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复合边界条件下功能梯度板 1:1 内共振的周期与混沌运动*

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摘要 以两对边简支另两对边自由的功能梯度材料板为研究对象,首先建立了考虑材料物性参数与温度相关的、在热/机械载荷共同作用下的几何非线性动力学方程,采用渐进摄动法对系统在 1:1 内共振 - 主参数共振 - 1/2 亚谐共振情况下的非线性动力学行为进行了摄动分析,得到系统的四自由度平均方程,并对平均方程进行数值计算,分析外激励对系统非线性动力学行为的影响,发现在一定条件下通过改变外激励可以改变系统的运动形式,产生混沌运动. 另外,第二阶模态的幅值远比第一阶模态的幅值大,这应该是两阶模态耦合产生内共振的结果,因此,研究该类结构的非线性动力学行为时不应该只考虑一阶模态,而应考虑到前两阶甚至更多阶模态的相互作用,以便于更好地利用或控制其运动形式.

关键词 功能梯度材料板, 复合边界条件, 混沌运动, 内共振

引 言

功能梯度复合材料(Functionally Graded Materials, 简称 FGMs)是指通过将不同性能的两种或两种以上的材料按一定的设计规律组合起来,使其结构的两侧分别由性能各异的材料组成,中间部分的材料组分则呈梯度连续变化的一种非均匀复合材料. 目前,功能梯度材料在大型空间站、航天飞行器等领域得到逐步的应用[1]. 由于功能梯度材料经常使用在高温环境中,与均质材料相比较,其在热机环境中的动力学行为也更加复杂,因此关于这类结构的宏观动力学行为研究具有重要的理论和工程应用价值.

近来,国内外学者对功能梯度材料矩形板振动和动力响应的研究逐渐增多,例如 Cheng 等[2] 用 Reddy 高阶剪切变形理论研究了具有 Winkler Pasternak 弹性基础的四边简支功能梯度材料板的稳态振动,Ng 等[3] 研究了受面内谐激励作用下的参数共振问题,Yang 等[4] 研究了具有初始应力功能梯度材料板的动力响应问题,对比分析了是否具有弹性基的影响,Qian 等[5] 用高阶变形理论和 Petrov - Galerkin 法分析了功能梯度材料板的弯曲变形、自由振动和强迫振动. Senthil 等[6] 给出了功能梯度材料简支板自由振动和强迫振动三维精确解. Hao 等[7,8] 对受热机载荷共同作用下的、功能梯度材料简支板的非线性动力学行为进行了分析. 国内外多数学者的研究大多集中在四边简支边界条件下功能梯度板结构的自由振动频率问题、瞬态响应问题等. 然而很少有对于具有两对边简支两对边自由(SFSF)边界条件的、功能梯度材料板的非线性动力行为进行分析研究.

本文以两对边简支另两对边自由的功能梯度材料板为研究对象,对前期建立的、考虑材料物性参数与温度相关的、在热/机械载荷共同作用下的几何非线性动力学

方程,采用渐进摄动法对系统在 1:1 内共振 - 主参数共振 - 1/2 亚谐共振情况下的非线性动力学行为进行了摄动分析,得到系统的四自由度平均方程,并对平均方程进行数值计算.

1 动力学模型和方程

1.1 热物参数

假设功能梯度材料矩形板的上表面为陶瓷,下表面为金属. 考虑金属体积含量沿厚度方向按幂律变化,表达式如下

$$V_m(z) = \left(\frac{2z+h}{2h}\right)^N \tag{1}$$

陶瓷材料的体积含量则为

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$$V_c(z)=1-V_m=1-\left(\frac{2z+h}{2h}\right)^N \tag{2}$$

上两式中,h 表示板的厚度,Z 为厚度方向的坐标,下标‘c’、‘m’分别代表陶瓷材料和金属材料,N 为金属的体积分数指数($0\leqslant N\leqslant\infty$),它表征了组分材料的体积分布规律,$N=0$ 时退化为均匀的各向同性材料.

