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高阶非完整系统的共形不变性与 Noether 守恒量*

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摘要　研究了高阶非完整系统的共形不变性与 Noether 守恒量,给出了与高阶非完整系统相应的完整系统的共形不变性的定义及其确定方程,通过系统共形不变性与 Lie 对称性的关系,推导出了系统运动方程具有共形不变性并且是 Lie 对称性的共形因子,利用限制方程和附加限制方程,给出了高阶非完整系统的弱 Lie 对称性和强 Lie 对称性的共形不变性,得到了共形不变性导致的 Noether 守恒量,举例说明了结果的应用.

关键词　高阶非完整系统,　共形不变性,　Noether 守恒量

引 言

动力学系统对称性与守恒量的研究在现代数理科学中占有重要地位,是分析力学的一个近代发展方向. 分析力学的近代对称性主要有 Noether 对称性[1]、Lie 对称性[2]和 Mei 对称性[3-4],得到的守恒量主要有 Noether 守恒量、Hojman 守恒量和 Mei 守恒量. 梅凤翔研究了完整系统的这三类对称性与三类守恒量[5]. 文献[6]则对各种约束力学系统的上述三种主要对称性与三类守恒量进行了全面、系统的研究. 1997 年, Galiullin A. S. 等在研究 Birkhoff 系统分析动力学时提出了 Birkhoff 方程的共形不变性和共形因子的概念,并讨论了共形不变性与 Lie 对称性之间的关系[7]. 文献[8]利用几何方法研究了 Hamilton 系统的共形不变性,讨论了共形不变性的几何结构及其与一般对称性的关系. 梅凤翔等研究了一阶运动微分方程的共形不变性,并将其转化为 Birkhoff 系统的共形不变性,进而得到共形不变性导致的 Noether 守恒量[9]. 随后,人们又将其推广到 Hamilton 系统,变质量力学系统,非完整力学系统等约束力学系统[10-16].

本文基于群变换理论,研究高阶非完整系统的共形不变性,导出无限小群变换下系统是共形不变的且是 Lie 对称性的共形因子表达式,得到系统共形不变性导致的 Noether 守恒量. 最后举例说明结果的应用.

1　系统的运动微分方程

假设力学系统的位形由 n 个广义坐标 $q_s(s=1,2,\cdots,n)$ 来确定,它的运动受 g 个理想双面 m 阶非完整约束

$$\overset{(m)}{q}_{\varepsilon+\beta}=\varphi_\beta(t,q_s,\dot q_s,\cdots,\overset{(m-1)}{q}_s,\overset{(m)}{q}_\sigma)\qquad(1)$$

$$\begin{pmatrix}s=1,\cdots,n;\beta=1,\cdots,g;\\ \sigma=1,\cdots,\varepsilon;\varepsilon=n-g\end{pmatrix}$$

系统的运动方程可表示为 Routh 形式,有

$$\begin{cases}\dfrac{d}{dt}\dfrac{\partial}{\partial\dot q_\sigma}-\dfrac{\partial L}{\partial q_\sigma}=Q''_\sigma-\lambda_\beta\dfrac{\partial\varphi_\beta}{\partial\overset{(m)}{q}_\sigma},(\sigma=1,2,\cdots,\varepsilon)\\ \dfrac{d}{dt}\dfrac{\partial}{\partial\dot q_{\varepsilon+\beta}}-\dfrac{\partial L}{\partial q_{\varepsilon+\beta}}=Q''_{\varepsilon+\beta}+\lambda_\beta,(\sigma=1,2,\cdots,\varepsilon)\end{cases}\qquad(2)$$

其中 L 为系统的 Lagrange 函数,Q'' 为非势广义力,λ_β 为约束乘子. 由方程(2)消去 λ_β,得到

$$f_{\varepsilon+\beta}a_{\beta\sigma}+f_\sigma=0\quad(\sigma=1,\cdots,\varepsilon)\qquad(3)$$

其中

$$\begin{cases}f_s(t,q_k,\dot q_k,\ddot q_k)\equiv\dfrac{d}{dt}\dfrac{\partial}{\partial\dot q}-\dfrac{\partial L}{\partial q_s}-Q''_s\\ a_{\beta\sigma}(t,q_s,\dot q_s,\cdots,\overset{(m-1)}{q}_s,\overset{(m)}{q}_\nu)\equiv\dfrac{\partial\varphi_\beta}{\partial\overset{(m)}{q}_s}\end{cases}\qquad(4)$$

