_0 l^2},$$

$$\alpha = \frac{\eta I}{\varepsilon l^3\sqrt{\rho AP_0}}, c_0 = \gamma_0\sqrt{\frac{\rho A}{P_0}}, c_1 = \gamma_1\sqrt{\frac{\rho A}{\varepsilon^2 P_0}},$$

$$f = \frac{P_1}{\varepsilon P_0}, \omega\leftrightarrow\omega\sqrt{\frac{\rho Al^2}{P_0}} \tag{6}$$

其中, ε 为无量纲小参数,不仅表示小量而且在将得出近似解的过程中作为一根拐杖或者是一种记帐的手段. 由方程(6)可导出控制方程(5)无量纲形式

$$\begin{aligned}&M\nu_{,tt} + G\nu_{,t} + K\nu = -\varepsilon c_1[\omega\nu_{,x}\cos\omega t + 2(\nu_{,xt} + \\ &c_0\nu_{,xx})\sin\omega t] - \varepsilon\alpha(\nu_{,xxxxt} + c_0\nu_{,xxxxx}) + \\ &\varepsilon f\nu_{,xx}\cos\omega t - \varepsilon^2 c_1\sin\omega t(c_1\sin\omega t\nu_{,xx} + \alpha\nu_{,xxxxx})\end{aligned} \tag{7}$$

其中,质量算子 M、陀螺算子 G,以及刚度算子 K 被定义为

$$M = I, G = 2c_0\frac{\partial}{\partial x}, K = (c_0^2 - 1)\frac{\partial^2}{\partial x^2} + \nu_f^2\frac{\partial^4}{\partial x^4} \tag{8}$$

使用多尺度法,将方程(8)的近似解表示成下面不同尺度时间的函数,这里仅讨论一阶近似

$$\nu(x,t;\varepsilon) = \nu_0(x,t,T) + \varepsilon\nu_1(x,t,T) \tag{9}$$

其中,$T = \varepsilon t$ 为慢时间尺度,用于描述由于黏弹性和可能发生的共振引起的振幅和相位的模态. 将近似解(9)代入方程(5),并令 ε^0 和 ε 的系数分别等于零,导出

$$M\nu_{0,tt} + G\nu_{0,t} + K\nu_0 = 0 \tag{10}$$

$$\begin{aligned}&M\nu_{1,tt} + G\nu_{1,t} + K\nu_1 = -2c_0\nu_{0,xT} + f\nu_{0,xx}\cos\omega t - \\ &c_1\omega\nu_{0,x}\cos\omega t - \alpha(\nu_{0,xxxxt} + c_0\nu_{0,xxxxx}) - 2\nu_{0,tT} - \\ &2c_1\sin\omega t(\nu_{0,xt} - 2c_0c_1\nu_{0,xx})\end{aligned} \tag{11}$$

方程(10)的解为[3]

$$\begin{aligned}&\nu_0(x,t,T) = \sum_{k=0,1,\cdots}^{\infty}[\phi_k(x)A_k(T)e^{i\omega_k t} + \\ &\bar{\phi}_k(x)\bar{A}_k(T)e^{-i\omega_k t}]\end{aligned} \tag{12}$$

其中,$\omega_k(k = m,\ n)$为线性方程(10)的第 k 阶固有频率.

2.1　组合共振

如果扰动频率 ω 接近线性系统任意两个固有频率之和,将会发生组合共振. 为了定量描述 ω 和 $\omega_m + \omega_n$ 的接近程度,我们引进一个由

$$\omega = \omega_m + \omega_n + \varepsilon\sigma \tag{13}$$

定义的解谐参数 $\sigma = O(1)$. 假设方程(11)解的形式为

$$\begin{aligned}&\nu_0(x,t,T) = \phi_m(x)A_m(T)e^{i\omega_m t} + \\ &\phi_n(x)A_n(T)e^{i\omega_n t} + cc\end{aligned} \tag{14}$$

其中,cc 表示前面所有项的共轭;将方程(13)和(14)代入(11),得到

$$\begin{aligned}&M\nu_{1,tt} + G\nu_{1,t} + K\nu_1 = -\{2\dot{A}_m(i\omega_m\phi_m + c_0\phi'_m) - \\ &\frac{1}{2}[c_1(\omega_n - \omega_m)\bar{\phi}'_n + i(2c_0c_1 - \\ &f)\bar{\phi}''_n]\bar{A}_n e^{i\sigma t}\}e^{i\omega_m t} - \alpha A_m[i\omega_m\phi_m^{(4)} + \\ &c_0\phi_m^{(5)}]e^{i\omega_m t} - \{2\dot{A}_n(i\omega_n\phi_n + c_0\phi'_n) - \frac{1}{2}[c_1(\omega_m - \\ &\omega_n)\bar{\phi}'_m + i(2c_0c_1 - f)\bar{\phi}''_m]\bar{A}_m e^{i\sigma t}\}e^{i\omega_n t} - \\ &\alpha A_n[i\omega_n\phi_n^{(4)} + c_0\phi_n^{(5)}]e^{i\omega_n t}\end{aligned} \tag{15}$$