根据(1)与(2)式可以确定功能梯度材料板内任一点处的等效热物参数如弹性模量、热膨胀系数、密度、热传导系数分别表示为

$$E=(E_m-E_c)V_m+E_c \tag{3a}$$

$$\alpha=(\alpha_m-\alpha_c)V_m+\alpha_c \tag{3b}$$

$$\rho=(\rho_m-\rho_c)V_m+\rho_c \tag{3c}$$

$$\kappa=(\kappa_m-\kappa_c)V_m+\kappa_c \tag{3d}$$

1.2 动力学方程的建立

如图 1 所示为功能梯度材料矩形板的力学模型,直角坐标系位于功能梯度材料矩形板的中面,板沿 x、y、z 方向的几何尺寸分别为 a、b、h. 此功能梯度材料板受到横向载荷 $F_0+F\cos\Omega_1 t$ 与 y 方向面内简谐动载荷 $p_1=-(p_{10}-p_{11}\cos(\Omega_2 t))$ 的共同作用,同时还考虑温度场沿板厚度方向变化的热载荷以及横向位移方向上线性阻尼的作用.

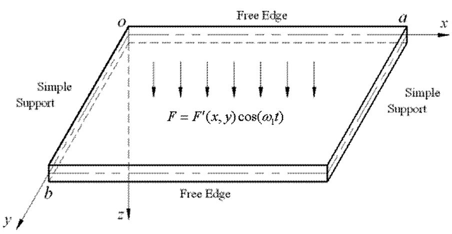

图 1 系统研究对象模型示意图

Fig. 1 The model of a FGMs plate and the coordinate system

根据薄板理论和 von Karman 非线性几何关系,利用 Hamilton 原理可以得到由位移形式表示的功能梯度材料矩形板的非线性动力学偏微分方程为

$$A_{11}\frac{\partial^2 u_0}{\partial x^2}+A_{66}\frac{\partial^2 v_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}+A_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}+(A_{12}+A_{66})\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x\partial y}+A_{66}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}-B_{11}\frac{\partial^3 w_0}{\partial x^3}-(B_{12}+2B_{66})\frac{\partial^3 w_0}{\partial x\partial y^2}=I_0\ddot{u}_0-I_1\frac{\partial\ddot{w}_0}{\partial x} \tag{4a}$$

$$A_{66}\frac{\partial^2 v_0}{\partial x^2}+A_{22}\frac{\partial^2 v_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}+A_{66}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x^2}+(A_{12}+A_{66})\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}+A_{22}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}-B_{11}\frac{\partial^3 w_0}{\partial x^3}-(B_{12}+2B_{66})\frac{\partial^3 w_0}{\partial x\partial y^2}=I_0\ddot{v}_0-I_1\frac{\partial\ddot{w}_0}{\partial y} \tag{4b}$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3}+(B_{12}+2B_{66})\frac{\partial^3 u_0}{\partial x\partial y^2}+B_{22}\frac{\partial^3 v_0}{\partial y^3}+(B_{12}+2B_{66})\frac{\partial^3 v_0}{\partial x^2\partial y}+A_{11}\frac{\partial u_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}+A_{12}\frac{\partial u_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}\frac{\partial w_0}{\partial y}+A_{22}\frac{\partial^2 v_0}{\partial y^2}\frac{\partial w_0}{\partial y}+A_{11}\frac{\partial^2 u_0}{\partial x^2}\frac{\partial w_0}{\partial x}+2A_{66}\frac{\partial u_0}{\partial y}\frac{\partial^2 w_0}{\partial x\partial y}+A_{66}\frac{\partial^2 u_0}{\partial y^2}\frac{\partial w_0}{\partial x}+(A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x\partial y}\frac{\partial w_0}{\partial x}+A_{12}\frac{\partial v_0}{\partial y}\frac{\partial^2 w_0}{\partial x^2}-D_{11}\frac{\partial^4 w_0}{\partial x^4}+A_{22}\frac{\partial v_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}+2A_{66}\frac{\partial v_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}+A_{66}\frac{\partial^2 v_0}{\partial x^2}\frac{\partial w_0}{\partial y}-D_{22}\frac{\partial^4 w_0}{\partial y^4}-(4D_{66}+2D_{12})\frac{\partial^4 w_0}{\partial x^2\partial y^2}+2(B_{12}-B_{66})\frac{\partial^2 w_0}{\partial y\partial x}\frac{\partial^2 w_0}{\partial y\partial x}+2(B_{66}-B_{12})\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}+2(A_{12}+2A_{66})\frac{\partial^2 w_0}{\partial y\partial x}\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}+(\frac{1}{2}A_{12}+A_{66})\frac{\partial^2 w_0}{\partial x^2}\frac{\partial w_0}{\partial y}\frac{\partial w_0}{\partial y}-\frac{2}{3}A_{12}\frac{\partial w_0}{\partial y}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}+(\frac{1}{2}A_{12}+A_{66})\frac{\partial^2 w_0}{\partial y^2}\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial x}+\frac{2}{3}A_{11}\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}+N_{xx}^T\frac{\partial^2 w}{\partial x^2}+(N_{yy}^T-p)\frac{\partial^2 w}{\partial y^2}+F\cos(\Omega t)-\gamma\dot{w}_0=I_0\ddot{w}_0-I_2\ddot{w}_{xx}+I_1(\ddot{u}_x+\ddot{v}_y) \tag{4c}$$

