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耦合的含立方非线性项系统的鞍结分岔控制*

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摘要　讨论了一个两自由度的含立方非线性项的受迫振动系统,设计了反馈控制器,对弱非线性系统用近似解析方法求出了控制系统的幅值控制方程,得到了控制参数与幅值的函数关系,实现了反馈控制法在多自由度非线性系统的鞍结分岔控制中的应用,证实了多尺度摄动法对多自由度非线性系统鞍结分岔控制的有效性和适用性.

关键词　单频外激励,　耦合,　鞍结分岔,　振动幅值,　反馈控制

引言

分岔是非线性系统的重要特征,是动力学复杂现象的必然反映[1-4].鞍结分岔是一种常见的静态分岔类型,在土木建筑、电力[5]等许多关系民生大计的基础建设领域普遍存在.电力系统中的鞍结分岔,至今已经被广泛地分析和研究[6-10].大量事实和研究成果均表明,研究鞍结分岔,特别是研究鞍结分岔的控制,对实际工程更是具有重要的理论指导意义和应用价值.本文采用的方法也可以在其它分岔控制中应用[11-13].

1　多自由度立方非线性系统的鞍结分岔

1.1　含立方非线性项的受迫振动系统

考虑一个含立方非线性项的受迫振动系统:

$$\begin{cases}\ddot{u}_1+\omega_1^2u_1=-2\hat{\mu}_1\dot{u}_1+\alpha_1u_1^3+\alpha_2u_1^2u_2+\\ \quad\alpha_3u_1u_2^2+\alpha_4u_2^3+F_1\cos(\Omega t+\tau_1)\\ \ddot{u}_2+\omega_2^2u_2=-2\hat{\mu}_2\dot{u}_2+\alpha_5u_1^3+\alpha_6u_1^2u_2+\\ \quad\alpha_7u_1u_2^2+\alpha_8u_2^3+F_2\cos(\Omega t+\tau_2)\end{cases}\tag{1}$$

研究这类两个自由度含立方非线性项系统的周期响应,有两种方法:第一类方法就是对系统用多尺度法或平均法进行摄动分析;第二类方法包括谐波平衡法、Galerkin. Ritz 法和 Lindstedt - Poincare 方法.这里我们用多尺度法对系统进行分析,并且只讨论系统在内共振前提条件下发生主共振的情况,即:在 $\omega_2=3\omega_1+\varepsilon^2\sigma_1$ 的前提下,讨论 $\Omega\approx\omega_1$ 以及 $\Omega\approx\omega_2$ 的情况,其中 σ_1 为解谐参数.

用多尺度法对方程(1)进行摄动求其一次近似解,令:

$$\begin{cases}u_1=\varepsilon u_{11}(T_0,T_2)+\varepsilon^3u_{13}(T_0,T_2)+\cdots\\ u_2=\varepsilon u_{21}(T_0,T_2)+\varepsilon^3u_{23}(T_0,T_2)+\cdots\end{cases}\tag{2}$$

其中 ε 为一幅值相关的小参数,$T_n=\varepsilon^nt$.此处,由于系统(1)的非线性只体现在 $O(\varepsilon^3)$ 项上,因此,在近似解解析式(2)中省略了对原系统非线性不产生影响的 $O(\varepsilon^2)$ 与时间尺度 T_1.设定 $\hat{\mu}_n=\varepsilon^2\mu_n$ 和 $F_n=\varepsilon^2f_n$,系统发生主共振时系统的阻尼、非线性和激励在同一个扰动方程里得到体现.当 $\Omega=\omega_n$ 时,系统产生主共振.

将式(2)代入式(1),展开后令 ε 的同次幂系数为零,得到各阶近似的线性偏微分方程组:

$$\begin{cases}D_0^2u_{11}+\omega_1^2u_{11}=0\\ D_0^2u_{21}+\omega_2^2u_{21}=0\end{cases}\tag{3}$$

$$\begin{cases}D_0^2u_{13}+\omega_1^2u_{13}=-2D_0(D_1u_{11}+u_1u_{11})+\alpha_1u_{11}^3+\\ \quad\alpha_2u_{11}^2u_{21}+\alpha_3u_{11}u_{21}^2+\alpha_4u_{21}^3+f_1\cos(\Omega T_0+\tau_1)\\ D_0^2u_{23}+\omega_2^2u_{23}=-2D_0(D_1u_{21}+u_2u_{21})+\alpha_5u_{11}^3+\\ \quad\alpha_6u_{11}^2u_{21}+\alpha_7u_{11}u_{21}^2+\alpha_8u_{21}^3+f_1\cos(\Omega T_0+\tau_2)\end{cases}\tag{4}$$