方程(3)的阶依赖于 $\partial\varphi_\beta/\partial\overset{(m)}{q}_\sigma$,可由 2ε 阶降到 $m\varepsilon$ 阶. 如果在这些偏导数中广义坐标对 t 的导数的最高阶为 $l(0\leqslant l\leqslant m)$,那么将方程(3)对 t 求 $(m-2)$ 阶导数 $(l\leqslant2)$ 或 $(m-l)$ 阶导数 $(l\geqslant2)$,则其阶成为 $m\varepsilon$. 将所得方程与约束方程(1)联合,并

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假设可以解出 $\overset{(m)}{q}_s$,简记作[17]

$$\overset{(m)}{q}_s = h_s(t, q, \dot{q}, \cdots, \overset{(m-1)}{q}), (s = 1, 2, \cdots, n)(m > 2) \tag{5}$$

称方程(5)为与非完整系统(1)、(3)相应的完整系统的运动方程.

2　系统的共形不变性及其确定方程

引入时间和广义坐标的无限小群变换

$$t^* = t + \Delta t, q_s^*(t^*) = q_s(t) + \Delta q_s \tag{6}$$

或其展开式为

$$t^* = t + \varepsilon\xi_0(t, q), q_s^*(t^*) = q_s(t) + \varepsilon\xi_s(t, q) \tag{7}$$

其中 ε 为无限小参数,ξ_0, ξ_s 为无限小群变换的生成元. 取无限小生成元向量

$$X^{(0)} = \xi_0 \frac{\partial}{\partial t} + \xi_s \frac{\partial}{\partial q_s} \tag{8}$$

以及它的一次扩展至 m 次扩展

$$\begin{cases} X^{(1)} = X^{(0)} + \overset{(1)}{\eta}_s \dfrac{\partial}{\partial \dot{q}_s}, \overset{(1)}{\eta}_s = \dot{\xi}_s - \dot{q}_s \dot{\xi}_0 \\ X^{(2)} = X^{(1)} + \overset{(2)}{\eta}_s \dfrac{\partial}{\partial \ddot{q}_s}, \overset{(2)}{\eta}_s = (\overset{(2)}{\eta}_s)^{\cdot} - \ddot{q}_s \dot{\xi}_0 \\ \vdots \\ X^{(m)} = X^{(m-1)} + \overset{(m)}{\eta}_s \dfrac{\partial}{\partial \overset{(m)}{q}_s}, \overset{(m)}{\eta}_\alpha = (\overset{(m-1)}{\eta}_s)^{\cdot} - \overset{(m)}{q}_s \dot{\xi}_0 \end{cases} \tag{9}$$

令

$$F_s = \overset{(m)}{q}_s - h_s(t, q, \dot{q}, \cdots, \overset{(m-1)}{q}) \tag{10}$$

定义 1　对于 F_s,在无限小生成元 $\xi_0(t,q)$,$\xi_s(t,q)$ 的变换下,若存在 Γ_s^k 满足

$$X^{(m)} F_s = \Gamma_s^k F_k \tag{11}$$

则称方程(10)在无限小变换(7)下是共形不变的,Γ_s^k 称为共形因子,式(11)称为方程(10)共形不变性的确定方程.

3　共形不变性与 Lie 对称性

定义 2　如果无限小变换的生成元 ξ_0, ξ_s 满足确定方程

$$X^{(m)} F_s |_{F_s = 0} = 0 \tag{12}$$

即

$$\overset{(m)}{\xi}_s - \dot{q}_s \overset{(m)}{\xi}_0 - C_m^1 \ddot{q}_s \overset{(m-1)}{\xi}_0 - C_m^2 \dddot{q}_s \overset{(m-2)}{\xi}_0 - \cdots - C_m^{m-2} \overset{(m-1)}{q}_s \ddot{\xi}_0 - C_m^{m-1} \dot{\xi}_0 h_s = X^{(m-1)}(h_s) \tag{13}$$

其中

$$C_m^k = \frac{m!}{(m-k)!\ k!} \tag{14}$$

则称相应对称性为与非完整系统(1)、(3)相应的完整系统(5)的 Lie 对称性.

