刚弹耦合质量摆减振结构的动力学建模*

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摘要 利用 Hamilton 变分原理建立高层建筑质量摆减振结构在考虑刚弹耦合作用下的动力学模型. 此模型中把高层建筑当作竖直悬臂梁即无线自由度结构处理,模型具有 2 个独立变量,为梁横向位移和质量摆的转角,同时考虑了几何非线性的约束条件,形成了一个非线性刚弹耦合方程. 最后采用 Galerkin 方法对其离散化,导出有限维离散化动力学模型,并给出仿真算例,验证了该模型的正确性.

关键词 高层建筑,　质量摆,　Hamilton 原理,　刚弹耦合,　数学建模

引言

高层建筑结构利用悬吊质量摆减振的方法由于其具有简单、经济、易于实施等优点而应用广泛,但随着科学技术的发展和经济实力的提高,越来越多的建筑采用轻质高强的材料以及其高度的不断增加,使得高层建筑结构在风荷载和地震作用下的各种响应增大,为了更好地控制其响应,采用传统的建模方法已达不到预期的效果,文献[1]-[4]中的传统的建模只考虑剪力的影响而把高层建筑简化为剪切型结构,但实际中考虑得更多的应是弯曲变形,为此需要将高层建筑结构当作弹性体即竖直悬臂连续梁来考虑,这样建立一个刚弹耦合的非线性方程更符合高层建筑质量摆减振结构的实际受力情况.

利用 Hamilton 原理和 Euler – Bernoulli 梁的基本方程建立了考虑刚弹耦合作用的质量摆减振结构的非线性动力学模型,该模型与目前的 MDOF 线性模型相比,具有明显的优越性.

1 基本概念与假设

设梁高为 H,横截面面积为 A,单位质量密度为 ρ（即单位体积的质量）,弹性模量为 EI,质量摆的质量为 m,摆长为 l,与竖直面的摆角为 $\theta(t)$. 根据悬臂梁的边界条件,梁的中线不考虑伸长,即梁无轴向变形而只有横向变形. 如图 1,假设梁最初沿着 x 轴,随后在 $x - y$ 平面内振动,沿着梁中线的

曲线坐标为 s,于是振动时梁的坐标表示为$(x(s,t),y(s,t))$（后面将梁的轴向坐标和横向坐标分别简写为 x 和 y,质量摆的摆角简写为 θ）,质量摆在距离梁底面 L 处$(0 < L \leqslant H)$.

基本假设[5-10]为:（1）梁的截面尺寸远远小于其高度;（2）系统在 $x - y$ 平面内运动;（3）忽略梁的转动惯量和剪切变形的影响;（4）忽略梁的阻力的影响.

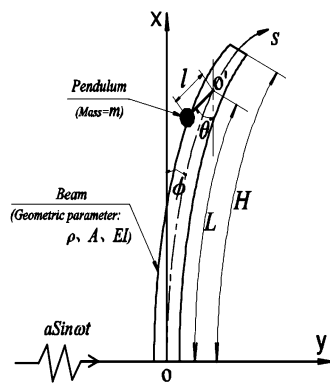

图 1　悬臂梁质量摆减振结构模型

Fig. 1　The model of the damping structure for cantilever beam with pendulum

1.1 弯曲曲率 κ

设悬臂梁的截面转角为 ϕ,则弯曲曲率可表示为:

$$\kappa = \frac{\partial \phi}{\partial s} \tag{1}$$

而 ϕ、$x(s,t)$ 和 $y(s,t)$ 分别满足:

$$\cos\phi = \frac{\partial x}{\partial s}, \sin\phi = \frac{\partial y}{\partial s} \tag{2}$$

$$\left(\frac{\partial x}{\partial s}\right)^2 + \left(\frac{\partial y}{\partial s}\right)^2 = 1 \tag{3}$$

将方程（2）、（3）对变量 s 求导分别得到：

$$\frac{\partial \phi}{\partial s} = \frac{\partial x}{\partial s}\frac{\partial^2 y}{\partial s^2} - \frac{\partial y}{\partial s}\frac{\partial^2 x}{\partial s^2} \tag{4}$$

$$\frac{\partial x}{\partial s}\frac{\partial^2 x}{\partial s^2} + \frac{\partial y}{\partial s}\frac{\partial^2 y}{\partial s^2} = 0 \tag{5}$$