式(3)的解为:

$$\begin{cases}u_{11}=A_1(T_2)\exp(i\omega_1T_0)+cc\\ u_{21}=A_2(T_2)\exp(i\omega_2T_0)+cc\end{cases}\tag{5}$$

其中,A_1 和 A_2 为满足这一阶精度要求的待定函数.把式(5)代入式(4),得到:

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$$\begin{cases} D_0^2u_{13}+\omega_1^2u_{13}=[-2i\omega_1(A'_1+\mu_1A_1)+3\alpha_1A_1^2\bar{A}_1+2\alpha_3A_2\bar{A}_2A_1]\exp(i\omega_1T_0)+(2\alpha_2A_1\bar{A}_1+3\alpha_4A_2\bar{A}_2)\times A_2\exp(i\omega_2T_0)+\alpha_1A_1^3\exp(3i\omega_1T_0)+\alpha_4A_2^3\exp(3i\omega_2T_0)+\alpha_2A_1^2A_2\exp[i(2\omega_1+\omega_2)T_0]+\alpha_2\bar{A}_1^2A_2\exp[i(\omega_2-2\omega_1)T_0]+\alpha_3A_1A_2^2\exp[i(\omega_1+2\omega_2)T_0]+\alpha_3A_1\bar{A}_2^2\exp[i(\omega_1-2\omega_2)T_0]+\frac{1}{2}f_1\exp[i(\Omega T_0+\tau_1)]+cc \\ D_0^2u_{23}+\omega_2^2u_{23}=[-2i\omega_2(A'_2+\mu_2A_2)+3\alpha_8A_2^2\bar{A}_2+2\alpha_6A_1\bar{A}_1A_2]\exp(i\omega_2T_0)+(2\alpha_7A_2\bar{A}_2+3\alpha_5A_1\bar{A}_1)\times A_1\exp(i\omega_1T_0)+\alpha_5A_1^3\exp(3i\omega_1T_0)+\alpha_8A_2^3\exp(3i\omega_2T_0)+\alpha_6A_1^2A_2\exp[i(2\omega_1+\omega_2)T_0]+\alpha_6\bar{A}_1^2A_2\exp[i(\omega_2-2\omega_1)T_0]+\alpha_7A_1A_2^2\exp[i(\omega_1+2\omega_2)T_0]+\alpha_7A_1\bar{A}_2^2\exp[i(\omega_1-2\omega_2)T_0]+\frac{1}{2}f_2\exp[i(\Omega T_0+\tau_2)]+cc \end{cases} \tag{6}$$

当 $\omega_2\approx3\omega_1$ 时,系统出现一阶内共振,引入解谐系数 σ_1,则 $\omega_2=3\omega_1+\varepsilon^2\sigma_1$. 接下来,本文就 $\omega_1\approx\Omega$ 时系统的鞍结分岔进行讨论.

1.2　$\omega_1\approx\Omega$ 时原系统的鞍结分岔

引入第二个解谐系数 σ_2,则有:

$$\Omega=\omega_1+\varepsilon^2\sigma_2 \tag{7}$$

将(7)式带入(6)式,并消去久期项,得到:

$$\begin{cases} -2i\omega_1(A'_1+\mu_1A_1)+3\alpha_1A_1^2\bar{A}_1+2\alpha_3A_2\bar{A}_2A_1+\alpha_2\bar{A}_1^2A_2\exp(i\sigma_1T_2)+\frac{1}{2}f_2\exp[i(\sigma_1T_2+\tau_2)]=0 \\ -2i\omega_2(A'_2+\mu_2A_2)+5\alpha_5A_1^3\exp(-i\sigma_1T_2)+3\alpha_8A_2^2\bar{A}_2+2\alpha_6A_1\bar{A}_1A_2=0 \end{cases} \tag{8}$$

引入极坐标表达 $A_n=\frac{1}{2}a_n\exp(i\theta_n)$,并分开实部虚部:

$$\begin{cases} 8\omega_1(a'_1+\mu_1a_1)=\alpha_2a_1^2a_2\sin\gamma_1+4f_1\sin\gamma_2 \\ 8\omega_2(a'_2+\mu_2a_2)=-\alpha_5a_1^3\sin\gamma_1 \\ 8\omega_1a_1\theta'_1=-(3\alpha_1a_1^2+2\alpha_3a_2^2)a_1-\alpha_2a_1^2a_2\cos\gamma_1-4f_1\cos\gamma_2 \\ 8\omega_2a_2\theta'_2=-(3\alpha_8a_2^2+2\alpha_6a_1^2)a_2-\alpha_5a_1^3\cos\gamma_1 \end{cases} \tag{9}$$

