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微扰 Kepler 系统的守恒量与对称性

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摘要　用直接积分法和 Noether 法研究微扰 Kepler 系统的守恒量,都得到了一个不同于 Hamilton 函数的守恒量,此守恒量与 Runge-Lenz 矢量有相同的量纲,可以称其为“类 Runge-Lenz 矢量守恒量”. 文中还讨论了守恒量的 Noether 对称性、Lie 对称性与 Mei 对称性,结果表明:与守恒量相应的无限小变换同时是 Noether 对称变换、Lie 对称变换和 Mei 对称变换.

关键词　微扰 Kepler 系统,　直接积分法,　Noether 法,　守恒量,　Noether 对称性,　Lie 对称性,　Mei 对称性

引言

力学系统的对称性与守恒量紧密地联系在一起,关于力学系统对称性与守恒量的研究已渗透到现代数学、力学、物理学等各个领域. 寻求力学系统的对称性和守恒量已成为近代分析力学的一大热点问题. 求守恒量常用的方法有 Noether 方法, Lie 方法, Mei 方法[1-6], Ermakov 方法[7-9],Poisson 括号方法[10-13]和直接积分法[14-18]. 对于给定势函数的系统,用直接积分法可以求得除 Hamilton 函数以外的守恒量,并且求守恒量比较直接、方便,它不必计算对称性,也不需要用 Noether 定理,当然也没法讨论守恒量的对称性. 而 Noether 方法既能确定守恒量也能讨论对称性,但必须解耦合的微分方程组. 微扰 Kepler 系统是一典型的力学系统,此系统在天体物理、原子与分子物理、量子力学中受到普遍的关注[19-21],很多文献比较关注其数值解、轨道的形状与封闭性问题,而对于其对称性与守恒量的研究较少见. 本文用直接积分法和 Noether 方法确定其守恒量(不同于 Hamilton 函数),并讨论守恒量的物理意义及其 Noether 对称性与 Lie 对称性.

1　确定微扰 Kepler 系统守恒量的两种方法

首先用直接积分法确定微扰 Kepler 系统的守恒量, 微扰 Kepler 系统的 Hamilton 函数为[14]

$$H=\frac{p_1^2}{2}+\frac{p_2^2}{2}-\frac{K}{(q_1^2+q_2^2)^{1/2}}+Aq_1+\frac{B}{q_2^2} \tag{1}$$

这里 K, A, B 为正常数.

由 Hamilton 正则方程可得下列运动方程

$$\frac{dq_1}{dt}=\frac{\partial H}{\partial p_1}=p_1=g_1 \tag{2a}$$

$$\frac{dq_2}{dt}=\frac{\partial H}{\partial p_2}=p_2=g_2 \tag{2b}$$

$$\frac{dp_1}{dt}=-\frac{\partial H}{\partial q_1}=-V_{q_1}=h_1 \tag{2c}$$

$$\frac{dp_2}{dt}=-\frac{\partial H}{\partial q_2}=-V_{q_2}=h_2 \tag{2d}$$

这里 p_1, p_2 是广义动量, V 是系统的势能, V 的下标 q_1、q_2 表示 V 对 q_1、q_2 的偏导数, g_1, g_2, h_1, h_2 是为方便下文讨论 *Lie* 对称性的简写符号(式(17)中用到),分别代表广义速度 $\dot{q}_1$, $\dot{q}_2$ 和广义动量对时间的变化率 $\dot{p}_1$, $\dot{p}_2$.

$$V_{q_1}=\frac{Kq_1}{(q_1^2+q_2^2)^{3/2}}+A \tag{3a}$$

$$V_{q_2}=\frac{Kq_2}{(q_1^2+q_2^2)^{3/2}}-\frac{2B}{q_2^3} \tag{3b}$$

式 (2a)-(2d) 可以写成下面 6 个 1 - 形式的微分式

$$p_2dq_1-p_1dq_2=0 \tag{4a}$$

$$V_{q_1}dq_1+p_1dp_1=0 \tag{4b}$$

$$V_{q_2}dq_1+p_1dp_2=0 \tag{4c}$$

$$V_{q_1}dq_2+p_2dp_1=0 \tag{4d}$$

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$$V_{q_2}dq_2+p_2dp_2=0 \tag{4e}$$

$$V_{q_2}dp_1-V_{q_1}dp_2=0 \tag{4f}$$

以上6个1－形式的微分式中仅含4个独立变量的微分 dq_1,dq_2,dp_1 和 dp_2，因此不是相互独立的，只有其中4个是相互独立的.