联立方程（1）、（3）、（4）和（5）可得：

$$\kappa = \frac{\partial^2 y/\partial s^2}{\sqrt{1 - (\partial y/\partial s)^2}} = \frac{y''}{\sqrt{1 - (y')^2}} \tag{6}$$

1.2 哈密顿原理

全文运用能量方法哈密顿原理，表述为：

$$\int_{t_0}^{t1}\delta R \mathrm{d}t = \int_{t_0}^{t1}\delta(T - V)\mathrm{d}t = 0 \tag{7}$$

式中：R 为哈密顿函数，$T = T_1 + T_2$，$V = V_1 + V_2 + V_3$，T_1、T_2 分别为悬臂梁和质量摆的动能，V_1、V_2 和 V_3 分别为悬臂梁的弯曲应变能、悬臂梁的重力势能和质量摆的重力势能.

1.3 考虑的阶数

梁作大范围运动，其横向位移认为很大，故横向坐标也很大，即

$$y \sim \vartheta(\varepsilon) \tag{8}$$

而梁的轴向位移认为比横向高一阶，所以轴向坐标比横向坐标高一阶，即：

$$x \sim \vartheta(\varepsilon^2) \tag{9}$$

其中：$\varepsilon \ll 1$.

由于系统结构的对称性，非线性方程将考虑至奇数阶，这里取至 $\vartheta(\varepsilon^3)$，然而为了获得更为准确的方程通常比要求的高一阶，所以以下的在对变量的积分方程都取至 $\vartheta(\varepsilon^4)$.

2 系统的运动方程

2.1 系统动能

按二维平面状态表示，梁的动能为：

$$T_1 = \frac{1}{2}\int_0^H \rho A(\dot{x}^2 + \dot{y}^2)\mathrm{d}s \tag{10}$$

质量摆的摆质点的运动速度由梁转动和自身转动的牵连复合组成，其动能为：

$$T_2 = \frac{1}{2}m\left[(\dot{x}|_{s=L} + l\,\dot\theta\sin\theta)^2 + (\dot{y}|_{s=L} - l\,\dot\theta\cos\theta)^2\right] \tag{11}$$

式中：$(\ )'$ 表示对曲线坐标 s 的导数，$(\ \dot{})$ 表示对时间 t 的导数，并均用于以下各式.

2.2 系统势能

梁的弯曲应变能为：

$$V_1 = \frac{EI}{2}\int_0^H \kappa^2 \mathrm{d}s \tag{12}$$

将式（6）代入上式得：

$$V_1 = \frac{EI}{2}\int_0^H \frac{(y'')^2}{1 - (y')^2}\mathrm{d}s =$$
$$\frac{EI}{2}\int_0^H (y'')^2(1 + y'^2)\mathrm{d}s + \vartheta(\varepsilon^5) \tag{13}$$

令梁底面的重力势能为零势能位置，则梁的重力势能为：

$$V_2 = \int_0^H (\rho A g)x\mathrm{d}s \tag{14}$$

质量摆的重力势能为：

$$V_3 = mg(x|_{s=L} - l\cos\theta) \tag{15}$$

2.3 运动方程

分别对方程（10）～（15）取变分，并在区间（t_0, t_1）上对 t 积分，同时代入边界条件

$$y(0,t) = a\sin\omega t,$$
$$y'(0,t) = y''(H,t) = y'''(H,t) = 0 \tag{16}$$

得：

$$\int_{t_0}^{t1}\delta T_1 \mathrm{d}t = -\int_{t_0}^{t1}\int_0^H \rho A(\ddot{x}\delta x + \ddot{y}\delta y)\mathrm{d}s\mathrm{d}t \tag{17}$$

