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具有损伤扁球面网壳非线性稳定性*

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摘要　基于Lematire等效应变损伤原理,计及扁球面网壳各个杆件的损伤影响,根据薄壳非线性动力学理论推导出含有损伤扁球面网壳非线性动力学方程和协调方程,在固定夹紧边界条件下,用Galerkin方法得到一个含二次和三次非线性振动微分方程,并对具有损伤扁球面网壳的非线性自由振动方程求解.用Floquet指数法研究系统分叉问题给出了平衡点的状态.并通过数字仿真绘出了不同损伤状态下系统的分叉图和平衡点的相对位置图,发现损伤对系统的平衡点的状态影响较大.

关键词　损伤,　分叉,　扁球面网壳,　非线性

引言

网壳结构是近些年来在建筑工程中应用极为广泛的工程结构形式,特别是在大型空间与公共建筑结构中可以说非他莫属了,尤其是网壳结构应用在大空间、大跨度结构中更是屡见不鲜.

众说周知,网壳结构通常属于薄壁轻钢结构,它们的变形大都属于大变形范围.在相当的跨度和载荷条件下,网壳结构的刚度较小于其它网格结构并在加载过程中刚度还会削弱,研究表明,在某些情况下倘若不考虑结构的非线性将会导致难以接受的误差,尤其在网壳的稳定性方面,因此用几何非线性分析网壳结构比较合理.近些年其非线性方面得到一定的研究[1-5].文献[6,7,8]研究了网格结构分岔与混沌问题.文献[9]初次提出了初始缺陷对扁球面网壳稳定性的影响.特别是由于网壳在制作杆件材料不可避免的都带有一定的损伤,因此对于带损伤的网壳的非线性分析的研究实属必要,笔者注意到关于这方面的研究尚不多见.作者正是在这方面做了一些尝试.首先基于Lematire等效应变损伤原理,计及扁球面网壳各个杆件的损伤影响,根据薄壳非线性动力学理论推导出含有损伤扁球面网壳非线性动力学方程和协调方程,在固定夹紧边界条件下,用Galerkin方法得到一个含二次和三次非线性微分方程,并对具有损伤扁球面网壳的非线性自由振动方程求解.用Floquet指数法研究系统分叉问题给出了平衡点的状态.最后发现损伤对系统的平衡点的状态影响较大.

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1　具有损伤扁球面网壳的非线性动力学控制方程的建立

1.1　具有损伤扁球面网壳的本构方程

首先利用拟板法建立具有损伤壳体的本构方程如下:

$$[N]=[D]\{\varepsilon\}=\frac{9E(1-\omega)}{8a}\times\begin{bmatrix}S_1+S_2 & \text{对} & \\ \frac{S_1+S_2}{3} & S_1+S_2 & \text{称}\\ 0 & 0 & \frac{S_1+S_2}{3}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix}\tag{1}$$

$$[M]=[B]\{\chi\}=(1-\omega)\times\begin{bmatrix}\frac{9EI}{8a} & \text{对} & \\ \frac{3EI}{8a} & \frac{9EI}{8a} & \text{称}\\ 0 & 0 & \frac{3EI}{8a}\end{bmatrix}\begin{Bmatrix}-\frac{\partial w}{\partial x^2}\\ -\frac{\partial w}{\partial y^2}\\ -2\frac{\partial w}{\partial x\partial y}\end{Bmatrix}\tag{2}$$

由(1)式可以得到:

$$\{\varepsilon\}=(\frac{1}{1-\omega})\times$$

$$\begin{bmatrix} \frac{a}{E(S_1+S_2)} & 对 & \\ \frac{a}{3E(S_1+S_2)} & \frac{a}{E(S_1+S_2)} & 称 \\ 0 & 0 & \frac{8a}{3E(S_1+S_2)} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \quad (3)$$

其中:E 为弹性模量,a 为弦杆长度,ω 为损伤变量 $\omega=\frac{S_d}{S}$,当 $\omega=1$ 时,杆件完全损伤;当 $\omega=0$ 时,杆件没有损伤 S_1,S_2 是上下弦杆的截面积,$I=\frac{S_1S_2}{S_1+S_1}h^2$,$h=h_1+h_2$,$h_1$,$h_2$ 板的上下表层到板中面的距离.