对高阶非完整系统的 Lie 对称性,还要考虑一些限制. 这些限制依赖于在得到方程(5)过程中对方程(3)的求导次数. 例如 $l = 1, m = 4$,为得到方程(5),需对方程(3)求两次导数. 这就要求方程(3)保持不变,它对 t 的一次导数也保持不变,即有

$$X^{(2)}(f_{\varepsilon+\beta} a_{\beta\sigma} + f_\sigma) = 0 \tag{15}$$

$$X^{(3)}(\dot{f}_{\varepsilon+\beta} a_{\beta\sigma} + f_{\varepsilon+\beta} \dot{a}_{\beta\sigma} + \dot{f}_\sigma) = 0 (\sigma = 1, \cdots, \varepsilon) \tag{16}$$

称这些方程为限制方程.

定义 3　如果无限小变换的生成元 ξ_0, ξ_s 满足确定方程(13)和限制方程,则相应对称性为高阶非完整系统的弱 Lie 对称性.

考虑到约束对生成元 ξ_0, ξ_s 的限制,即

$$\xi_{\varepsilon+\beta} - \dot{q}_{\varepsilon+\beta} \xi_0 - \frac{\partial \varphi_\beta}{\partial \overset{(m)}{q}_\sigma} (\xi_\sigma - \dot{q}_\sigma \xi_0) = 0 \quad (\beta = 1, \cdots, g) \tag{17}$$

定义 4　如果无限小变换的生成元 ξ_0, ξ_s 满足确定方程(13)、限制方程和附加限制方程(17),则相应对称性为高阶非完整系统的强 Lie 对称性.

命题 1　对方程(10),若无限小变换的生成元 ξ_0, ξ_s 是 Lie 对称性的,且存在矩阵 β_s^l 满足下列条件

$$X^{(m)} F_s - X^{(m)} F_s |_{F_s = 0} = \beta_s^l F_l \tag{18}$$

则共形不变性同时是 Lie 对称性的充分必要条件为

$$\Gamma_s^l = \beta_s^l \tag{19}$$

证明: 方程(10)的 Lie 对称性满足

$$X^{(m)} F_s |_{F_s = 0} = 0 \tag{20}$$

若存在一矩阵 β_s^l 满足方程(18),则有

$$X^{(m)} F_s = \beta_s^l F_l \tag{21}$$

根据共形不变性定义方程(11),系统的共形因子为 $\Gamma_s^l = \beta_s^l$.

另一方面,由方程(11)和(18),容易得到

$$(\Gamma_s^l - \beta_s^l) F_l = X^{(m)} F_s |_{F_s = 0} \tag{22}$$

如果 $\Gamma_s^l = \beta_s^l$,那我们有 $X^{(m)} F_s |_{F_s = 0} = 0$,则系统是 Lie 对称性的.

故共形不变性同时是 Lie 对称性的充分必要条件为 $\Gamma_s^l=\beta_s^l$.

4 共形因子

为得到方程共形不变性的共形因子表达式,需要计算

$$X^{(m)}(F_s)-X^{(m)}(F_s)|_{F_s=0} \tag{23}$$

利用

$$\dot{\xi}_s=\frac{\partial\xi_s}{\partial q_k}\dot{q}_k+\frac{\partial\xi_s}{\partial t} \tag{24}$$

$$\ddot{\xi}_s=\frac{\partial\xi_s}{\partial q_k}\ddot{q}_k+2\frac{\partial^2\xi_s}{\partial q_k\partial t}\dot{q}_k+\frac{\partial^2\xi_s}{\partial q_k\partial q_j}\dot{q}_k\dot{q}_j+\frac{\partial^2\xi_s}{\partial t^2} \tag{25}$$

$$\dddot{\xi}_s=\frac{\partial\xi_s}{\partial q_k}\dddot{q}_k+3\frac{\partial^2\xi_s}{\partial q_k\partial q_j}\ddot{q}_k\dot{q}_j+3\frac{\partial^2\xi_s}{\partial q_k\partial t}\ddot{q}_k+3\frac{\partial^3\xi_s}{\partial q_k\partial t^2}\dot{q}_k+3\frac{\partial^3\xi_s}{\partial q_k\partial q_j\partial t}\dot{q}_k\dot{q}_j+\frac{\partial^3\xi_s}{\partial q_k\partial q_j\partial q_\gamma}\dot{q}_k\dot{q}_j\dot{q}_\gamma+\frac{\partial^3\xi_s}{\partial t^3} \tag{26}$$

$$\ddddot{\xi}_s=\frac{\partial\xi_s}{\partial q_k}\ddddot{q}_k+4\frac{\partial^2\xi_s}{\partial q_k\partial q_j}\dddot{q}_k\dot{q}_j+4\frac{\partial^2\xi_\alpha}{\partial q_k\partial t}\dddot{q}_k+\cdots \tag{27}$$