$$\int_{t_0}^{t1}\delta T_1 \mathrm{d}t = -m \cdot \int_{t_0}^{t1}\left[\ddot{x}|_{s=L} + (l\ddot\theta\sin\theta + l\dot\theta^2\cos\theta)\right]\delta(x|_{s=L})\mathrm{d}t - m \cdot \int_{t_0}^{t1}\left[\ddot{y}|_{s=L} - (l\ddot\theta\cos\theta - l\dot\theta^2\sin\theta)\right]\delta(y|_{s=L})\mathrm{d}t -$$
$$m \cdot \int_{t_0}^{t1}\left[\ddot{x}|_{s=L}(l\sin\theta) - \ddot{y}|_{s=L}(l\cos\theta) + l^2\ddot\theta\right]\delta\theta\mathrm{d}t \tag{18}$$

$$\int_{t_0}^{t1}\delta V_1 \mathrm{d}t = EI\int_{t_0}^{t1}\int_0^H\left[y''''(1 + y'^2) + 4y'y''y''' + y''^3\right]\delta y\mathrm{d}s\mathrm{d}t \tag{19}$$

$$\int_{t_0}^{t1}\delta V_2 \mathrm{d}t = \rho A g\int_{t_0}^{t1}\int_0^H\delta x\mathrm{d}s\mathrm{d}t \tag{20}$$

$$\int_{t_0}^{t1}\delta V_3 \mathrm{d}t = mg\int_{t_0}^{t1}\delta(x|_{s=L})\mathrm{d}t + mgl\int_{t_0}^{t1}\sin\theta\delta\theta\mathrm{d}t \tag{21}$$

将（3）式分别对 s 和 t 求导得：

$$x' = \sqrt{1 - y'^2},$$
$$\dot{x}' = -\frac{y'\dot{y}'}{x'} = -\frac{y'\dot{y}'}{\sqrt{1 - y'^2}} = -y'\dot{y}'(1 + \frac{1}{2}y'^2) + \vartheta(\varepsilon^5)$$

由上面两式进一步求得：

$$\delta x^{'} = -\frac{y^{'}\delta y^{'}}{\sqrt{1-y^{'2}}} = -y^{'}(1+\frac{1}{2}y^{'2})\delta y^{'} + \vartheta(\varepsilon^{4})$$

$$\ddot{x}^{'} = -\dot{y}^{'2}(1+\frac{3}{2}y^{'2}) - \ddot{y}^{'}(y^{'}+\frac{1}{2}y^{'2})$$

所以,

$$\delta x = \int_{0}^{s}\delta x^{'}\mathrm{d}s = -y^{'}(1+\frac{1}{2}y^{'2})\delta y + \int_{0}^{s}y^{''}(1+\frac{3}{2}y^{'2})\delta y\mathrm{d}s + \vartheta(\varepsilon^{4}) \tag{22}$$

$$\ddot{x} = \int_{0}^{s}\ddot{x}^{'}\mathrm{d}s = -\int_{0}^{s}[\dot{y}^{'2}(1+\frac{3}{2}y^{'2}) + \ddot{y}^{'}(y^{'}+\frac{1}{2}y^{'2})]\mathrm{d}s \tag{23}$$

将方程(17)至(23)代入(7)式中,并应用 Dirac 函数的性质

$$f(s)\mid_{s=L} = f(L) = \int_{0}^{H}f(s)\Delta(s-L)\mathrm{d}s^{[11]}$$

(为了区别变分符号 δ 与 Dirac 函数,将 Dirac 函数表示为 Δ(s)),以及关系式

$$\int_{0}^{H}g(s)(\int_{0}^{H}f(s)\delta y\mathrm{d}s)\mathrm{d}s = \int_{0}^{H}(\int_{0}^{H}g(s)\mathrm{d}s)f(s)\delta y\mathrm{d}s^{[5]}$$

