

基础位移激励下碳纤维矩形索网的非线性振动*

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摘要 研究碳纤维材料矩形单层索网在基础位移激励下的非线性振动问题,是索网抗震防震设计的基础.在考虑温度变化的基础上,建立了碳纤维矩形索网在基础位移激励下的非线性振动方程,采用 Galerkin 原理及 KBM 法求得碳纤维材料矩形单层索网非线性振动的近似解.在把碳纤维索网与钢丝索网比较的基础上,讨论分析了温度、位移激励、振幅、阻尼等因素对矩形单层索网非线性振动的影响,得到了碳纤维索网振动特性优于钢丝索网振动特性的结论.

关键词 基础, 位移激励, 碳纤维, 索网, 非线性振动

引言

由于碳纤维增强复合材料的徐变和松弛等重要指标均优于钢材,且碳纤维材料还具有耐久性好、抗腐蚀、自重轻、富有柔性、高强度等优点,因此在 20 世纪末随着军工技术的解禁,碳纤维增强复合材料在机械、土木建筑等实际工程中得到了应用,另外,钢丝绳因其使用条件而极易产生腐蚀退化和振动疲劳等问题,所以采用碳纤维索将是解决这一问题的根本途径.早在 1987 年,就有资深专家提出在直布罗陀海峡最窄处建造 8400m 的全碳纤维斜拉桥的伟大构想和理论可行性^[1].此后,瑞士、日本、丹麦、美国等国家都竞相开展碳纤维斜拉桥的研究,现已建成两座全碳纤维拉索的人行桥和两座部分采用碳纤维拉索的公路桥^[2-3],我国于 2005 年 5 月也在江苏大学建成了国内首座碳纤维拉索的人行桥.随着科学技术的发展,未来空间结构有采用碳纤维索网替代钢丝索网的趋势,可以预见在不远的将来,碳纤维材料将会推广到大跨度空间结构中,文献[4-8]研究了钢丝索网体系的非线性的自振特性,文献[9-11]首次研究了碳纤维双层索及索网的非线性自振特性.但是,至今未见到研究碳纤维索网在基础位移激励下非线性振动的文献.由于地震主要是靠基础位移激励来对结构产生破坏的,因此本文研究了碳纤维矩形单层索网在基础位移激励下的非线性振动问题,为碳纤维矩形单层索

网的抗震及防震设计进行前期理论研究.

1 索网非线性振动近似解

对于图 1 所示矩形索网,为了研究其非线性振动特性,可做如下基本假设:①索网的索是理想柔性的;②索的变形是小垂度的;③索材料满足虎克定律.

由弹性振动理论可以得到,矩形单层索网在基础位移激励下的非线性振动方程为

$$H_x \frac{\partial(w+w_s)}{\partial x^2} + H_y \frac{\partial(w+w_s)}{\partial y^2} + \Delta H_x \left[\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2(w+w_s)}{\partial x^2} \right] + \Delta H_y \left[\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2(w+w_s)}{\partial y^2} \right] = m \frac{\partial^2(w+w_s)}{\partial t^2} + \mu \frac{\partial(w+w_s)}{\partial t} \quad (1)$$

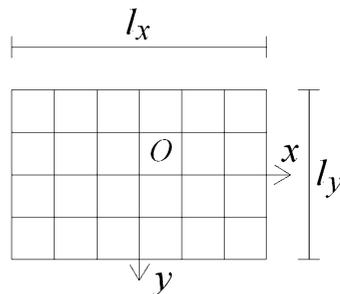


图 1 矩形单层索网

Fig. 1 The rectangular single cable net

式中, H_x 、 H_y 分别为索网 x 、 y 方向单位宽度内索拉力水平分量初值, ΔH_x 、 ΔH_y 分别为索网 x 、 y 方向单

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位宽度内索拉力增量水平投影, $z(x, y)$ 为索网在初始状态的曲面形状函数, $w(x, y, t)$ 为索网的振动位移函数, $w_s(x, y, t)$ 为地震时的基础位移函数, m 为

$$\begin{cases} \Delta H_x = \frac{EA_x}{l_x} \int_{-\frac{l_x}{2}}^{\frac{l_x}{2}} \left\{ \frac{\partial z}{\partial x} \frac{\partial(w+w_s)}{\partial x} + \frac{1}{2} \left[\frac{\partial(w+w_s)}{\partial x} \right]^2 \right\} dx - \alpha_s EA_x \Delta T \\ \Delta H_y = \frac{EA_y}{l_y} \int_{-\frac{l_y}{2}}^{\frac{l_y}{2}} \left\{ \frac{\partial z}{\partial y} \frac{\partial(w+w_s)}{\partial y} + \frac{1}{2} \left[\frac{\partial(w+w_s)}{\partial y} \right]^2 \right\} dy - \alpha_s EA_y \Delta T \end{cases} \quad (2)$$

