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平面粘性流体扰动问题的变分原理及双正交关系*

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摘要　通过引入不同的对偶变量，将粘性流体的扰动问题化为具有良好结构特性的可解耦 Hamilton 系统. 利用可解耦 Hamilton 系统微分形式与积分形式的等价性，导出了粘性流体扰动问题的 Hamilton 混合能变分原理，并建立了本征函数系之间的双正交关系.

关键词　哈密顿体系，　粘性流体，　变分原理，　双正交关系

引言

钟万勰[1]利用结构力学与最优控制相模拟的理论，将 Hamilton 体系引入弹性力学，建立了弹性力学求解新体系. 在此之后，诸多学者拓展了新体系方法，理性求解了许多力学问题[2-19]. 对于各向同性平面弹性问题，罗建辉发现文献[1]中的正交关系可以分解为两个对称的、独立的子正交关系[9-10]，后来将这种正交关系推广到厚板[11]及薄板理论[12-13]并导出了相应的 Hamilton 混合能变分原理. 罗指出在恰当选择对偶向量后，弹性力学的新正交关系可以推广到三维弹性力学[14]和三维应力偶问题[15].

分析平面粘性流体的波扰动解是研究湍流产生机制的一条途径，传统研究方法是在 Lagrange 体系欧式空间中进行的. 马坚伟将哈密顿体系通过变分原理引入平面粘性流体扰动问题[16]中. 本文通过利用变换矩阵，将上述粘性流体的 Hamilton 系统化为可解耦 Hamilton 系统，即 $\dot{U}=\begin{pmatrix}0 & S\\ W & 0\end{pmatrix}$ 的形式，其中 $H=\begin{pmatrix}0 & S\\ W & 0\end{pmatrix}$ 是斜对角 Hamilton 算子. 与文献[16]中的算子矩阵相比，斜对角 Hamilton 算子的主对角元是零算子，而且斜对角元是对称算子. 基于上述特性，可以建立粘性流体波扰动问题的 Hamilton 混合能变分原理，可理性推导可解耦 Hamilton 系统本征函数系间的双正交关系.

1　哈密顿体系与平面粘性流体问题

在平面直角坐标系下，不可压粘性流体的波扰动问题具有如下 Hamilton 形式[16]：

$$\dot{V}=\overline{H}V \tag{1}$$

其中

$$(\cdot)=\partial(\,)/\partial z,$$

$$\overline{H}=\begin{pmatrix}0 & -\dfrac{\partial}{\partial x} & 0 & 0\\ -\dfrac{\partial}{\partial x} & 0 & 0 & \dfrac{1}{\mu}\\ \rho\omega i & 0 & 0 & -\dfrac{\partial}{\partial x}\\ 0 & -4\mu\dfrac{\partial^2}{\partial x^2}+\rho\omega i & -\dfrac{\partial}{\partial x} & 0\end{pmatrix},$$

$$V=\begin{pmatrix}\psi_Z\\ \psi_X\\ \kappa_Z\\ \kappa_{XZ}\end{pmatrix}$$

这里 ψ_X,ψ_Z 分别是 X 和 Z 方向的速度分量；κ_{XZ}，κ_Z 为应力分量.

按文献[17]的思路，引入变换矩阵

$$P=\begin{pmatrix}0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}(P=P^T=P^{-1}).$$

令 $U=PV$，则系统(1)可化为如下可解耦 Hamilton 形式[17]：

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$$\dot{U}=HU \tag{2}$$

其中

$$H=P\overline{H}P^T=\begin{pmatrix}0 & S\\ W & 0\end{pmatrix},$$

$$U=(\kappa_Z,-\psi_X,\psi_Z,\kappa_{XZ})^T$$

且

$$S=\begin{pmatrix}\rho\omega i & -\dfrac{\partial}{\partial x}\\ \dfrac{\partial}{\partial x} & -\dfrac{1}{\mu}\end{pmatrix},$$

$$W=\begin{pmatrix}0 & \dfrac{\partial}{\partial x}\\ -\dfrac{\partial}{\partial x} & 4\mu\dfrac{\partial^2}{\partial x^2}-\rho\omega i\end{pmatrix}.$$

记 $U_f=(\kappa_z,-\psi_x)^T,U_g=(\psi_z,\kappa_{xz})^T$，可得到：

$$\dot{U}_f=SU_g,\dot{U}_g=WU_f \tag{3}$$

(3)式即为原微分系统对应的维数较低的微分方程组,它是可以被解耦的.