这里,γ 为板受到的阻尼系数薄膜刚度 A_{ij}、耦合刚度 B_{ij}、弯曲刚度 D_{ij} 由下式计算

$$(A_{ij},B_{ij},D_{ij})=\int_{-h/2}^{h/2}Q_{ij}(1,z,z^2)\,\mathrm{d}z,\quad(i,j=1,2,6) \tag{5a}$$

$$A_{ij}=\int_{-h/2}^{h/2}Q_{ij}\,\mathrm{d}z,\ (i,j=4,5) \tag{5b}$$

功能梯度材料板的各阶广义惯量为

$$I_i=\int_{-h/2}^{h/2}z^i\rho\,\mathrm{d}z,\ (i=0,1,2) \tag{6}$$

其各种弹性参数分别为

$$Q_{11}=Q_{22}=\frac{E}{1-v^2},Q_{12}=Q_{21}=\frac{vE}{1-v^2},Q_{66}=\frac{E}{2(1-v)} \quad (7)$$

热膨胀系数为

$$\alpha_{xx}=\alpha_{yy}=\alpha,\alpha_{xy}=0 \quad (8)$$

为了便于分析,引入如下无量纲量

$$\bar{u}=\frac{u_0}{a},\bar{v}=\frac{v_0}{b},\bar{w}=\frac{w_0}{h},\bar{N}_{yy}^T=\frac{1-v^2}{EL},$$

$$\bar{F}=\frac{(ab)^{7/2}}{\pi^4 Eh^7}F,\bar{N}_{xx}^T=\frac{1-v^2}{EL},\bar{\gamma}=\frac{(ab)^2}{\pi^2h^4}\left(\frac{1}{\rho E}\right)^{1/2}\gamma,$$

$$\bar{P}_0=\frac{b^2}{Eh^3}p_0,\bar{P}_1=\frac{b^2}{Eh^3}p_1,\bar{t}=\pi^2\left(\frac{E}{ab\rho}\right)^{1/2},$$

$$\bar{\Omega}_i=\frac{1}{\pi}\left(\frac{ab\rho}{E}\right)^{1/2}\Omega_i(i=1,2),\bar{A}_{ij}=\frac{(ab)^{1/2}}{Eh^2}A_{ij},$$

$$\bar{B}_{ij}=\frac{(ab)^{1/2}}{Eh^3}B_{ij},\bar{D}_{ij}=\frac{(ab)^{1/2}}{Eh^4}D_{ij},$$