式中, E 为索网材料的弹性模量, A_x, A_y 分别为 x, y 方向单位长度内索网的横截面积。

该索网初始状态下的曲面形状函数、索网的振动位移函数, 基础位移函数表达式分别为

$$\begin{cases} Z(x, y) = f \sin \frac{\pi(x + \frac{l_x}{2})}{l_x} \sin \frac{\pi(y + \frac{l_y}{2})}{l_y} \\ w(x, y, t) = T(t) \sin \frac{\pi(x + \frac{l_x}{2})}{l_x} \sin \frac{\pi(y + \frac{l_y}{2})}{l_y} \\ w_s(x, y, t) = A \cos \Omega t \end{cases} \quad (3)$$

把式(2)、式(3)代入式(1)中, 利用伽辽金原理可以得到下式

$$\frac{d^2 T}{dt^2} + \varepsilon \eta \frac{dT}{dt} + \omega_0^2 T + \varepsilon \alpha T^2 + \varepsilon \beta T^3 = \varepsilon F_0 + \varepsilon A_{11} \cos \Omega t + \varepsilon A_{22} \sin \Omega t \quad (4)$$

式中

$$\begin{aligned} \omega_0^2 &= \frac{\pi^2}{m} \left(\frac{H_x}{l_x^2} + \frac{H_y}{l_y^2} \right) + \frac{3\pi^4 E f^2}{8m} \left(\frac{A_x}{l_x^4} + \frac{A_y}{l_y^4} \right) - \\ &\quad \frac{\pi^2 \alpha_s E \Delta T}{m} \left(\frac{A_x}{l_x^4} + \frac{A_y}{l_y^4} \right) \\ \varepsilon \eta &= \frac{\mu}{m}, \varepsilon \alpha = \frac{9\pi^4 E f}{16m} \left(\frac{A_x}{l_x^4} + \frac{A_y}{l_y^4} \right), \\ \varepsilon \beta &= \frac{3\pi^4 E}{16m} \left(\frac{A_x}{l_x^4} + \frac{A_y}{l_y^4} \right), \\ \varepsilon F_0 &= -\frac{\pi^2 \alpha_s E \Delta T f}{m} \left(\frac{A_x}{l_x^2} + \frac{A_y}{l_y^2} \right), \\ \varepsilon A_{11} &= \frac{16}{\pi^2} A \Omega^2, \varepsilon A_{22} = \frac{16\mu A \Omega}{\pi^2 m} \end{aligned}$$

为了研究碳纤维矩形单层索网在基础位移激励下的非共振动力行为可设

$$\begin{cases} \frac{da}{dt} = \varepsilon A_1(a) + \varepsilon^2 A_2(a) + \dots \\ \frac{d\phi}{dt} = \omega_0 + \varepsilon \omega_1(a) + \varepsilon^2 \omega_2(a) \dots \end{cases} \quad (5)$$

把式(4)的解表示为如下形式

索网在单位面积质量, μ 为阻尼系数。

以图1单层索网中心 O 为坐标原点, 由弹性理论可知索网的拉力增量 $\Delta H_x, \Delta H_y$ 分别为

$$\begin{aligned} T(t) &= a \cos \phi + \varepsilon T_1(a, \phi, \Omega t) + \\ &\quad \varepsilon^2 T_2(a, \phi, \Omega t) + \dots \end{aligned} \quad (6)$$

把式(5)、式(6)代入(4)中可得

$$\begin{aligned} \omega_0^2 \frac{d^2 T_1}{d\phi^2} + 2\omega_0 \frac{d^2 T_1}{d\phi dt} + \frac{d^2 T_1}{dt^2} + \omega_0^2 T_1 &= F_0 - \frac{\alpha a^2}{2} + \\ &\quad A_{11} \cos \Omega t + A_{22} \sin \Omega t + (\eta \omega_0 a + 2\omega_0 A_1) \sin \phi + \\ &\quad (2\omega_0 \omega_1 a - \frac{3\beta a^3}{4}) \cos \phi - \frac{\alpha a^2}{2} \cos 2\phi - \\ &\quad \frac{\beta a^3}{2} \cos 3\phi \end{aligned} \quad (7)$$