整理后得系统运动微分方程[5,6]为:

$$\rho A\ddot{y} + m\ddot{y}\mid_{s=L}\Delta(s-L) + ml(\ddot{\theta}\cos\theta - \dot{\theta}^{2}\sin\theta)\Delta(s-L) + EI(y^{''''} + y^{'''}y^{'2} + 4y^{'}y^{''}y^{'''} + y^{''3}) + \rho Ag[-y^{'}(1+\frac{1}{2}y^{'2}) + (H-s)y^{''}(1+\frac{3}{2}y^{'2})] + m[g + l(\ddot{\theta}\sin\theta + \dot{\theta}^{2}\cos\theta)]\cdot[-y^{'}(1+\frac{1}{2}y^{'2}) + (H-s)y^{''}(1+\frac{3}{2}y^{'2})]\mid_{s=L}\Delta(s-L) + m\{[y^{'} - (H-s)y^{''}]\int_{0}^{s}(\dot{y}^{'2} + \ddot{y}^{'}y^{'})\mathrm{d}s\}\mid_{s=L}\Delta(s-L) + \rho A[y^{'} - (H-s)y^{''}]\int_{0}^{s}(\dot{y}^{'2} + \ddot{y}^{'}y^{'})\mathrm{d}s = 0 \tag{24}$$

$$l\ddot{\theta} - (\cos\theta)\ddot{y}\mid_{s=L} + g\sin\theta - (\sin\theta)\cdot\{\int_{0}^{s}[\dot{y}^{'2}(1+\frac{3}{2}y^{'2}) + \ddot{y}^{'}(y^{'}+\frac{1}{2}y^{'3})]\mathrm{d}s\}\mid_{s=L} = 0 \tag{25}$$

　　由于边界 s = 0 处为动位移,故为了能直接运用一端固支另一端自由的悬臂梁的振型函数,必须先将边界条件齐次化[8,11].

　　设 $y(s,t) = \xi(s,t) + a\sin\omega t$ (26)
则边界条件(16)变为:

$$\xi(0,t) = \xi^{'}(0,t) = \xi^{''}(H,t) = \xi^{'''}(H,t) = 0 \tag{27}$$

满足条件.

　　另外,初始条件变为:

$$\xi(s,0) = 0, \dot{\xi}(s,0) = -a\omega, \theta(0) = 0, \dot{\theta}(0) = 0 \tag{28}$$

将式(26)代入到方程(24)和(25)中,则边界条件齐次化后的系统运动微分方程为:

$$\rho A\ddot{\xi} + m\ddot{\xi}\mid_{s=L}\Delta(s-L) - ml(\ddot{\theta}\cos\theta - \dot{\theta}^{2}\sin\theta)\Delta(s-L) + EI(\xi^{''''} + \xi^{'''}\xi^{'2} + 4\xi^{'}\xi^{''}\xi^{'''} + \xi^{''3}) + \rho Ag[-\xi^{'}(1+\frac{1}{2}\xi^{'2}) + (H-s)\xi^{''}(1+\frac{3}{2}^{'2})] + m[g + l(\ddot{\theta}\sin\theta + \dot{\theta}^{2}\cos\theta)]\cdot[-\xi^{'}(1+\frac{1}{2}\xi^{'2}) + (H-s)\xi^{''}(1+\frac{3}{2}\xi^{'2})]\mid_{s=L}\Delta(s-L) + \rho A[\xi^{'} - (H-s)\xi^{''}]\int_{0}^{s}(\dot{\xi}^{'2} + \ddot{\xi}^{'}\xi^{'})\mathrm{d}s + m\{[\xi^{'} - (H-s)\xi^{''}]\int_{0}^{s}(\dot{\xi}^{'2} + \ddot{\xi}^{'}\xi^{'})\mathrm{d}s\}\mid_{s=L}\Delta(s-L) = \rho Aa\omega^{2}\sin\omega t + m\Delta(s-L)a\omega^{2}\sin\omega t \tag{29}$$

$$l\ddot{\theta} - (\sin\theta)\{\int_{0}^{s}[\dot{\xi}^{'2}(1+\frac{3}{2}\xi^{'2}) + \ddot{\xi}^{'}(\xi^{'}+\frac{1}{2}\xi^{'3})]\mathrm{d}s\}\mid_{s=L} - (\cos\theta)\ddot{\xi}\mid_{s=L} + g\sin\theta = -(\cos\theta)a\omega^{2}\sin\omega t \tag{30}$$

2.4 离散化方程

　　采用 Galerkin 方法对方程(29)、(30)进行离散化,设其解的形式为

$$\xi(s,t) = \sum_{i=1}^{n}\varphi_{i}(s)q_{i}(t) \tag{31}$$

其中:$\varphi_{i}(s)$ 为满足边界条件(27)的悬臂梁横向振动的第 i 阶模态振型函数,$q_{i}(t)$ 为对应的第 i 阶模态广义坐标,$i \in [1,n]$.