其中,方程(15)必须满足如下可解性条件才能得到有界解[14].

$$\begin{aligned}&\langle -2\dot{A}_m(i\omega_m\phi_m + c_0\phi'_m) + \frac{1}{2}[c_1(\omega_n - \\ &\omega_m)\bar{\phi}'_n + i(2c_0c_1 - f)\bar{\phi}''_n]\bar{A}_n e^{i\sigma t} - \\ &\alpha A_m[i\omega_m\phi_m^{(4)} + c_0\phi_m^{(5)}]e^{i\omega_m t}\rangle\end{aligned} \tag{16}$$

$$\begin{aligned}&\langle -2\dot{A}_n(i\omega_n\phi_n + c_0\phi'_n) - \frac{1}{2}[c_1(\omega_m - \\ &\omega_n)\bar{\phi}'_m + i(2c_0c_1 - f)\bar{\phi}''_m]\bar{A}_m e^{i\sigma t} - \\ &\alpha A_n[i\omega_n\phi_n^{(4)} + c_0\phi_n^{(5)}]e^{i\omega_n t}\rangle\end{aligned} \tag{17}$$

这里,在[0,1]的范围内,函数 $g_1(x)$ 和 $g_2(x)$ 的内积被定义为

$$\langle g_1, g_2\rangle = \int_0^1 g_1(x)\bar{g}_2(x)\mathrm{d}x \tag{18}$$

根据可解性条件(16)和(17),导出

$$\dot{A}_m + \alpha\mu_m A_m + (c_1\chi_m + f\kappa_m)\bar{A}_n e^{i\sigma t} = 0 \tag{19}$$

$$\dot{A}_n + \alpha\mu_n A_n + (c_1\chi_n + f\kappa_n)\bar{A}_m e^{i\sigma t} = 0 \tag{20}$$

其中,

$$\mu = \frac{1}{2}\frac{c_0\int_0^1 \bar{\phi}_k\phi_k^{(5)}\mathrm{d}x + i\omega_k\int_0^1 \bar{\phi}_k\phi_k^{(4)}\mathrm{d}x}{c_0\int_0^1 \bar{\phi}_k\phi'_k\mathrm{d}x + i\omega_k\int_0^1 \bar{\phi}_k\phi_k\mathrm{d}x},\ (k=m,n) \tag{21}$$

$$\chi_m = -\frac{1}{4}\frac{(\omega_n-\omega_m)\int_0^1 \bar{\phi}_m\bar{\phi}'_n\mathrm{d}x + 2ic_0\int_0^1 \bar{\phi}_m\bar{\phi}''_n\mathrm{d}x}{c_0\int_0^1 \bar{\phi}_m\bar{\phi}'_m\mathrm{d}x + i\omega_m\int_0^1 \bar{\phi}_m\bar{\phi}''_m\mathrm{d}x} \tag{22}$$

$$\kappa_m = -\frac{1}{4}\frac{\int_0^1 \bar{\phi}_m\bar{\phi}''_n\mathrm{d}x}{c_0\int_0^1 \bar{\phi}_m\phi'_m\mathrm{d}x + i\omega_m\int_0^1 \bar{\phi}_m\phi_m\mathrm{d}x} \tag{23}$$

$$\chi_n = -\frac{1}{4}\frac{(\omega_m-\omega_n)\int_0^1 \bar{\phi}_n\bar{\phi}'_m\mathrm{d}x + 2ic_0\int_0^1 \bar{\phi}_n\bar{\phi}''_m\mathrm{d}x}{c_0\int_0^1 \bar{\phi}_n\bar{\phi}'_n\mathrm{d}x + i\omega_n\int_0^1 \bar{\phi}_n\bar{\phi}''_n\mathrm{d}x} \tag{24}$$

$$\kappa_n = -\frac{1}{4}\frac{\int_0^1 \bar{\phi}_n\bar{\phi}''_m\mathrm{d}x}{c_0\int_0^1 \bar{\phi}_n\phi'_n\mathrm{d}x + i\omega_n\int_0^1 \bar{\phi}_n\phi_n\mathrm{d}x} \tag{25}$$