2 哈密顿混合能变分原理的推导

令 $U=\phi(x)e^{\lambda z},\phi(x)=(\phi_f,\phi_g)^T$,则

$$U_f=\phi_f(x)e^{\lambda z},\quad U_g=\phi_g(x)e^{\lambda z} \tag{4}$$

在区域 Ω 引入内积

$$<U_1^T,U_2>=\int_{\Omega_x}U_1U_2\mathrm{d}x \tag{5}$$

由(2),(3),(4)和(5)可得

$$U_g^{*T}\dot{U}_f=\psi_x\frac{\partial\kappa_{xz}^*}{\partial z}+\psi_z^*\frac{\partial\kappa_z}{\partial z}-\frac{\partial}{\partial z}(\kappa_{xz}^*\psi_x) \tag{6}$$

$$U_g^{*T}SU_g=\frac{\partial(\psi_z\kappa_{xz}^*)}{\partial x}-\frac{\partial(\psi_z^*\kappa_{xz})}{\partial x}-\frac{\kappa_{xz}^*\kappa_{xz}}{\mu}+\rho\omega i\psi_z^*\psi_z \tag{7}$$

$$U_f^{*T}\dot{U}_g=-\psi_x^*\frac{\partial\kappa_{xz}}{\partial z}-\psi_z\frac{\partial\kappa_z^*}{\partial z}+\frac{\partial}{\partial z}(\kappa_z^*\psi_z) \tag{8}$$

$$U_f^{*T}WU_f=-\frac{\partial(\kappa_z^*\psi_x)}{\partial x}-\rho\omega i\psi_x^*\psi_x+4\mu\frac{\partial\psi_x^*}{\partial x}\frac{\partial\psi_x}{\partial x}+\frac{\partial(\psi_x^*\kappa_z)}{\partial x} \tag{9}$$

在区域 Ω 的边界 $S(S=S_\psi+S_\kappa)$ 上,考虑如下边界条件:

速度边界(在 S_ψ 上) $\psi_n=\overline{\psi}_n,\psi_s=\overline{\psi}_s$; (10)

应力边界(在 S_κ 上) $\kappa_n=\overline{\kappa}_n,\kappa_s=\overline{\kappa}_s$ (11)

这里

$$\kappa_n=l\kappa_{xz}+m\kappa_z,\kappa_s=m\kappa_{xz}+1\kappa_z,\psi_n=\psi_z,\psi_s=\psi_x \tag{12}$$

其中 n 和 s 分别表示边界 S 的法向和切向.

由(6),(7),(8),(9),(10)和(11)可得

$$Q(u,u^*)=\iint_\Omega[U_g^{*T}(\dot{U}_f-SU_g)-U_f^{*T}(\dot{U}_g-WU_f)]\mathrm{d}x\mathrm{d}z+\int_S[(\overline{\psi}_n-\psi_n)\kappa_s^*-(\overline{\psi}_s-\psi_s)\kappa_n^*]\mathrm{d}S=\iint\{(\psi_x^*\frac{\partial\kappa_{xz}}{\partial z}+\psi_x\frac{\partial\kappa_{xz}^*}{\partial z})+(\psi_z^*\frac{\partial\kappa_z}{\partial z}+\psi_z\frac{\partial\kappa_z^*}{\partial z})+(\frac{\partial(\psi_x\kappa_z^*)}{\partial x}+\frac{\partial(\psi_x^*\kappa_z)}{\partial x})+(\frac{\partial(\psi_z\kappa_{xz}^*)}{\partial x}+\frac{\partial(\psi_z^*\kappa_{xz})}{\partial x})-\rho\omega i\psi_x^*\psi_x+4\mu\frac{\partial\psi_x^*}{\partial x}\frac{\partial\psi_x}{\partial x}+\frac{\kappa_{xz}^*\kappa_{xz}}{\mu}-\rho\omega i\psi_z^*\psi_z-2[\frac{\partial(\psi_x\kappa_z^*)}{\partial x}+\frac{\partial(\psi_z\kappa_z^*)}{\partial x}]-\frac{\partial(\psi_x\kappa_{xz}^*)}{\partial z}+\frac{\partial(\psi_z\kappa_z^*)}{\partial z}\}\mathrm{d}x\mathrm{d}z+\int_S[(\overline{\psi}_n-\psi_n)\kappa_s^*+(\overline{\psi}_s-\psi_s)\kappa_n^*]\mathrm{d}S \tag{13}$$