$\vdots$

$$\overset{(m)}{\xi}_s=\frac{\partial\xi_s}{\partial q_k}\overset{(m)}{q}_k+m\frac{\partial^2\xi_s}{\partial q_k\partial q_j}\overset{(m-1)}{q}_k\dot{q}_j+m\frac{\partial^2\xi_\alpha}{\partial q_k\partial t}\overset{(m-1)}{q}_k+\cdots \tag{28}$$

有

$$X^{(m)}(F_s)=X^{(m)}(\overset{(m)}{q}_s-h_s(t,q,\dot{q},\cdots,\overset{(m-1)}{q}))=\overset{(m)}{\xi}_k-\dot{q}_k\overset{(m)}{\xi}_0-C_m^1\ddot{q}_k\overset{(m-1)}{\xi_0}-C_m^2\dddot{q}_k\overset{(m-2)}{\xi_0}-\cdots-C_m^{m-2}\overset{(m-1)}{q}_k\ddot{\xi}_0-C_m^{m-1}\overset{(m)}{q}_k\dot{\xi}_0-X^{(m-1)}(h_s)=\frac{\partial\xi_k}{\partial q_r}\overset{(m)}{q}_r-\dot{q}_k\frac{\partial\xi_0}{\partial q_r}\overset{(m)}{q}_r-C_m^{m-1}\overset{(m)}{q}_k(\frac{\partial\xi_0}{\partial t}+\frac{\partial\xi_0}{\partial q_r}\dot{q}_r)-X^{(m-1)}(h_s)+D(t,q,\dot{q},\cdots,\overset{(m-1)}{q}) \tag{29}$$

其中 $D(t,q,\dot{q},\cdots,\overset{(m-1)}{q})$ 为其余不含 $\overset{(m)}{q}_\alpha$ 项的代数和. 同理可得

$$X^{(m)}F_s|_{F_s=0}=\frac{\partial\xi_k}{\partial q_r}h_r-\dot{q}_k\frac{\partial\xi_0}{\partial q_r}h_r-C_m^{m-1}h_k(\frac{\partial\xi_0}{\partial t}+\frac{\partial\xi_0}{\partial q_r}\dot{q}_r)-X^{(m-1)}(h_s)+D(t,q,\dot{q},\cdots,\overset{(m-1)}{q}) \tag{30}$$

式(29)减去式(30)得到

$$X^{(m)}(F_s)-X^{(m)}(F_s)|_{F_s=0}=(\frac{\partial\xi_k}{\partial q_r}-\dot{q}_k\frac{\partial\xi_0}{\partial q_r})(\overset{(m)}{q}_r-h_r)-C_m^{m-1}(\overset{(m)}{q}_k-h_k)(\frac{\partial\xi_0}{\partial t}+\frac{\partial\xi_0}{\partial q_r}\dot{q}_r)=[(\frac{\partial\xi_k}{\partial q_r}-\dot{q}_k\frac{\partial\xi_0}{\partial q_r})-C_m^{m-1}\delta_s^l(\frac{\partial\xi_0}{\partial t}+\frac{\partial\xi_0}{\partial q_r}\dot{q}_r)]F_l=\beta_s^lF_l \tag{31}$$

则共形因子为

$$\Gamma_s^l=\beta_s^l=(\frac{\partial\xi_k}{\partial q_r}-\dot{q}_k\frac{\partial\xi_0}{\partial q_r})-C_m^{m-1}\delta_s^l(\frac{\partial\xi_0}{\partial t}+\frac{\partial\xi_0}{\partial q_r}\dot{q}_r) \tag{32}$$

定义 5 如果无限小变换的生成元 ξ_0 和 ξ_s 满足确定方程(12)及限制方程,则相应共形不变性为高阶非完整系统的弱 Lie 对称性的共形不变性.