$$\bar{I}_i=\frac{1}{(ab)^{(i+1)/2}\rho}I_i \quad (9)$$

在下面的研究中为了简便将无量纲变量中的横线去掉. 主要考虑功能梯度材料板的横向振动,取满足位移和力边界条件的二阶模态进行 Galerkin 截断,这样横向位移可以表示如下[9]

$$w_0=w_1((\xi_{11}+\xi_{12}y)\cosh(\lambda_1 y)+(\xi_{13}+\xi_{14}y)\sinh(\lambda_1 y)+w_{1p})\sin\left(\frac{\pi x}{a}\right)+w_2((\xi_{31}+\xi_{32}y)\cosh(\lambda_3 y)+(\xi_{33}+\xi_{34}y)\sinh(\lambda_3 y)+w_{3p})\sin\left(\frac{3\pi x}{a}\right) \quad (10)$$

这里 w_1 和 w_2 分别为两阶模态的幅值,$w_{1p}=\frac{4F_0a^4}{D_{11}\pi^5}$ 和 $w_{3p}=\frac{4F_0a^4}{3^5D_{11}\pi^5}$ 为只有静态横向载荷作用时的特解,其中为静态横向载荷部分,系数 ξ_{11}、ξ_{12}、ξ_{13}、ξ_{14}、ξ_{31}、ξ_{32}、ξ_{33} 和 ξ_{34} 由自由边的边界界条件决定.

假设横向载荷只沿 x 方向变化,将其展开成单三角级数形式

$$F(x,y,t)=F_1(t)\sin\frac{\pi x}{a}+F_2(t)\sin\frac{3\pi x}{a} \quad (11)$$

这里 F_1 和 F_2 表示与两个激励幅值.

和文献[10]一样, 忽略掉方程式(4a)和(4b)中所有惯性项,并将(10)和(11)式代入(4a)和(4b)中,可以将和用 w 表示出来. 同时用运 Galerkin 法将式(4)的偏微分方程转换为如下式所示的无量纲非线性常微分方程

$$\ddot{w}_1+c_1\dot{w}_1+\omega_1^2w_1+P_1\cos(\Omega_2 t)w_1+g_{102}w_1^2+g_{103}w_1w_2+g_{104}w_2^2+g_{105}w_1^3+g_{106}w_1^2w_2+g_{107}w_2^2w_1=f_1\cos\Omega_1 t \quad (12a)$$

$$\ddot{w}_2+c_2\dot{w}_2+\omega_2^2w_2+P_2\cos(\Omega_2 t)w_2+g_{202}w_1^2+g_{203}w_1w_2+g_{204}w_2^2+g_{205}w_1^3+g_{206}w_1^2w_2+g_{207}w_2^3w_1=f_2\cos\Omega_1 t \quad (12b)$$

其中 $\omega_1^2=g_{101}+N'_1$ 和 $\omega_2^2=g_{201}+N'_2$,N'_1 和 N'_2 是和温度相关的热载荷.

1.3　摄动分析

大量的理论和实验研究表明,内共振在多自由度非线性振动系统的模态之间建立了一个能量交换机制,将名义上的耦合模态真正耦合在一起[11]. 另外,根据非线性振动理论,对于具有参数激励的系统,系统有较大振动幅值的振动出现在 1/2 亚谐共振附近[12],而共振问题往往是结构产生破坏的主要原因,文献[13]研究了四边简支功能梯度材料板的多种内共振及响应. 本文主要关注系统发生 1:1 内共振 – 主参数共振 – 1/2 亚谐共振情况下的非线性动力行为. 共振关系表示如下

$$\omega_1=\frac{\Omega}{2}+\varepsilon^2\sigma_1,\omega_2=\frac{\Omega}{2}+\varepsilon^2\sigma_2,$$

$$\Omega_2=\Omega_1,\omega_1=\omega_2 \quad (13)$$

式中 ω_1 和 ω_2 是两个不同的线性频率,σ_1 和 σ_2 是两个不同的调谐参数.