设定边界为直线段,利用奥氏公式得:

$$\iint\{-2[\frac{\partial(\psi_x\kappa_z^*)}{\partial x}+\frac{\partial(\psi_z\kappa_{xz}^*)}{\partial x}]-\frac{\partial(\psi_x\kappa_{xz}^*)}{\partial z}-\frac{\partial(\psi_z\kappa_z^*)}{\partial z}\}\mathrm{d}x\mathrm{d}z=\int_S(l\psi_x\kappa_z^*+l\psi_z\kappa_{xz}^*+m\kappa_{xz}^*\psi_x+m\psi_z\kappa_z^*)\mathrm{d}S=\int_S l(\kappa_n^*\psi_s+\kappa_s^*\psi_n)\mathrm{d}S \tag{14}$$

将(14)代入(13)中，我们有：

$$Q(u,u^*)=\iint\{(\psi_x^*\frac{\partial\kappa_{xz}}{\partial z}+\psi_x\frac{\partial\kappa_{xz}^*}{\partial z})+(\psi_z^*\frac{\partial\kappa_z}{\partial z}+\psi_z\frac{\partial\kappa_z^*}{\partial z})+(\frac{\partial(\psi_x\kappa_z^*)}{\partial x}+\frac{\partial(\psi_x^*\kappa_z)}{\partial x})+(\frac{\partial(\psi_z\kappa_{xz}^*)}{\partial x}+\frac{\partial(\psi_z^*\kappa_{xz})}{\partial x})-\rho\omega i\psi_x^*\psi_x+4\mu\frac{\partial\psi_x^*}{\partial x}\frac{\partial\psi_x}{\partial x}+\frac{\kappa_{xz}^*\kappa_{xz}}{\mu}-\rho\omega i\psi_z^*\psi_z\}\mathrm{d}x\mathrm{d}z+\int(\overline{\psi}_n\kappa_s^*+\overline{\psi}_s\kappa_n^*)\mathrm{d}S \tag{15}$$

令

$$Q(u,u)=\iint\{(\psi_x\frac{\partial\kappa_{xz}}{\partial z}+\psi_x\frac{\partial\kappa_{xz}}{\partial z})+(\psi_z\frac{\partial\kappa_z}{\partial z}+\psi_z\frac{\partial\kappa_z}{\partial z})+(\frac{\partial(\psi_x\kappa_z)}{\partial x}+\frac{\partial(\psi_x\kappa_z)}{\partial x})+$$

$$\left(\frac{\partial(\psi_z\kappa_{xz})}{\partial x}+\frac{\partial(\psi_z\kappa_{xz})}{\partial x}\right)-\rho\omega i\psi_x\psi_x+4\mu\frac{\partial\psi_x}{\partial x}\frac{\partial\psi_x}{\partial x}+\frac{\kappa_{xz}\kappa_{xz}}{\mu}-\rho\omega i\psi_z\psi_z\}\mathrm{d}x\mathrm{d}z+\int_S(\bar{\psi}_n\kappa_s+\bar{\psi}_s\kappa_n)\mathrm{d}S \quad (16)$$

引入变分运算

$\delta\kappa_z=\kappa_z^*-\kappa_z$,

$\delta\kappa_{xz}=\kappa_{xz}^*-\kappa_{xz}$,

$\delta\psi_x=\psi_x^*-\psi_x$,

$\delta\psi_z=\psi_z^*-\psi_z$,

$\delta\kappa_n=\kappa_n^*-\kappa_n$,

$\delta\kappa_s=\kappa_s^*-\kappa_s$,

$\delta\psi_n=\psi_n^*-\psi_n$,

$\delta\psi_s=\psi_s^*-\psi_s$.

(15)减去 (16)得

$$Q(u,u^*)-Q(u,u)=\iint\{(\psi_x\frac{\partial\delta\kappa_{xz}}{\partial z}+\delta\psi_x\frac{\partial\kappa_{xz}}{\partial z})+(\delta\psi_z\frac{\partial\kappa_z}{\partial z}+\psi_z\frac{\partial\delta\kappa_z}{\partial z})+\left(\frac{\partial(\psi_x\delta\kappa_z)}{\partial x}+\frac{\partial(\delta\psi_x\kappa_z)}{\partial x}\right)+\left(\frac{\partial(\psi_z\delta\kappa_{xz})}{\partial x}+\frac{\partial(\delta\psi_z\kappa_{xz})}{\partial x}\right)-\rho\omega i\delta\psi_x\psi_x+4\mu\frac{\partial\delta\psi_x}{\partial x}\frac{\partial\psi_x}{\partial x}+\frac{\kappa\delta_{xz}\kappa_{xz}}{\mu}-\rho\omega i\delta\psi_z\psi_z\}\mathrm{d}x\mathrm{d}z+\int_S(\bar{\psi}_n\delta\kappa_s+\bar{\psi}_s\delta\kappa_n)\mathrm{d}S \quad (17)$$