定义 6 如果无限小变换的生成元 ξ_0 和 ξ_s 满足确定方程(12)、限制方程和附加限制方程(17),则相应共形不变性为高阶非完整系统的强 Lie 对称性的共形不变性.

5 结构方程与守恒量

共形不变性在一定条件下可以导致相应的守恒量.

命题 2[17] 如果无限小变换的生成元 ξ_0 和 ξ_s 满足方程(32),且存在规范函数 $G=G(t,q,\dot{q})$ 满足结构方程

$$L\dot{\xi}_0+X^{(1)}(L)+(Q_s''+\Lambda_s)(\xi_s-\dot{q}_s\xi_0)+\dot{G}=0 \tag{33}$$

其中

$$\Lambda_\sigma=-\lambda_\beta\frac{\partial\varphi_\beta}{\partial\overset{(m)}{q}_\sigma},\Lambda_{\varepsilon+\beta}=\lambda_\beta \tag{34}$$

则与高阶非完整系统(1)和(3)相应的完整系统(5)的共形不变性拥有如下形式的守恒量

$$I=L\xi_0+\frac{\partial L}{\partial\dot{q}_s}(\xi_s-\dot{q}_s\xi_0)+G=\mathrm{const} \tag{35}$$

命题 3 对于满足高阶非完整系统(1)和(3)的弱(强)Lie 对称性的无限小变换的生成元 ξ_0 和 ξ_s,若存在规范函数 $G=G(t,q,\dot{q})$ 满足结构方程(33),则高阶非完整系统的弱(强)Lie 对称性的共形不变性可导致守恒量(35).

6 算例

设系统的 Lagrange 函数为

$$L=\frac{1}{2}(\dot{q}_1^2+\dot{q}_2^2+\dot{q}_3^2)-q_3 \tag{36}$$

所受约束是三阶的,有形式

$$\dddot{q}_1-t\dddot{q}_2+t^2\dddot{q}_3=0 \tag{37}$$

研究系统的共形不变性与守恒量.

系统的运动微分方程为

$$\begin{cases}\ddot{q}_1=\lambda\\ \ddot{q}_2=-\lambda t\\ \ddot{q}_3+1=\lambda t^2\end{cases}\tag{38}$$

由此消去 λ,得

$$t\ddot{q}_1+\ddot{q}_2=0,\ t^2\ddot{q}_1-(\ddot{q}_3+1)=0\tag{39}$$

将方程(39)对 t 求一次导数,并将结果与约束方程(37)联合,解得

$$\begin{cases}\dddot{q}_1=-\dfrac{t(1+2t^2)}{1+t^2+t^4}\ddot{q}_1=h_1\\ \dddot{q}_2=-\dfrac{1-t^4}{1+t^2+t^4}\ddot{q}_1=h_2\\ \dddot{q}_3=\dfrac{t(2+t^2)}{1+t^2+t^4}\ddot{q}_1=h_3\end{cases}\tag{40}$$

并且有

$$\Lambda_1=\lambda=\ddot{q}_1,\Lambda_2=-\lambda t=-t\ddot{q}_1,\Lambda_3=\lambda t^2=t^2\ddot{q}_1\tag{41}$$

取　$\xi_0=0,\xi_1=q_1,\xi_2=q_2,\xi_3=q_3+\dfrac{1}{2}t^2$　(42)

则有

$$X^{(3)}=\xi_1\frac{\partial}{\partial q_1}+\xi_2\frac{\partial}{\partial q_2}+\xi_3\frac{\partial}{\partial q_3}+\dot{\xi}_1\frac{\partial}{\partial \dot{q}_1}+\dot{\xi}_2\frac{\partial}{\partial \dot{q}_2}+\dot{\xi}_3\frac{\partial}{\partial \dot{q}_3}+\ddot{\xi}_1\frac{\partial}{\partial \ddot{q}_1}+\ddot{\xi}_2\frac{\partial}{\partial \ddot{q}_2}+\ddot{\xi}_3\frac{\partial}{\partial \ddot{q}_3}+\dddot{\xi}_1\frac{\partial}{\partial \dddot{q}_1}+\dddot{\xi}_2\frac{\partial}{\partial \dddot{q}_2}+\dddot{\xi}_3\frac{\partial}{\partial \dddot{q}_3}\tag{43}$$