考虑到系统含有平方和立方非线性项,故此利用渐进摄动法[14,15]对 SFSF 边界条件的功能梯度材料矩形板进行摄动分析,得到系统的平均方程为

$$\frac{dx_1}{d\tau}=-\frac{1}{2}c_1x_1+(\sigma_1+\frac{1}{2}\omega P_1+\frac{4}{3\omega}g_{102}f_1+\frac{2}{3\omega}g_{102}f_2)x_2+t_{110}x_1x_3x_4+(\frac{2}{3\omega}g_{103}f_1+\frac{4}{3\omega}g_{104}f_2)x_4+t_{101}x_1^2x_2+t_{102}x_3^2x_4+t_{103}x_1^2x_4+t_{104}x_2^3+t_{105}x_4^2x_2+t_{106}x_2^2x_4+t_{107}x_3^2x_2+t_{108}x_4^3+t_{109}x_1x_2x_3 \quad (14a)$$

$$\frac{dx_2}{d\tau}=-\frac{1}{2}c_1x_2+(\frac{2}{3\omega}g_{103}f_2-\sigma_1-\frac{1}{2}\omega P_1+\frac{4}{3\omega}g_{102}f_1)x_1+t_{209}x_2x_3x_4+(\frac{2}{3\omega}g_{103}f_1+\frac{4}{3\omega}g_{104}f_2)x_3+t_{201}x_2^2x_3+t_{202}x_4^2x_1+t_{203}x_1x_2x_4+t_{204}x_2^2x_1+t_{205}x_1^3+t_{206}x_1^2x_3+t_{207}x_3^2x_1+t_{209}x_4x_2x_3+t_{208}x_3^3 \quad (14b)$$

$$\frac{dx_3}{d\tau} = -\frac{1}{2}c_2x_3 + (\frac{2}{3\omega}g_{203}f_1 + \sigma_2 - \frac{1}{2}\omega P_2 + \frac{4}{3\omega}g_{204}f_2)x_4 + t_{309}x_1x_3x_4 + (\frac{2}{3\omega}g_{203}f_2 + \frac{4}{3\omega}g_{202}f_1)x_2 + t_{301}x_3^2x_4 + t_{302}x_4^3 + t_{310}x_4^2x_2 + t_{304}x_1^2x_4 + t_{305}x_1^2x_2 + t_{306}x_3^2x_6 + t_{307}x_2^2x_4 + t_{308}x_1x_2x_3 + t_{303}x_2^3 \quad (14c)$$

$$\frac{dx_4}{d\tau} = -\frac{1}{2}c_2x_4 + (\frac{2}{3\omega}g_{203}f_1 - \sigma_2 - \frac{1}{2}\omega P_2 + \frac{4}{3\omega}g_{204}f_2)x_3 + t_{409}x_2x_3x_4 + (\frac{2}{3\omega}g_{203}f_2 + \frac{4}{3\omega}g_{202}f_1)x_1 + t_{401}x_1^3 + t_{402}x_3^3 + t_{409}x_2x_3x_4 + t_{404}x_2^2x_1 + t_{405}x_1^2x_3 + t_{406}x_2^2x_3 + t_{407}x_3^2x_1 + t_{408}x_1x_2x_4 + t_{403}x_4^2x_3 \quad (14d)$$

限于篇幅这里没有给出上述方程式系数的表达式.

2 周期运动与混沌运动数值计算

本节利用 Runge-Kutta 算法对平均方程(14)进行数值计算. 选择横向激励作为控制参数,研究激励幅值对系统产生周期运动和混沌运动的影响. 图 2 所示为功能梯度材料板结构在参数和初始条件分别为