1.2　具有损伤扁球面网壳混合边值问题方程的推导

对于半径为 R,矢高为 f 的扁球面网壳,可以把它看作具有初始挠度为 $w_c=f(\frac{r^2}{R^2}-1)$ 的圆板,且 $\frac{dw_c}{dr}=\frac{2r}{R^2}f$ 则得到具有损伤扁球面网壳的平衡方程和协调方程为:

$$\frac{9EI(1-\omega)}{8a}\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(r\frac{dw}{dr})=q+\frac{1}{r}\frac{d}{dr}(rN_r(\frac{dw}{dr}+\frac{2r}{R^2}f)) \quad (4)$$

$$r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(r^2N_r)=-\frac{E(S_1+S_2)(1-\omega)}{a}\times\left[\frac{1}{2}\left(\frac{dw}{dr}\right)^2+\frac{2r}{R^2}f\frac{dw}{dr}\right] \quad (5)$$

引入无量纲量如下:

$\rho=\frac{r}{R}$,$W=\frac{w}{\beta}$,$N=\frac{8aR}{9EI}rN_r$,$Q=\frac{8aR^4q}{9EI\beta}$,$k=\frac{2f}{\beta}$,$\psi=\frac{4(S_1+S_2)}{9\beta^2}$其中 $I=\beta^4$ 则方程(4)和(5)可简化为:

$$\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}(\rho\frac{dW}{d\rho})=\frac{1}{1-\omega}(Q+\frac{1}{\rho}\frac{d}{d\rho}\left[T\left(k\rho+\frac{dW}{d\rho}\right)\right]) \quad (6)$$

$$\rho\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}(\rho N)=-\psi(1-\omega)\left(\frac{dW}{d\rho}+2k\rho\right)\frac{dW}{d\rho} \quad (7)$$

在具有损伤扁球面网壳的弯曲方程(6)右端加上无量钢化后的惯性项 $\Omega^2=\frac{8aR^4}{8EI}$和阻尼项 $c_1=\frac{8aR^4}{9EI}c$(c 为阻尼系数),其中设振动固有频率为 $\tilde{\omega}$,$\tilde{\omega}=\frac{2\pi}{T}$,$T=t_2-t_1=\frac{2\pi}{\tilde{\omega}}$为一个周期,取 $\tau=\tilde{\omega}t$ 就得到了具有损伤扁球面网壳的非线性动力学方程:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial W}{\partial\rho})=\frac{1}{1-\omega}(Q+\frac{1}{\rho}\frac{\partial}{\partial\rho}\left[N\left(k\rho+\frac{\partial W}{\partial\rho}\right)\right]-\Omega^2\frac{\partial^2W}{\partial\tau^2}-c_1\frac{\partial W}{\partial\rho}) \quad (8)$$

$$\rho\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho N)=-\psi(1-\omega)(\frac{\partial W}{\partial\rho}+2k\rho)\frac{\partial W}{\partial\rho} \quad (9)$$

选取固定夹紧边界条件:

$$当\ \rho=1\ 时,W=\frac{\partial W}{\partial\rho}=0,\frac{\partial}{\partial\rho}(N)-N/3=0 \quad (10)$$

$$当\ \rho=0\ 时,W,\frac{\partial W}{\partial\rho},N\ 有限 \quad (11)$$

初始条件为:$t=0$,$W=W_0$,$\frac{\partial W}{\partial t}=0$

$$选取\ W=f(t)(1-\rho^2)^2 \quad (12)$$

由方程(9)、(10)和(11)可以解得:

$$N=-\frac{1}{3}\frac{\psi}{1-\omega}(\rho^7-4\rho^5+6\rho^3-6\rho)f^2(t)-\frac{1}{3}k\psi(\rho^5-3\rho^3+5\rho)f(t) \quad (13)$$