其中:

$$\gamma_1=\sigma_1T_2+\theta_2-3\theta_1,\quad \gamma_2=\sigma_2T_2-\theta_1+\tau_1 \tag{10}$$

稳态响应时存在:$a'_n=\gamma'_n=0$. 于是可将式(9)化简为:

$$\begin{cases} 8\omega_1\mu_1a_1-\alpha_2a_1^2a_2\sin\gamma_1-4f_1\sin\gamma_2=0 \\ 8\omega_2\mu_2a_2+\alpha_5a_1^3\sin\gamma_1=0 \\ 8\omega_1a_1\sigma_2+(3\alpha_1a_1^2+2\alpha_3a_2^2)a_1+\alpha_2a_1^2a_2\cos\gamma_1+4f_1\cos\gamma_2=0 \\ 8\omega_2a_2(3\sigma_2-\sigma_1)+(3\alpha_8a_2^2+2\alpha_6a_1^2)a_2+\alpha_5a_1^3\cos\gamma_1=0 \end{cases} \tag{11}$$

将系统参数取为:$\alpha_1=0.01,\alpha_2=0.03,\alpha_3=0.015,\alpha_5=0.03,\alpha_6=0.015,\alpha_8=0.01,\mu_1=\mu_2=0.1,\omega_1=1,\omega_2=3,f_1=800$,固定 σ_1 以考察振幅 a_1 随 σ_2 的变化情况,如图 1 所示.

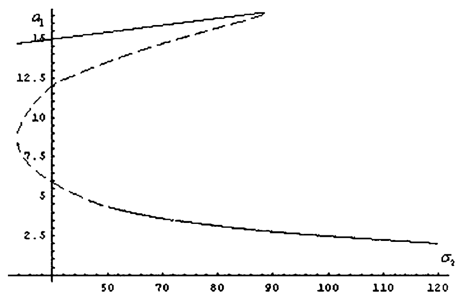

图 1　未控制系统(1)频响曲线:$a_1-\sigma_2$,在情况:$\omega_2\approx3\omega_1,\Omega\approx\omega_1$ 下

Fig. 1　Frequency response curve of uncontrolled system (1): $a_1-\sigma_2$ Under condition of: $\omega_2\approx3\omega_1,\Omega\approx\omega_1$

2　鞍结分岔控制

可以设计线性或非线性控制器,对原系统进行控制. 本文将反馈控制器设计为:

$$V_1(u_1,u_2)=K_1u_1,\quad V_2(u_1,u_2)=K_2u_2 \tag{12}$$

其中 $K_n=\varepsilon^2k_n$. 于是原系统(1)变为控制系统:

$$\begin{cases} \ddot{u}_1+\omega_1^2u_1=-2\hat{\mu}_1\dot{u}_1+\alpha_1u_1^3+\alpha_2u_1^2u_2+\alpha_3u_1u_2^2+\alpha_4u_2^3+F_1\cos(\Omega t+\tau_1)+K_1u_1 \\ \ddot{u}_2+\omega_2^2u_2=-2\hat{\mu}_2\dot{u}_2+\alpha_5u_1^3+\alpha_6u_1^2u_2+\alpha_7u_1u_2^2+\alpha_8u_2^3+F_2\cos(\Omega t+\tau_2)+K_2u_2 \end{cases} \tag{13}$$

按照之前的方法用式(2)对式(13)进行摄动,并消去久期项,得到:

$$\begin{cases} -2i\omega_1(A'_1+\mu_1A_1)+3\alpha_1A_1^2\bar{A}_1+2\alpha_3A_2\bar{A}_2A_1+\alpha_2\bar{A}_1^2A_2\exp(i\sigma_1T_2)+\frac{1}{2}f_2\exp[i(\sigma_1T_2+\tau_2)]+k_1A_1=0 \\ -2i\omega_2(A'_2+\mu_2A_2)+\alpha_5A_1^3\exp(-i\sigma_1T_2)+3\alpha_8A_2^2\bar{A}_2+2\alpha_6A_1\bar{A}_1A_2+k_2A_2=0 \end{cases} \tag{14}$$

同样,引入极坐标表达,并分开实部虚部,得到简化后稳态响应:

$$\begin{cases}8\omega_1\mu_1 a_1-\alpha_2 a_1^2 a_2\sin\gamma_1-4f_1\sin\gamma_2=0\\ 8\omega_2\mu_2 a_2+\alpha_5 a_1^3\sin\gamma_1=0\\ 8\omega_1 a_1\sigma_2+(3\alpha_1 a_1^2+2\alpha_3 a_2^2+8k_1)a_1+\alpha_2 a_1^2 a_2\cos\gamma_1+4f_1\cos\gamma_2=0\\ 8\omega_2 a_2(3\sigma_2-\sigma_1)+(3\alpha_8 a_2^2+2\alpha_6 a_1^2+8k_2)a_2+\alpha_5 a_1^3\cos\gamma_1=0\end{cases}\tag{15}$$

其中 γ_1,γ_2 同式(10).