易知,系数$\chi_m,\chi_n,\kappa_m,\kappa_n,\mu_k(k=m,n)$与线性系统(10)的固有频率和模态函数有关,与轴向运动平均速度c_0和梁的刚度v_f有关,而与黏性阻尼α,扰动速度幅度c_1及扰动拉力幅度无关. 基于方程(19)和(20),根据Chen和Yang的方法[5]可以得到组合参数共振的稳定性失稳边界的解析表达式

$$\left[1+\frac{(\mu_m-\mu_n)^2}{(\mu_m+\mu_n)^2}\right]\sigma = 4(c_1\chi_m+f\kappa_m)(c_1\bar{\chi}_n+f\bar{\kappa}_n)-4\alpha^2\mu_m\mu_n \tag{26}$$

2.2 主共振

如果扰动频率ω接近线性系统(10)的任何一阶固有频率的2倍,那么将会发生主参数共振. 这里ω表示为

$$\omega = 2\omega_k+\varepsilon\sigma \tag{27}$$

让方程(26)中的$m=n=k$,则第k阶主参数共振的稳定性失稳边界的解析表达式为

$$\sigma^2+4\alpha^2\mu_k^2 = 4(c_1\chi_k+f\kappa_k)(c_1\bar{\chi}_k+f\bar{\kappa}_k) \tag{28}$$

其中,μ_k的表达式见(21);将$m=n=k$代入式(22)至(25)得到χ_k和κ_k表达式

$$\chi_k = -\frac{1}{2}\frac{ic_0\int_0^1 \bar{\phi}_k\bar{\phi}''_k\mathrm{d}x}{c_0\int_0^1 \bar{\phi}_k\phi'_k\mathrm{d}x + i\omega_k\int_0^1 \bar{\phi}_k\phi_k\mathrm{d}x} \tag{29}$$

$$\kappa_k = -\frac{1}{4}\frac{\int_0^1 \bar{\phi}_k\bar{\phi}''_k\mathrm{d}x}{c_0\int_0^1 \bar{\phi}_k\phi'_k\mathrm{d}x + i\omega_n\int_0^1 \bar{\phi}_k\phi_k\mathrm{d}x} \tag{30}$$

考虑梁的两端均受到相同带有扭转弹簧的套筒铰支约束,将其边界条件表示为

$$\nu(0,t)=0,\quad \nu_{,xx}(0,t)-k\nu_{,x}(0,t)=0$$
$$\nu(1,t)=0,\quad \nu_{,xx}(1,t)+k\nu_{,x}(1,t)=0 \tag{31}$$

式中,k为扭转弹簧的刚度.

3 数值算例

基于组合共振的稳定性边界方程(26)和主共振的稳定性边界方程(28),考虑轴向运动黏弹性梁的参数$v_f=0.8$和$c=2.0$,则对于边界条件(31)的情况,并让扭转弹簧的刚度$k=2$,可求得线性系统的前两阶固有频率$\omega_1=8.157$和$\omega_2=32.944$,模态函数$\phi_k(x)$ $(k=m,\ n)$的表达式见[5,7]. 在$\sigma-c_1$平面内,失稳区域能被确定,如图1至图6所示,其中横坐标为σ,纵坐标为c_1.

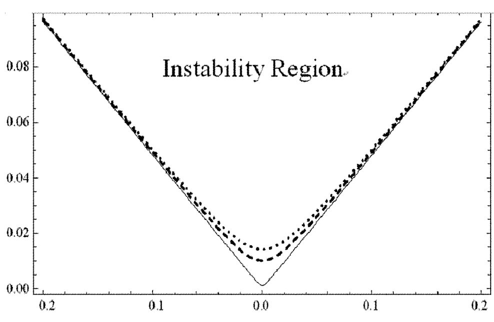

图1 第1阶主共振拉力影响

Fig. 1 Effect of tension for 1st principal resonance

图1为轴向拉力对第1阶主共振的影响且黏性阻尼$\alpha=0.0001$和拉力$f=0$(点线), 0.04(虚线), 0.057(实线). 图2为轴向黏性阻尼对第1阶主共振的影响且轴向拉力$f=0.0001$和黏性阻尼$\alpha=0.0001$(点线),0.0002(虚线),0.0003(实线). 可以看出:失稳区域会随着拉力增大而增大,随着黏性阻尼的增大失稳区域减小.