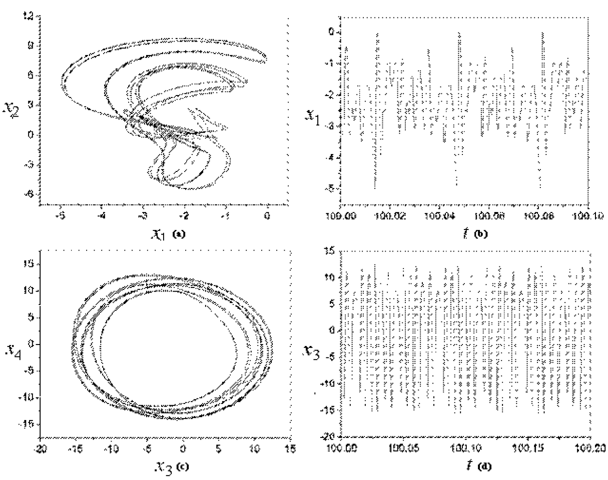

图 2 倍周期运动

Fig. 2 The multiple period motion

$f_1 = 76.85$, $f_2 = 7.02$, $\sigma_1 = 3.56$, $\sigma_2 = 3.5$, $c_1 = 0.64$, $c_2 = 0.64$, $P_1 = 8.87$, $P_2 = 12.12$, $t_{101} = 0.66$, $t_{102} = -0.36$, $t_{103} = -0.71$, $t_{104} = 0.96$, $t_{105} = -0.97$, $t_{106} = 6.4$, $t_{107} = -0.54$, $t_{108} = -0.37$, $t_{109} = 21.8$, $t_{110} = 12.48$, $t_{201} = 8.071$, $t_{202} = 7.06$, $t_{203} = -23.6$, $t_{204} = -0.946$, $t_{205} = -0.68$, $t_{206} = -16.32$, $t_{207} = 9.36$, $t_{208} = -0.676$, $t_{209} = 5.6$, $t_{210} = -3.676$, $t_{301} = -6.98$, $t_{302} = -8.21$, $t_{303} = -11.3$, $t_{304} = 6.15$, $t_{305} = -11.3$, $t_{306} = 4.55$, $t_{307} = -1.95$, $t_{308} = -8.3$, $t_{309} = 1.32$, $t_{310} = 4.119$, $t_{401} = 11.3$, $t_{402} = 6.98$, $t_{403} = 6.124$, $t_{404} = 11.3$, $t_{405} = 1.95$, $t_{406} = 11.35$, $t_{407} = -5.81$, $t_{408} = -8.3$, $t_{409} = 1.32$, $t_{410} = -9.927$, $x_{10} = 0.44$, $x_{20} = 1.55$, $x_{30} = 2.35$,