为了使 T_1 不出现分母为零的项可令 $\sin \phi, \cos \phi$ 的系数为零得

$$A_1 = -\frac{a\eta}{2}, \omega_1 = \frac{3\beta a^2}{8\omega_0} \quad (8)$$

利用式(5)、式(8)可以求得

$$a = a_0 e^{-\frac{\varepsilon \eta t}{2}}, \phi = \omega_0 t - \frac{3\varepsilon \beta a_0^2}{8\varepsilon \eta \omega_0} e^{-\varepsilon \eta t} + \frac{3\varepsilon \beta a_0^2}{8\varepsilon \eta \omega_0} + \phi_0 \quad (9)$$

式中, a_0 是振幅 a 的初始值, ϕ_0 是相位角 ϕ 的初始值。

再由式(7)可以求得

$$\begin{aligned} T_1(t) &= \frac{2F_0 - \alpha a^2}{2\omega_0^2} + \frac{A_{11}}{\omega_0^2 - \Omega^2} \cos \Omega t + \\ &\quad \frac{A_{22}}{\omega_0^2 - \Omega^2} \sin \Omega t + \frac{\alpha a^2}{6\omega_0^2} \cos 2\phi + \frac{\beta a^3}{32\omega_0^2} \cos 3\phi \end{aligned} \quad (10)$$

所以碳纤维矩形单层索网在基础位移激励下的非共振近似解为

$$\begin{aligned} w(x, y, t) &= \left[a \cos \phi + \frac{2\varepsilon F_0 - \varepsilon \alpha a^2}{2\omega_0^2} + \right. \\ &\quad \left. \frac{\varepsilon A_{11}}{\omega_0^2 - \Omega^2} \cos \Omega t + \frac{\varepsilon A_{22}}{\omega_0^2 - \Omega^2} \sin \Omega t + \frac{\varepsilon \alpha a^2}{6\omega_0^2} \cos 2\phi + \right. \\ &\quad \left. \frac{\varepsilon \beta a^3}{32\omega_0^2} \cos 3\phi \right] \sin \frac{\pi(x + \frac{l_x}{2})}{l_x} \sin \frac{\pi(y + \frac{l_y}{2})}{l_y} \end{aligned} \quad (11)$$

2 算例分析及讨论

为了讨论分析有关因素对碳纤维矩形索网非线性动力行为的影响,以便把碳纤维矩形索网与钢丝绳网进行比较研究,本文按照等轴向刚度准则、等强度准则分别构造了碳纤维矩形索网. 矩形索网构造参数见表1. 表1中索网单位面积质量 m 已计入了作用在索网上的恒载. 其中,1#为钢丝绳网,2#为与1#等截面的碳纤维索网,3#为按照等轴向刚度准则构造的碳纤维索网,4#为按照等强度准则构造的碳纤维索网. 四种结构相同的计算参数为: $l_x = 80m, l_y = 60m, H_x = 1.6 \times 10^5 N/m, H_y = 1.35 \times 10^5 N/m, f = 0.1m$.

表1 结构材料和截面特性

Table 1 The property of material and section

construction	$E(N/m^2)$	$m(kg/m^2)$	$A_x = A_y(m^2)$	$\alpha_s(^\circ C)$
1#	2.0×10^{11}	86.0115	5.28×10^{-4}	1.2×10^{-5}
2#	1.6×10^{11}	74.3453	5.28×10^{-4}	0.7×10^{-6}
3#	1.6×10^{11}	75.0337	6.6×10^{-4}	0.7×10^{-6}
4#	1.6×10^{11}	73.1907	3.19×10^{-4}	0.7×10^{-6}

把有关参数代入以上相关各式中,即可算得矩形索网时程曲线,具体计算结果如图2—图7所示. 图中 $x = y = 0$.