同样固定 σ_1 来考察控制系统(13)振幅 a_1 随 σ_2 的变化情况,如图 2 所示,此时,保持其他系统参数不变,将控制参数取 $k_1=12,k_2=15$. 对比图 1 和图 2,可以看到虽然此时尚未消除鞍结分岔现象,但是 a_1 不稳定区域与未控制系统(1)相比要小. 可见反馈控制器(12)是有效的.

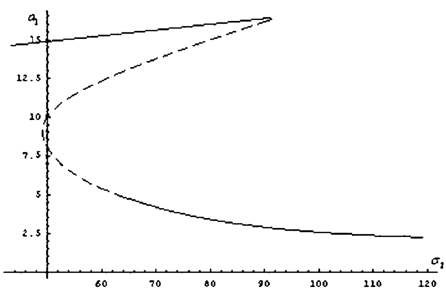

图 2 控制系统(13)频响曲线:$a_1-\sigma_2$ 在情况:$\omega_2\approx3\omega_1,\Omega\approx\omega_1$ 下

Fig. 2 Frequency response curve of controlled system (13): $a_1-\sigma_2$ Under condition of: $\omega_2\approx3\omega_1,\Omega\approx\omega_1$

3 非线性控制器

将反馈控制器设计为:

$$V_1(u_1,u_2)=K_3u_1u_2,V_2(u_1,u_2)=K_4u_1u_2\tag{16}$$

其中 $K_n=\varepsilon^2k_n$. 于是原系统式(1)变为控制系统:

$$\begin{cases}\ddot{u}_1+\omega_1^2u_1=-2\hat{\mu}_1\dot{u}_1+\alpha_1u_1^3+\alpha_2u_1^2u_2+\\ \quad\alpha_3u_1u_2^2+\alpha_4u_2^3+F_1\cos(\Omega t+\tau_1)+K_3u_1u_2\\ \ddot{u}_2+\omega_2^2u_2=-2\hat{\mu}_2\dot{u}_2+\alpha_5u_1^3+\alpha_6u_1^2u_2+\\ \quad\alpha_7u_1u_2^2+\alpha_8u_2^3+F_2\cos(\Omega t+\tau_2)+K_4u_1u_2\end{cases}\tag{17}$$

系统有稳态响应:

$$\begin{cases}8\omega_1\mu_1 a_1-\alpha_2 a_1^2 a_2\sin\gamma_1-4f_1\sin\gamma_2=0\\ 8\omega_2\mu_2 a_2+\alpha_5 a_1^3\sin\gamma_1=0\\ 8\omega_1 a_1\sigma_2+(3\alpha_1 a_1^2+2\alpha_3 a_2^2+k_3 a_2)a_1+\\ \quad\alpha_2 a_1^2 a_2\cos\gamma_1+4f_1\cos\gamma_2=0\\ 8\omega_2 a_2(3\sigma_2-\sigma_1)+(3\alpha_8 a_2^2+2\alpha_6 a_1^2+k_4 a_1)a_2+\alpha_5 a_1^3\cos\gamma_1=0\end{cases}\tag{18}$$

反馈控制器(16)也有较好的控制效果.

4 结论

本文研究一个单频外激励的含立方非线性的系统,用多尺度法对系统进行摄动分析,发现系统存在鞍结点分岔. 一般来说要获得控制参数与幅值的函数关系是比较困难的,需借助于数值分析,设计控制器,确定适合的控制参数. 本文分析了鞍结点分岔的发生原因,设计了简单可行的反馈控制器,成功的实现了在系统存在内共振条件下发生主共振时,对系统鞍结点分岔的控制,该研究工作可推广到其它多自由度非线性系统的分岔控制.

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CONTROL OF SADDLE-NODE BIFURCATION IN A COUPLED SYSTEM WITH CUBIC NONLINEAR TERMS *

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Abstract A forced two-degree- of-freedom coupled system with cubic nonlinear terms was analyzed, and a feedback controller of nonlinear was designed. With the aid of approximate analytical method, the relationship between the control parameters and the amplitude of the vibration system was obtained, which proves that the feedback control is suitable in saddle-node bifurcation control of nonlinear systems with multi-degrees of freedom, and the multi-scale method is also applicable.

Key words single-frequency excitation, coupled, saddle-node bifurcation, amplitude, feedback control

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