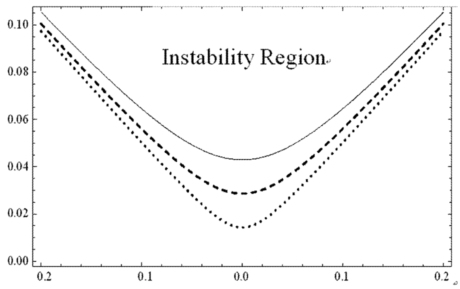

图 2　第 1 阶主共振黏性阻尼影响

Fig. 2　Effect of viscosity for 1st principal resonance

图 3 为轴向拉力对第 2 阶主共振的影响且黏性阻尼 $\alpha = 0.0001$ 和拉力 $f = 0$ (点线)，0.3 (虚线)，0.46 (实线). 图 4 为轴向黏性阻尼对第 2 阶主共振的影响且轴向拉力 $f = 0.1$ 和黏性阻尼 $\alpha = 0.0001$ (点线)，0.0002 (虚线)，0.0003 (实线). 可以看出：失稳区域会随着拉力增大而增大，随着黏性阻尼的增大失稳区域减小.

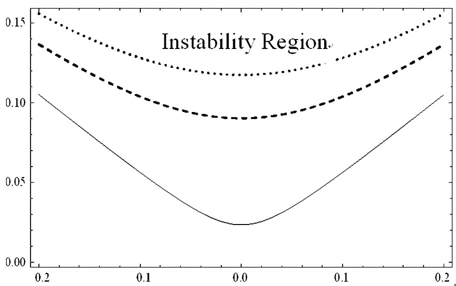

图 3　第 2 阶主共振拉力影响

Fig. 3　Effect of tension for 2nd principal resonance

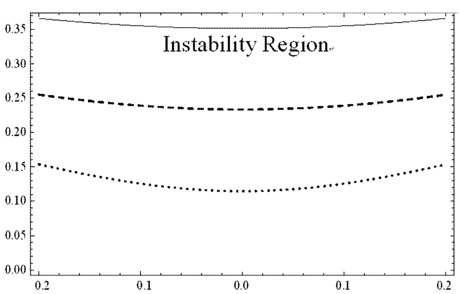

图 4　第 2 阶主共振黏性阻尼影响

Fig. 4　Effect of viscosity for 2nd principal resonance

图 5 为轴向拉力对组合共振的影响且黏性阻尼 $\alpha = 0.0001$ 和拉力 $f = 0.1$ (点线)，0.3 (虚线)，0.4 (实线). 图 6 为轴向黏性阻尼对组合共振的影响且轴向拉力 $f = 0.1$ 和黏性阻尼 $\alpha = 0.0001$ (点线)，0.0002 (虚线)，0.0003 (实线). 可以看出：失稳区域会随着拉力增大而增大，随着黏性阻尼的增大失稳区域减小.

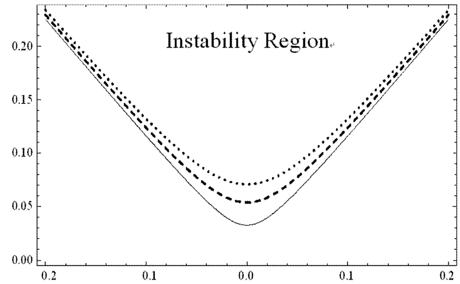

图 5　组合共振拉力影响

Fig. 5　Effect of tension for summation resonance

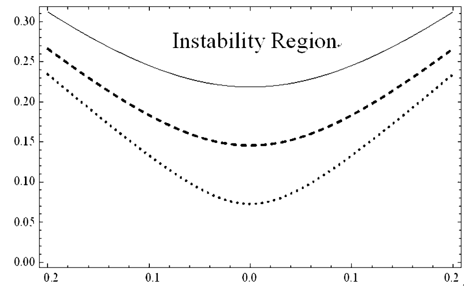

图 6　组合共振黏性阻尼影响

Fig. 6　Effect of viscosity for summation resonance

4　结论

本文使用多尺度法近似分析了受拉力扰动影响的轴向变速黏弹性梁的稳定性. 通过数值例子给出这样的结论：不论当主共振还是组合共振发生时，失稳区域会都会随着轴向拉力的增大而增大，随着黏性阻尼的增大失稳区域减小. 所以增大轴向拉力会更容易使参激振动失去稳定性. 而黏性阻尼可以起到抑制轴向运动黏弹性梁振动的作用.