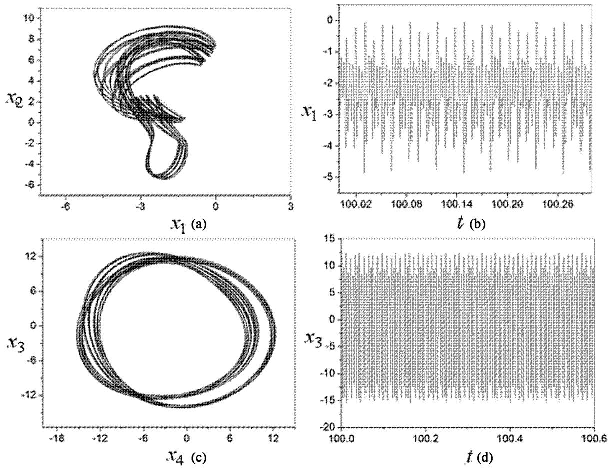

图 3 概周期运动

Fig. 3 The quasi - period motion

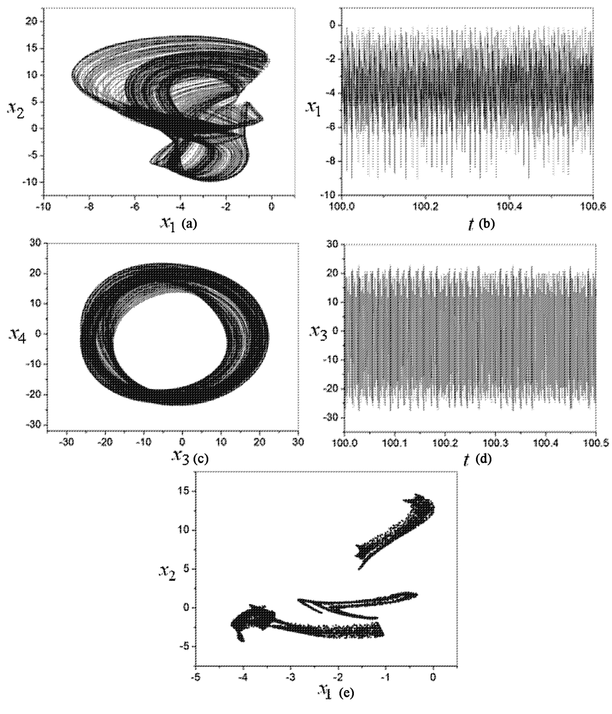

图 4 混沌运动

Fig. 4 The chaotic motion

时的数值结果,结果表明在该参数激励作用下,系统为多倍周期运动,第二阶模态的幅值远比第一阶模态的幅值大,这应该是模态耦合的结果,所以在研究其非线性动力学行为时不能只考虑一阶模态

的作用. 固定其他参数值,只改变激励大小,当激励幅值f_1 =88.85 时,系统的非线性动力学响应如图 3 所示,结果表明此时系统明显处于概周期的运动状态,继续增大激励到f_1 =126.85 时,系统由概周期运动转变为混沌运动,图 4 为此时系统的平面相图、波形图以及 Poincare 截面,其中图(4e)为 Poincare 截面. 从图 2 到图 4 的结果看,随着外激励的增加,功能梯度材料板结构会出现由倍周期→概周期→混沌运动的变化,而且系统的两阶模态的幅值都会随着增大,所以我们可以改变外激励以控制其运动形式.

3　结论

本文以两对边简支另两对边自由的功能梯度材料板为研究对象,建立了考虑材料物性参数与温度相关的、在热/机械载荷共同作用下的几何非线性动力学方程,采用渐进摄动法对系统在 1:1 内共振 - 主参数共振 - 1/2 亚谐共振情况下的非线性动力学行为进行了摄动分析,得到系统的四自由度平均方程. 利用数值分析的方法得到系统的平面相图、波形图和 Poincare 截面,研究了横向激励对系统周期运动和混沌运动的影响. 数值结果表明,选取一定的参数和初始条件,只改变外激励时,该非线性系统会出现倍周期 - 概周期 - 混沌运动. 由此可见该系统对外激励非常敏感,外激励是影响系统运动形式的重要控制参数.

另外,第二阶模态的幅值远比第一阶模态的幅值大,这应该是两阶模态耦合产生内共振的结果,因此,研究该类结构的非线性动力学行为时不应该只考虑一阶模态,而应考虑到前两阶甚至更多阶模态的相互作用,以便于更好地利用或控制其运动形式.

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PERIODIC AND CHAOTIC MOTION OF MEXIED BOUNDARY FGM PLATE WITH 1:1 INTERNAL RESONANCE *

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Abstract This paper studied the nonlinear vibration of FGM plate with two simply supported opposite and two free edges subjected to the in-plane and transversal excitations. The geometrical nonlinear system, whose material properties of the FGM plate are assumed to be temperature dependent, was used. The resonant case considered here is 1:1 internal resonance and principal parametric resonance-1/2 subharmonic resonance. The asymptotic perturbation method was utilized to obtain four-dimensional nonlinear averaged equation. Numerical method was used to find the nonlinear dynamic responses of the FGM rectangular plate. It is found that there exist periodic, quasiperiodic solutions and chaotic motions for the plates under certain conditions. It is thought that the forcing excitations can change the form of motions for the FGM rectangular plate. It is also found from the numerical simulations that the amplitude of the first mode for the FGM plate is much less than that of the second mode in the given parameters. It means that we must consider the second mode when the internal resonance occurs.

Key words functionally graded material plate, mixed boundary, chaotic motions, internal resonance

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