将上式执行变分的逆运算,得 Hamilton 混合能表达式 $\delta\Pi=0$,其中

$$\Pi=\iint\{\psi_x\frac{\partial\kappa_{xz}}{\partial z}+\psi_z\frac{\partial\kappa_z}{\partial z}+\frac{\partial(\psi_x\kappa_z)}{\partial x}+\frac{\partial(\psi_z\kappa_{xz})}{\partial x}-\frac{\rho\omega i\psi_x^2}{2}+2\mu\left(\frac{\partial\psi_x}{\partial z}\right)^2+\frac{\kappa_{xz}\kappa_{xz}}{2\mu}-\frac{\rho\omega i\psi_x^2}{2}\}\mathrm{d}x\mathrm{d}z+\int_S(\bar{\psi}_n\kappa_s+\bar{\psi}_s\kappa_n)\mathrm{d}S \quad (18)$$

(18)式给出了速度与应力边界条件的积分形式的能量泛函表达式,对(18)式进行变分即得可解耦 Hamilton 系统(2)及边界条件(10)和(11).

3　双正交关系的推导

考虑粘性流场满足的如下边界条件:

$\psi_x=0,\kappa_{xz}=0$, 当 $x=x_1$ 和 $x=x_2$

由(7)和(9)式得:

$$U_g^{*T}sU_g-U_g^TsU_g^*=2\frac{\partial(\psi_z\kappa_{xz}^*)}{\partial x}-2\frac{\partial(\psi_z^*\kappa_{xz})}{\partial x} \quad (19)$$

$$U_f^{*T}wU_f-U_f^TwU_f^*=2\frac{\partial(\psi_x^*\kappa_z)}{\partial x}-2\frac{\partial(\psi_x\kappa_z^*)}{\partial x} \quad (20)$$

由(6)和(7)式得:

$$\int_{x_1}^{x2}(U_g^{*T}sU_g-U_g^TsU_g^*)\mathrm{d}S=2(\psi_z\kappa_{xz}^*)\Big|_{x_1}^{x_2}-2(\psi_z^*\kappa_{xz})\Big|_{x_1}^{x_2} \quad (21)$$

$$\int_{x_1}^{x2}(U_f^{*T}wU_f-U_f^TwU_f^*)\mathrm{d}S=2(\psi_x^*\kappa_z)\Big|_{x_1}^{x_2}-2(\psi_x\kappa_z^*)\Big|_{x_1}^{x_2} \quad (22)$$

由(5)式和相应的边界条件,得:

$$\langle U_g^{*T},SU_g\rangle=\langle U_g^T,SU_g^*\rangle,\quad \langle U_f^{*T},WU_f\rangle=\langle U_f^T,WU_f^*\rangle \quad (23)$$

由(3)和(4)得:

$$\lambda\langle\phi_g^{*T},\phi_f\rangle=\lambda^*\langle\phi_g^T,\phi_f^*\rangle,\quad \lambda\langle\phi_g^{*T},\phi_f\rangle=\lambda\langle\phi_g^T,\phi_f^*\rangle \quad (24)$$

当 $\lambda^2\neq\lambda^{*2}$时,得到如下双正交关系:

$$\langle\phi_f^T,\phi_g^*\rangle=0,\quad \langle\phi_g^T,\phi_f^*\rangle=0 \quad (25)$$

上述双正交关系的物理意义是可解耦 Hamilton 系统的基本解系关于 x 坐标的对称性,对微分系统(2)的基本解系的完备性证明会有一定的意义.

4　结论

文中建立了平面粘性流体扰动问题的可解耦 Hamilton 形式,利用导出的斜对角 Hamilton 算子的结构特性,引入对偶变量,建立了维数较低的粘性流体微分方程组,导出了粘性流体哈密顿混合能变分原理,提出了不可压粘性流体本征向量的一种双正交关系.

不可压粘性流体本征向量双正交关系包含辛正交关系,其成立的条件是. 这个条件的物理意义是可解藕微分方程的基本解系关于 x 坐标的对称性. 双正交关系的建立对于平面粘性流体的半解析有限元等数值方法的发展具有一定参考价值.

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THE VARIATIONAL PRINCIPLE FOR DISTURBANCE OF PLANE VISCOUS FLOW AND ITS BIORTHOGONALITY RELATIONSHIP *

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Abstract　By constructing new dual variables, the disturbance problem of plane viscous flow was derived to an uncoupled Hamilton system, which has good structural characteristics. According to the equivalence between the integral form and differential form of the uncoupled Hamilton system, the mixed energy Hamiltonian variational principle of the problem was obtained, and a biorthogonal relationship of the eigenfunctions was established.

Key words　Hamiltonian system,　viscous flow,　variational principle,　biorthogonal relationship

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