方程(8)的能量变分方程为:

$$\int_0^{2\pi}\int_0^1\{L(W)-\frac{1}{1-\omega}(\left(Q+\frac{1}{\partial}\frac{\partial}{\partial\rho}\left[N\left(k\rho+\frac{\partial W}{\partial\rho}\right)\right]\right)-\Omega^2\frac{\partial^2W}{\partial\tau^2}-c_1\frac{\partial W}{\partial\rho})\}\rho\delta W\mathrm{d}\rho\mathrm{d}\tau=0 \quad (14)$$

其中:$L(W)=\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}$

把(12)和(13)式代入方程(14)用 Galerkin 方法得:

$$\int_0^{2\pi}\{f''(t)+\frac{c_1}{\Omega^2}f'(t)+\left[\frac{320}{3\Omega^2}(1-\omega)+\frac{38}{9\Omega^2}k^2\psi\right]f(t)-\left[\frac{80}{9\Omega^2}k\psi+\frac{40k\psi}{9\Omega^2(1-\omega)}\right]f^2(t)+\frac{200\psi}{21\Omega^2(1-\omega)}f^3(t)-\frac{50Q}{3\Omega^2}\}\delta f(t)\mathrm{d}t=0 \quad (15)$$

由于 $\delta f(t)$ 的任意性可以得到:

$$f''(t)+\frac{c_1}{\Omega^2}f'(t)+(1-\omega)\left[\frac{320}{3\Omega^2}+\frac{38}{9\Omega^2(1-\omega)}k^2\psi\right]f(t)-\frac{1}{(1-\omega)}\left[\frac{80}{9\Omega^2}(1-\omega)k\psi+\frac{40k\psi}{9\Omega^2}\right]f^2(t)+\frac{200\psi}{21\Omega^2(1-\omega)}f^3(t)-\frac{5Q}{3\Omega^2}=0 \quad (16)$$

为了简化公式我们令

$$c_2=\frac{c_1}{\Omega^2},\tilde{\omega}^2=\frac{320}{3\Omega^2}+\frac{38}{9\Omega^2(1-\omega)}k^2\psi,\Omega_1^2=\frac{80}{9\Omega^2}$$

$(1-\omega)k\psi+\dfrac{40k\psi}{9\Omega^2}, \alpha=\dfrac{200\psi}{21\Omega^2}, g=\dfrac{5Q}{3\Omega^2}, Q=G\cos\Omega' t$

那么方程(16)可简化为:

$$\frac{d^2f(t)}{dt^2}+c_2\frac{df(t)}{dt}+(1-\omega)\tilde{\omega}f(t)-\Omega_1^2\frac{1}{(1-\omega)}f^2(t)+\frac{1}{(1-\omega)}\alpha f^3(t)=g\cos(\Omega' t) \quad (17)$$

取 $\tau=\tilde{\omega}t, f(t)=\dfrac{\tilde{\omega}}{\sqrt{\alpha}}\eta(\tau)$ 则(17)式可变为:

$$\frac{d^2\eta(\tau)}{d\tau^2}+\frac{c_2}{\tilde{\omega}}\frac{d\eta(\tau)}{d\tau}+(1-\omega)\eta(\tau)-\frac{\Omega_1^2}{\tilde{\omega}\sqrt{\alpha}}\times\frac{1}{(1-\omega)}\eta^2(\tau)+\frac{1}{(1-\omega)}\eta^3(\tau)=\frac{\sqrt{\alpha}}{\tilde{\omega}^3}g\cos(\frac{\Omega'}{\tilde{\omega}}\tau) \quad (18)$$