参　考　文　献

1　Wickert J A, Mote C D Jr. Current research on the vibration and stability of axially moving materials. *Shock and Vibration Digest*, 1988, 20(5): 3 ~ 13

2　Pasin F. Ueber die Stabilitat der Beigeschwingungen von in Laengsrichtung periodisch hin und herbewegten Staben. *Ingenieur - Archiv*, 1972, 41: 387 ~ 393

3　Öz H R, Pakdemirli M, Ozkaya E. Transition behavior from string to beam for an axially accelerating material. *Journal of Sound and Vibration*, 1998, 215: 571 ~ 576

4　Özkaya E, Pakdemirli M. Vibrations of an axially accelerating beam with small flexural stiffness. *Journal of Sound and Vibration*, 2000, 234: 521 ~ 535

5　Chen L Q, Yang X D. Vibration and stability of an axially moving viscoelastic beam with hybrid supports. *European*

Journal of Mechanics A/Solids, 2006, 25(6): 996 ~ 1008

6 Yang X D, Chen L Q. Stability in parametric resonance of axially accelerating beams constituted by Boltzmann's superposition principle. *Journal of Sound and Vibration*, 2006, 289: 54 ~ 65

7 Ding H. Stability of axially accelerating viscoelastic beams: multi - scale analysis with numerical confirmations. *European Journal of Mechanics A/Solid*, 2008, 27: 1108 ~ 1120

8 Chen L Q, Wang B. Stability of axially accelerating viscoelastic beams: asymptotic perturbation analysis and differential quadrature validation. *European Journal of Mechanics A/Solids*, 2009, 28: 786 ~ 791

9 Wang B, Chen L Q. Asymptotic stability analysis with numerical confirmation of an axially accelerating beam constituted by the standard linear solid viscoelastic model. *Journal of Sound and Vibration*, 2009, 328: 456 ~ 466

10 冯志华,胡海岩.直线运动柔性梁非线性动力学—主参数共振与内共振联合激励. 振动工程学报, 2004, 17(2): 126 ~ 131 (Feng Z H, Hu H Y. Nonlinear dynamics of flexible beams undergoing a large linear motion of basement: combinational parametric and internal resonances. *Journal of Vibration Engineering*, 2004, 17(3): 126 ~ 131 (in Chinese))

11 陈树辉,黄建亮.轴向运动梁非线性振动内共振研究. 力学学报, 2005, 37: 57 – 63 (Chen S H, Huang J L. On internal resonance of nonlinear vibration of axially moving beams. *Acta Mechanica Sinica*, 2005, 37(1): 57 ~ 63(in Chinese))

12 张伟,温洪波,姚明辉. 黏弹性传动带 1:3 内共振时的周期和混沌运动. 力学学报, 2004, 36: 443 ~ 454 (Chen S H, Huang J L. On internal resonance of nonlinear vibration of axially moving beams. *Acta Mechanica Sinica*, 2005, 37(1): 57 ~ 63(in Chinese))

13 张红星,张伟,姚明辉,刘彦琦. 粘弹性传动带非线性振动实验研究. 动力学与控制学报, 2007, 5 (4): 361 ~ 364 (Zhang H X, Zhang W, Yao M H, Liu Y Q. Experimental research on nonlinear vibration of an axially moving viscoelastic belt. *Journal of Dynamics and Control*, 2007, 5(4): 361 ~ 364(in Chinese))

14 Chen L Q, Zu J W. Solvability condition in multi – scale analysis of gyroscopic continua. *Journal of Sound and Vibration*, 2008, 309: 338 ~ 342

STABILITY OF PARAMETRIC VIBRATION EXCITED OF AN AXIALLY ACCELERATING VISCOELASTIC BEAM SUBJECTED TO AXIAL DISTURBING TENSION *

Wang Bo

(*School of Mechanical Engineering, Shanghai Institute of Technology, Shanghai* 200235, *China*)

Abstract The stability of axially accelerating viscoelastic beam was investigated when the principal parametric resonanceor combination parametric resonance was excited. The material time derivative was used in the viscoelastic constitutive relation. When the axial speed variation frequency approaches the sum of two arbitrary natural frequencies or the twice of arbitrary natural frequency, the summation or principal parametric resonance may occur. The method of multiple scales was employed to directly solve the governing equation of axially accelerating viscoelastic beam. Analytical expressions of the instability boundary were obtained for summation and principal parametric resonance. Finally, numerical examples show the effects of viscosity and axial disturbing tension for instability region.

Key words axially accelerating beam, viscoelasticity, disturbing tension, parameter resonance, stability

Received 15 December 2010, revised 20 June 2011.

* The project supported by the National Natural Science Foundation of China (10972143), Shanghai Leading Academic Discipline Project (J51501), and the Start Foundation for Introducing Talent of Shanghai Institute of Technology (YJ2011 – 26)