$$X^{(3)}\begin{pmatrix}F_1\\F_2\\F_3\end{pmatrix}=X^{(3)}\begin{pmatrix}\dddot{q}_1+\dfrac{t(1+2t^2)}{1+t^2+t^4}\ddot{q}_1\\ \dddot{q}_2+\dfrac{1-t^4}{1+t^2+t^4}\ddot{q}_1\\ \dddot{q}_3+\dfrac{t(2+t^2)}{1+t^2+t^4}\ddot{q}_1\end{pmatrix}=$$

$$\begin{pmatrix}\dddot{q}_1+\dfrac{t(1+2t^2)}{1+t^2+t^4}\ddot{q}_1\\ \dddot{q}_2+\dfrac{1-t^4}{1+t^2+t^4}\ddot{q}_1\\ \dddot{q}_3+\dfrac{t(2+t^2)}{1+t^2+t^4}\ddot{q}_1\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}\dddot{q}_1+\dfrac{t(1+2t^2)}{1+t^2+t^4}\ddot{q}_1\\ \dddot{q}_2+\dfrac{1-t^4}{1+t^2+t^4}\ddot{q}_1\\ \dddot{q}_3+\dfrac{t(2+t^2)}{1+t^2+t^4}\ddot{q}_1\end{pmatrix}\tag{44}$$

因此我们得到共形因子为

$$\Gamma=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\tag{45}$$

由共形因子表达式(32)式我们也可得到

$$\Gamma=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

显然生成元满足限制方程

$$\begin{cases}X^{(2)}(t\ddot{q}_1+\ddot{q}_2)|_{t\ddot{q}_1+\ddot{q}_2=0}=0\\ X^{(2)}(t^2\ddot{q}_1-(\ddot{q}_3+1))|_{t^2\ddot{q}_1-(\ddot{q}_3+1)=0}=0\end{cases}\tag{46}$$

即

(47)

因此相应对称性为弱 Lie 对称性. 但是生成元不满足附加限制方程,即

$$\xi_1-\dot{q}_1\xi_0-t(\xi_2-\dot{q}_2\xi_0)+t^2(\xi_3-\dot{q}_3\xi_0)\neq 0\tag{48}$$

故相应对称性不是强 Lie 对称性的.

将方程(36)、(41)和(42)代入结构方程(33),得到

$$G=-(q_1\dot{q}_1+q_2\dot{q}_2+q_3\dot{q}_3+\frac{1}{2}t^2\dot{q}_3)\tag{49}$$

将方程(36)、(42)和(49)代入方程(35)得到守恒量为

$$I=-\frac{1}{2}t^2\dot{q}_3\tag{50}$$

7　结论

本文研究了与高阶非完整系统相应的完整系统的共形不变性,利用共形不变性的定义和 Lie 对称性的确定方程,得到了系统共形不变性的共形因子,该共形因子也就是系统的共形不变性同时是 Lie 对称性的充分必要条件. 如果无限小变换的生成元满足限制方程,则相应共形不变性为高阶非完整系统弱 Lie 对称性的共形不变性;如果无限小变换的生成元满足限制方程和附加限制方程,则相应共形不变性为高阶非完整系统强 Lie 对称性的共形不变性. 系统共形不变性在一定条件下可以导致 Noether 守恒量.

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CONFORMAL INVARIANCE AND NOETHER CONSERVED QUANTITY OF HIGHER-ORDER NONHOLONOMIC SYSTEM *

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Abstract This paper studied the conformal invariance and conserved quantity of the holonomic system, which corresponds to a higher-order nonholonomic system. Firstly, the definition and determining equation of conformal invariance of the system were presented. The conformal factor, which is the necessary and sufficient condition that conformal invariance of the system would be Lie symmetry, was deduced from conformal invariance and Lie symmetry. The conformal invariance of weak and strong Lie symmetry for the higher-order nonholonomic system was given using restriction equations and additional restriction equations. Secondly, the Noether conserved quantity of conformal invariance of the system was derived. Lastly, an example was given to illustrate the application of the results.

Key words higher-order nonholonomic system, conformal invariance, Noether conserved quantity

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