令 $\alpha'=\dfrac{c_2}{\tilde{\omega}}, \beta'=\dfrac{\Omega_1^2}{\tilde{\omega}\sqrt{\alpha}}, g'=\dfrac{\sqrt{\alpha}}{\tilde{\omega}^3}g, \tilde{\omega}'=\dfrac{\Omega'}{\tilde{\omega}}$ 则(18)式化为:

$$\frac{d^2\eta(\tau)}{d\tau^2}+\alpha'\frac{d\eta(\tau)}{d\tau}+(1-\omega)\eta(\tau)-\beta'\frac{1}{(1-\omega)}\eta^2(\tau)+\frac{1}{(1-\omega)}\eta^3(\tau)=g'\cos(\tilde{\omega}'\tau) \quad (19)$$

方程(19)的等价方程可写为:

$$\begin{cases}\eta_1'(\tau)=\eta_2(\tau)\\ \eta_2'(\tau)=-(1-\omega)\eta(\tau)+\beta'\dfrac{1}{1-\omega}\eta^2(\tau)-\dfrac{1}{1-\omega}\eta^3(\tau)+\varepsilon\left(g'\cos(\tilde{\omega}\tau)-\alpha'\dfrac{d\eta(\tau)}{d\tau}\right)\end{cases} \quad (20)$$

当 $\varepsilon=0$ 时,方程(20)是一未扰动的 Hamilton 系统,其 Hamilton 量为:

$$H(\eta_1(\tau),\eta_2(\tau))=\frac{(\eta_1'(\tau))^2}{2}+\frac{(\eta_1(\tau))^2}{2}\times(1-\omega)-\frac{\beta'(\eta_1(\tau))^3}{3(1-\omega)}+\frac{(\eta_1(\tau))^4}{4(1-\omega)}=H \quad (21)$$

对于不同的 H 值,系统将有不同的力学行为.

2 用 Floquet 指数法研究系统分叉问题

若无外激励作用时则方程(20)变为:

$$\begin{cases}\eta_1'(\tau)=\eta_2(\tau)\\ \eta_2'(\tau)=-(1-\omega)\eta_1(\tau)+\beta'\dfrac{1}{(1-\omega)}\eta_1^2(\tau)-\dfrac{1}{(1-\omega)}\eta_1^3(\tau)-\alpha'\dfrac{d\eta_2(\tau)}{d\tau}\end{cases} \quad (22)$$

给出方程(22)的 Jacobi 矩阵:

$$\begin{pmatrix}\dfrac{\partial f_1}{\partial\eta_1} & \dfrac{\partial f_1}{\partial\eta_2}\\ \dfrac{\partial f_2}{\partial\eta_1} & \dfrac{\partial f_2}{\partial\eta_2}\end{pmatrix}=\begin{pmatrix}0 & 1\\ -(1-\omega)+\dfrac{2\beta'}{(1-\omega)}\eta_1-\dfrac{3}{(1-\omega)}\eta_1^2 & -\alpha'\end{pmatrix} \quad (23)$$

当损伤变量 $\omega=0$ 时,系统处于理想无损状态,则式子(23)可变为:

$$\begin{pmatrix}\dfrac{\partial f_1}{\partial\eta_1} & \dfrac{\partial f_1}{\partial\eta_2}\\ \dfrac{\partial f_2}{\partial\eta_1} & \dfrac{\partial f_2}{\partial\eta_2}\end{pmatrix}=\begin{pmatrix}0 & 1\\ -1+2\beta'\eta_1-3\eta_1^2 & -\alpha'\end{pmatrix} \quad (24)$$

令 $\begin{cases}f_1(\eta_1,\eta_2)=0\\ f_2(\eta_1,\eta_2)=0\end{cases}$,当 $\beta'\geqslant 2$ 时系统存在三个平衡点 $(0,0)$,和 $\left\{\dfrac{\beta'\pm\sqrt{(\beta')^2-4(1-\omega)}}{2},0\right\}$