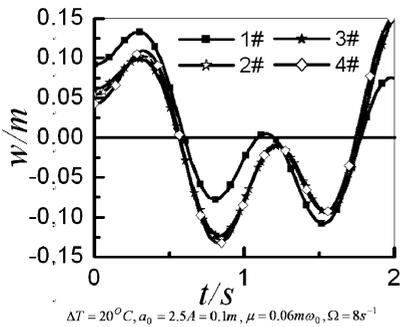


图2 矩形单层索网

Fig.2 The rectangular single cable net

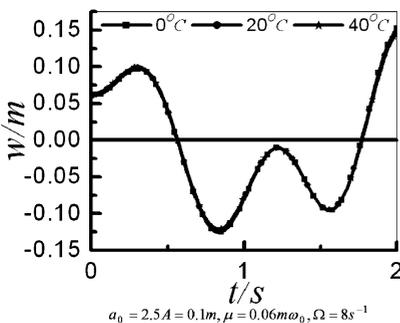


图3 矩形单层索网

Fig.3 The rectangular single cable net

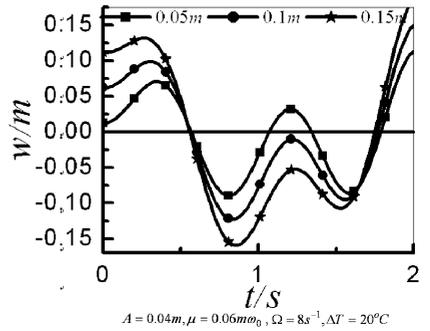


图4 矩形单层索网

Fig.4 The rectangular single cable net

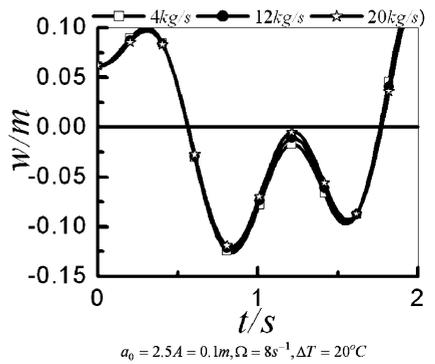


图5 矩形单层索网

Fig.5 The rectangular single cable net

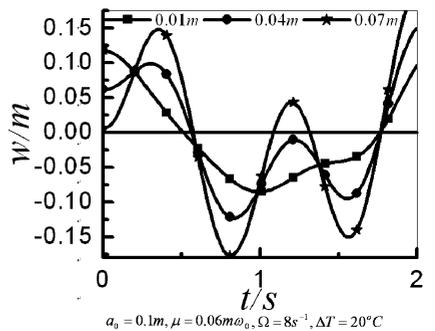


图6 矩形单层索网

Fig.6 The rectangular single cable net

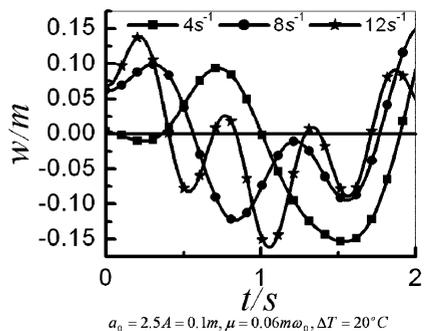


图7 矩形单层索网

Fig.7 The rectangular single cable net

对图2—图7进行分析可知:

(1) 在相同的外激励作用下, 钢丝绳网的振幅最大, 且钢丝绳网的振动周期也比碳纤维索网的振动周期大, 这说明碳纤维索网的振动频率比钢丝绳网的振动频率大. 外激励振幅增大, 碳纤维索网的振动周期变小、振幅增大; 而外激励频率的增大, 碳纤维索网的振幅也增大、振动周期变小.

(2) 对碳纤维索网而言, 4#索网的振幅最大、振动周期最小、振动频率最大, 3#索网的振幅最小、振动周期最大、振动频率最小, 这表明按照等轴向刚度准则设计的碳纤维索网优于按照等强度准则设计的碳纤维索网. 在温度变化的条件下, 碳纤维索网的振幅、振动周期、振动频率几乎不受温度的影响, 这说明碳纤维索网对温度变化不敏感, 而钢丝绳网受温度的影响却很大^[9-11].

(3) 随着碳纤维索网振幅增大, 其振动周期则变小. 但是, 阻尼的变化对碳纤维索网振动的影响却不大.

综合以上分析并考虑碳纤维具有耐久性好、抗腐蚀、自重轻、富有柔性、高强度等优点, 可以得出碳纤维索网振动特性优于钢丝绳网振动特性的结论.

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(in Chinese))

THE NONLINEAR VIBRATION OF RECTANGULAR CABLE NET USING CARBON FIBER CABLES UNDER FOUNDATION DISPLACEMENT EXCITATION *

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Abstract This paper studied that the nonlinear vibration of rectangular cable net using carbon fiber cables under foundation displacement excitation, and the nonlinear vibration equation was presented on the basis of considering temperature changing. The approximate solution of nonlinear vibration of rectangular cable mesh was given using Galerkin principle and KBM method. Then some factors, which affect the nonlinear vibration, such as the amount of temperature changing, foundation displacement excitation, amplitude as well as damping, were discussed with the comparison between steel cable and carbon fiber cable. At last the conclusions can be obtained that the vibration characteristics of cable net using carbon fiber cables is superior to cable net using steel.

Key words foundation, displacement excitation, carbon, fiber cables material, cable net, nonlinearity vibration