　　将式(31)代入方程(29)和(30)中,并在两端同乘以 $\varphi_{j}(s)$,且在区间[0,H]上积分,同时利用悬臂梁振型函数的正交性可得系统如下的离散化动力学方程:

$$(M_{0} + M_{cij})\ddot{q}_{i} + (K_{0ij} + K_{cij})q_{i} + R_{ij}q_{i}^{3} + P_{ij}q_{j}(q_{i}^{2} + q_{i}\ddot{q}_{i}) + f_{1j}(\ddot{\theta},\dot{\theta},t) = F_{1j}(s,t) \tag{32}$$

$$\ddot{\theta} + \frac{g}{l}\sin\theta + f_{2i}(\theta,q_{i},\dot{q}_{i},\ddot{q}_{i},t) = F_{2}(t) \tag{33}$$

式中:$M_{0} = \rho A$, 　$M_{cij} = m\varphi_{i}(L)\varphi_{j}(L)$,

$$K_{0ij} = EI\int_{0}^{H}\varphi_{i}^{''''}\varphi_{j}\mathrm{d}s$$

$$K_{cij} = \rho Ag[\int_{0}^{H}(H-s)\varphi_{i}^{''}\varphi_{j}\mathrm{d}s - \int_{0}^{H}\varphi_{i}^{'}\varphi_{j}\mathrm{d}s] +$$

$$m[g + l(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)] \cdot \{[(H - L)\varphi_i''(L) - \varphi_i'\varphi_j(L)]\varphi_j(L)\},$$

$$R_{ij} = EI\int_0^H (\varphi_i'''\varphi_i'^2\varphi_j + 4\varphi_i'\varphi_i''\varphi_i'''\varphi_j + \varphi_i''^3\varphi_j)\mathrm{d}s +$$

$$\rho Ag[\int_0^H (H - s)\frac{3}{2}\varphi_i''\varphi_i'^2\varphi_j\mathrm{d}s - \int_0^H \frac{1}{2}\varphi_i'^3\varphi_j\mathrm{d}s] +$$

$$m[g + l(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)] \cdot [(H - L) \times$$

$$\frac{3}{2}\varphi_i''(L)\varphi_i'^2(L)\varphi_j(L) - \frac{1}{2}\varphi_i'^3(L)\varphi_j(L)],$$

$$P_{ij} = \rho A\{\int_0^H [\varphi_i' - (H - s)\varphi_i''](\int_0^s \varphi_i'^2\mathrm{d}s)\varphi_j\mathrm{d}s\} +$$

$$m\varphi_j(L)(\int_0^L \varphi_i'^2\mathrm{d}s)[\varphi_i'(L) - (H - L)\varphi_i''(L)],$$

$$f_{1j}(\ddot{\theta}, \dot{\theta}, t) = -ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)\varphi_j(L)$$

$$f_{2i}(\theta, q_i, \dot{q}_i, \ddot{q}_i, t) = -\frac{\cos\theta}{l}\ddot{q}_i - (\frac{\sin\theta}{l}) \times$$

$$[(\int_0^L \varphi_i'^2\mathrm{d}s)(\dot{q}_i^2 + \ddot{q}_iq_i) + (\int_0^L \frac{1}{2}\varphi_i'^4\mathrm{d}s) \times$$

$$(3\dot{q}_i^2q_i^2 + \ddot{q}_iq_i^3)],$$

$$F_{1j}(s, t) = [\rho A\int_0^H \varphi_j\mathrm{d}s + m\varphi_j(L)]a\omega^2\sin\omega t,$$