讨论系统在平衡点的稳定情况

1)在平衡点(0,0)处的 Jacobi 矩阵为: $\begin{pmatrix}0 & 1\\ -1 & -\alpha'\end{pmatrix}$ 求得其特征根为: $\lambda_{1,2}=\dfrac{-\alpha'\pm\sqrt{(\alpha')^2-4}}{2}$

当 $0<\alpha'<2$ 时,特征根 λ 为两个不相等的复数,在复平面上有稳定的焦点. 如图 1 和图 2

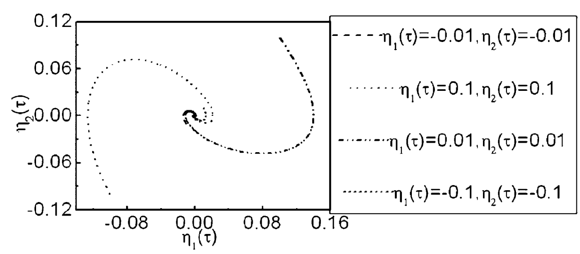

图 1 相图 $(\omega=0.2, \alpha'=1, \beta'=2)$

Fig. 1 phase plane $(\omega=0.2, \alpha'=1, \beta'=2)$

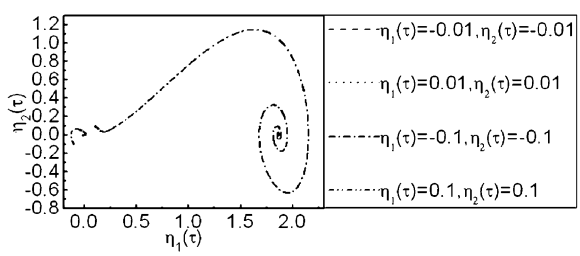

图 2 相图 $(\omega=0.5, \alpha'=1, \beta'=2)$

Fig. 2 phase plane $(\omega=0.5, \alpha'=1, \beta'=2)$

当 $\alpha'=2$ 时,特征根 λ 为两个相等的负数,平衡点为临界结点. 如图 3 和图 4

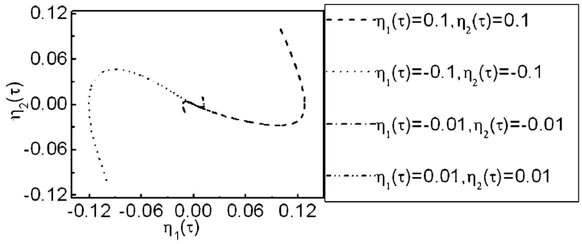

图 3　相图 ($\omega=0.2,\alpha'=2,\beta'=2$)

Fig. 3　phase plane($\omega=0.2,\alpha'=2,\beta'=2$)

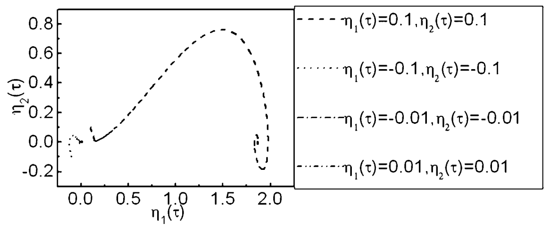

图 4　相图 ($\omega=0.5,\alpha'=2,\beta'=2$)

Fig. 4　phase plane($\omega=0.5,\alpha'=2,\beta'=2$)

当 $\alpha'>2$ 时,特征根 λ 为两个不相等的负数,平衡点为稳定结点. 如图 5 和图 6

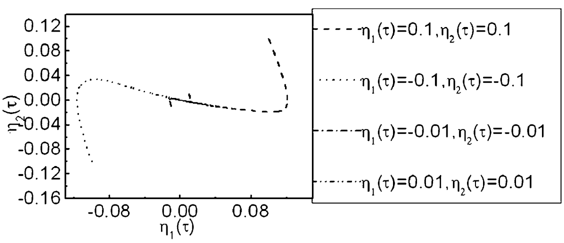

图 5　相图 ($\omega=0.2,\alpha'=3,\beta'=2$)

Fig. 5　phase plane($\omega=0.2,\alpha'=3,\beta'=2$)

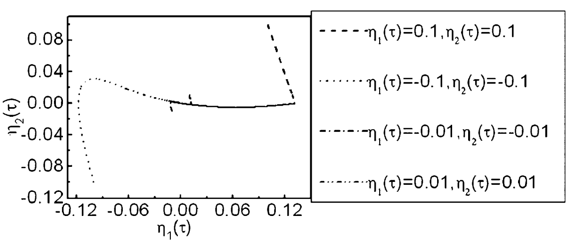

图 6　相图 ($\omega=0.5,\alpha'=3,\beta'=2$)

Fig. 6　phase plane($\omega=0.5,\alpha'=3,\beta'=2$)

当 $\alpha'=0$ 时,特征根 λ 为纯虚数,解的曲线为极限环,发生 Hopf 分叉. 如图 7 和图 8

从图 1 到图 8 分别给出了不同损伤值情况下的分叉图,从图中很明显看出损伤对系统平衡点稳定状态的影响,因此工程实际中不能忽视损伤的存在,应给予重视.

2)在平衡点处与平衡点处非常相似,这里不再一一鳌述.

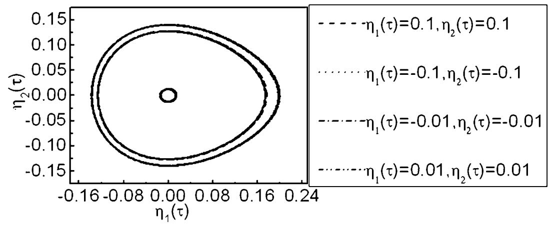

图 7　相图 ($\omega=0.2,\alpha'=0,\beta'=2$)

Fig. 7　phase plane($\omega=0.2,\alpha'=0,\beta'=2$)

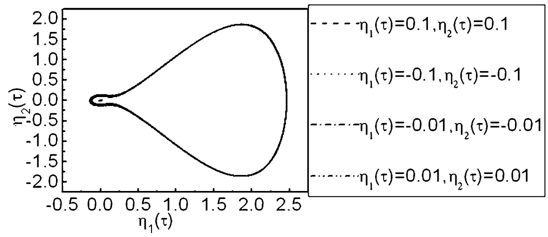

图 8　相图 ($\omega=0.5,\alpha'=0,\beta'=2$)

Fig. 8　phase plane($\omega=0.5,\alpha'=0,\beta'=2$)

由平衡点我们可以很清楚地看到损伤对系统平衡点的影响是应给予重视的. 我们通过绘出平衡点与损伤之间的关系曲线来进一步说明这一问题.

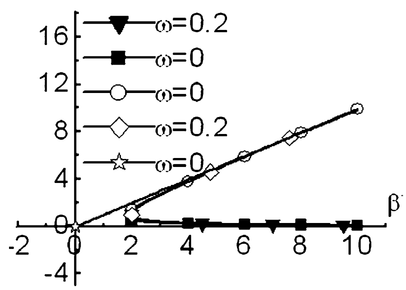

图 9　平衡点的相对位置

Fig. 9　relative position of equilibrium point

由图 9 可以看出损伤对平衡点相对位置的影响,损伤使平衡点远离无损伤理想状态平衡点($\beta'+\sqrt{\beta'^2-4}/2,0$)和($\beta'+\sqrt{\beta'^2-4}/2,0$).

3　结论

通过对具有损伤扁球面网壳分叉问题的分析,可以看出损伤对平衡点相对位置的影响,损伤使平衡点远离无损伤理想状态平衡点($\beta'+\sqrt{\beta'^2-4}/2,0$)和($\beta'+\sqrt{\beta'^2-4}/2,0$),而且随着损伤变量的不断增加其对平衡点的影响越来越明显. 然而其对平衡点(0,0)点相对位置没有影响. 由此可见损伤对结构稳定性的影响是不容忽视的,为今后的工程实际应用问题提供了一定的理论依据.