$$F_2(t) = -\frac{\cos\theta}{l}a\omega^2\sin\omega t.$$

3 仿真算例

选取文献[3]（实验）中的悬臂梁结构,梁的几何尺寸 $H = 956\mathrm{mm}$, $\rho = 2.85\mathrm{g/cm}^3$, $A = 25\mathrm{mm} \times 3\mathrm{mm} = 75\mathrm{mm}^2$, $E = 72\mathrm{GPa}$, 质量摆摆长为 $l = 35\mathrm{mm}$, 并取 $L = H$, ω 为悬臂梁结构基频,系数 $a = 0.536\mathrm{mm}$, 截取一阶模态($i = j = 1$), 代入悬臂梁的振型函数以及方程(32)和(33)中,经计算并整理得到如下方程:

$$\ddot{q}_1 + 243.69q_1 + 923.68q_1^3 + 6.2q_1(\dot{q}_1^2 + \ddot{q}_1q_1) - (0.00373q_1 + 0.01624q_1^3)(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) -$$

$$0.00126(\ddot{\theta}\cos\theta + \dot{\theta}^2\sin\theta) =$$

$$0.7451a\omega^2\sin(16.746t) \quad (34)$$

$$\ddot{\theta} - 58.443(\cos\theta)\ddot{q}_1 + 280\sin\theta - (\sin\theta) \times$$

$$(145.299q_1^2 + 1980.309\dot{q}_1^2q_1^2 + 145.299\dot{q}_1q_1 +$$

$$660.103\dot{q}_1q_1^3) = -28.571a\omega^2\sin(16.746t) \quad (35)$$

无质量摆时悬臂梁的运动方程为:

$$\ddot{q}_1 + 264.32q_1 + 1002.47q_1^3 + 6.66q_1(\dot{q}_1^2 + \ddot{q}_1q_1) =$$

$$0.7656a\omega^2\sin(16.746t) \quad (36)$$
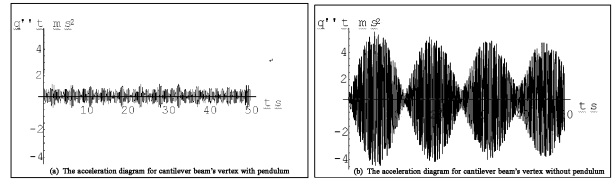
动力学仿真结果如图 2 所示.

图 2 悬臂梁顶点加速度变化曲线

Fig. 2 The acceleration diagram for cantilever beam's vertex

可见,和文献[3]中的结果相同,当结构加上质量摆后悬臂梁的振动明显减小了,且有无质量摆时的梁的加速度值的数量级与文献[3]中的数量级相同. 而且由方程(34)和(35)得到的悬臂梁、质量摆频率均约为 $\omega = 16.746rad/s$, 和动位移频率一致,即系统发生共振,由此证明了基于 Hamilton 原理建立的考虑刚弹耦合作用质量摆减振结构的动力学模型的有效性.

4 结论

利用 Hamilton 原理推导出了高层建筑质量摆减振结构在受到弯曲振动作用下的模型,该模型考虑了刚弹耦合作用,是一个非线性的动力学方程,又运用 Galerkin 方法,将系统在弹性梁轴向不可伸长的几何约束条件下的模型离散化,得到了一个有限维动力学模型. 基于实验基础上的算例验证了该模型的有效性.

同时,模型中将质量的位置设为梁上的任意处,相对 MDOF 模型更具有一般性.

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THE DYNAMIC MODELING FOR THE DAMPING STRUCTURE WITH A MASS PENDULUM CONSIDERING THE RIGID-ELASTIC COUPLING EFFECTS *

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Abstract　Based on the Hamilton's principle, a dynamic model of high – storey building damping structure with a mass pendulum considering the rigid – elastic coupling effects is discussed. In the model, the high – storey building was regarded as an upright cantilever beam namely infinite degrees of freedom. And the model had two variables, which were lateral displacement of the beam and angle of rotation of the mass pendulum. Then, a nonlinear rigid – elastic coupling equation was formed considering the constraint condition of geometric nonlinearity. Finally, The finite dimensions dynamic equations considering the rigid – elastic coupling effects were obtained by taking advantage of Galerkin method, and the validity of the theories presented in this paper was verified by a numerical simulation example.

Key words　high-storey building,　mass pendulum,　Hamilton principal,　rigid-elastic coupling,　mathematical modeling