参 考 文 献

1 Seung-Deogkim etal.. Dynamic instability of shell-like shallow trusses considering damping. *Computers and structure*, 1997, 64(1-4):481 ~ 489

2 Yasushi Uematsu, Osama Kuribara etc.. Wind-induced dynamic behavior and its load estimation of a single-layer latticed dome with a long-span. *Journal of wind Engineering and Industrial Aerodynamics*, 2001, 89:1671 ~ 1687

3 Mircea Puta, Razvan Tudoran. Controllability Stability and the n-dimensional Toda lattice. *Bullentin Dessciences Mathematiques*, 2002, 126: 241 ~ 247

4 Li Zhongxue, Shen Zuyan. Shaking table tests of two shallow reticulated shells. *International journal of solids and structures*, 2001, 38:7875 ~ 7884

5 Q S Li, J M Chen. Nonlinear elastoplastic analysis of single-layer reticulated shells subjected earthquake excitation. *Computers and Structures*, 2003, 81:177 ~ 188

6 王新志,梁从兴,韩明君,叶开沅,王 钢. 扁柱面网壳的非线性动力学行为. 应用数学和力学,2007,28(27):135 ~ 140(Wang X Zh, Liang C X, Han M J, etal. Nonlinear dynamical behavior of shallow cylindrical reticulated shells. *Applied Mathematics and Mechanics*, 2007, 28(27):135 ~ 140(in Chinese))

7 王新志,梁从兴,栗蕾,韩明君,丁雪兴. 扁锥面单层网壳的非线性动力学特性. 动力学与控制学报,2004,2(3):14 ~ 17(Wang X Zh, Liang C X, Li L, etal. Nonlinear dynamic characteristics of single-layer shallow conical lattice shells. *Journal of Dynamics and Control*, 2004,2(3):14 ~ 17(in Chinese))

8 王新志,梁从兴,丁雪兴,韩明君,赵永刚. 单层扁锥面网壳非线性动力稳定性分析. 工程力学,2005,22:172 ~ 176(Wang X Z, Liang C X, Ding X X, etal.. Nonlinear dynamic stability analysis single-layer conical lattice shells. *Engineering and Mechanics*, 2005,22:172 ~ 176(in Chinese))

9 栗蕾,黄义. 具有初始缺陷扁球面网壳的非线性稳定性. 西安建筑科技大学学报,2009,41(1):32 ~ 36 (Li L, Huang Y. Non-linear stability of the shallow reticulated spherical shells considering initial imperfection. *Journal of XI'AN University of Architecture & Technology*, 2009, 41(1):32 ~ 36(in Chinese))

NONLINEAR STABILITY OF THE SHALLOW RETICULATED SPHERICAL SHELLS WITH DAMAGE *

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Abstract Based on the theory of Lematire's equivalent strain of the damage, taking into account the damage of bars of the shallow reticulated spherical shell, and according to nonlinear dynamical theory of thin shells, the nonlinear dynamical equations and the consistency equation of the shallow reticulated shells with damage were obtained by quasi – shell method. Under the fixed and clamped boundary conditions, a nonlinear differential oscillation equation with quadric and cubic items was presented by the Galerkin method, and a nonlinear free oscillation equation of the shallow reticulated shells with damage was solved. Then the bifurcation of the system was discussed by Floquet exponent method, and the state of the equilibrium point was given. Lastly the bifurcation map and the relative position map of the equilibrium point were plotted by numerical emulation under the different damage state. It is founded that the damage of the bars of the shells greatly impacts on the state of the equilibrium point.

Key words damage, bifurcation, the shallow reticulated spherical shell, nonlinear

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