用未知函数 W,Q,R,S,T 和 U 分别乘式(4a)-(4f)，求和可得(其中未知函数 W,Q,R,S,T 和 U 都是关于 q_1,q_2,p_1,p_1 的函数)

$$\begin{aligned}\omega=&W(p_2dq_1-p_1dq_2)+Q(V_{q_1}dq_1+p_1dp_1)+\\&R(V_{q_2}dq_1+p_1dp_2)+S(V_{q_1}dq_2+p_2dp_1)+\\&T(V_{q_2}dq_2+p_2dp_2)+U(V_{q_2}dp_1-V_{q_1}dp_2)=\\&A_1dq_1+A_2dq_2+A_3dp_1+A_4dp_2=0\end{aligned} \tag{5}$$

其中 $A_i(i=1,2,3,4)$ 均是关于 q_1,q_2,p_1,p_2 的函数，且

$$A_1=Wp_2+QV_{q_1}+RV_{q_2},$$

$$A_2=-Wp_1+SV_{q_1}+TV_{q_2}$$

$$A_3=Qp_1+Sp_2+UV_{q_2},A_4=Rp_1+Tp_2-UV_{q_1}.$$

若存在 $I(q_1,q_2,p_1,p_2)$ 使

$$dI=A_1dq_1+A_2dq_2+A_3dp_1+A_4dp_2=\omega \tag{6}$$

则由式(5)知 $dI=0$，即 I 为守恒量.

由微分几何[22]知 $d\cdot d=0$. 即若存在 $I(q_1,q_2,p_1,p_2)$ 使 $dI=\omega$，则 $d(dI)=d\omega=0$. 反之，由 Poincaré 引理[22]知，若 $d\omega=0$，则 ω 在哈密顿函数 H 的定义域内恰当. 式(1)中的 q_1,q_2 为笛卡尔坐标，p_1,p_2 是与广义坐标 q_1,q_2 相应的广义动量，故 $p_1\subset R,p_2\subset R$. 但由于 $q_2\neq0$，系统的相空间为 $R^4-\{q_2=0$ 的平面上的点$\}$，即 $q_2=0$ 的平面将相空间分为不连通的两个流形 $M_+(q_2>0)$ 和 $M_-(q_2<0)$. 可通过1－1可微映射 $\tilde{x}_1=q_1,\tilde{x}_2=\pm\tan(q_2-\frac{\pi}{2}),\tilde{x}_3=p_1,\tilde{x}_4=p_2$ 分别将 M_+ 和 M_- 映射到 R^4. 由文献[23]知，ω 在 R^4 上闭则一定恰当，则 ω 在 M_+ 和 M_- 上闭则一定恰当，即 ω 在系统的相空间闭则一定恰当. 因此，存在 $I(q_1,q_2,p_1,p_2)$ 使 $dI=\omega$ 成立的充要条件为

$$d\omega=0 \tag{7}$$

令 $x_1\equiv q_1,x_2\equiv q_2,x_3\equiv p_1,x_4\equiv p_2$，则

$$\begin{aligned}d\omega=&d(\sum_{i=1}^4A_idx_i)=\sum_{i=1}^4dA_i\wedge dx_i=\\&\sum_{i=1}^4\sum_{j=1}^4\frac{\partial A_i}{\partial x_j}dx_j\wedge dx_i=(\frac{\partial A_2}{\partial x_1}-\frac{\partial A_1}{\partial x_2})dx_1\wedge dx_2+\\&(\frac{\partial A_3}{\partial x_1}-\frac{\partial A_1}{\partial x_3})dx_1\wedge dx_3+(\frac{\partial A_4}{\partial x_1}-\frac{\partial A_1}{\partial x_4})dx_1\wedge dx_4+\\&(\frac{\partial A_2}{\partial x_3}-\frac{\partial A_3}{\partial x_2})dx_2\wedge dx_3+(\frac{\partial A_2}{\partial x_4}-\frac{\partial A_4}{\partial x_2})dx_2\wedge dx_4+\\&(\frac{\partial A_4}{\partial x_3}-\frac{\partial A_3}{\partial x_4})dx_3\wedge dx_4=0\end{aligned}$$

的充要条件(即存在 $I(q_1,q_2,p_1,p_2)$ 使 $dI=\omega$ 的充要条件)为

$$\begin{aligned}&\frac{\partial A_2}{\partial q_1}=\frac{\partial A_1}{\partial q_2},\frac{\partial A_3}{\partial q_1}=\frac{\partial A_1}{\partial p_1},\frac{\partial A_4}{\partial q_1}=\frac{\partial A_1}{\partial p_2},\\&\frac{\partial A_2}{\partial p_1}=\frac{\partial A_3}{\partial q_2},\frac{\partial A_2}{\partial p_2}=\frac{\partial A_4}{\partial q_2},\frac{\partial A_4}{\partial p_1}=\frac{\partial A_3}{\partial p_2}\end{aligned} \tag{8}$$

另有

$$V_{q_1q_2}=V_{q_2q_1},V_{q_1p_1}=V_{q_1p_2}=V_{q_2p_1}=V_{q_2p_2}=0.$$

可得未知函数应满足的6个相互耦合的偏微分方程组

$$\begin{aligned}&\frac{\partial W}{\partial q_2}p_2+\frac{\partial W}{\partial q_1}p_1+\frac{\partial Q}{\partial q_2}V_{q_1}+\frac{\partial R}{\partial q_2}V_{q_2}-\frac{\partial S}{\partial q_1}V_{q_1}-\\&\quad\frac{\partial T}{\partial q_1}V_{q_2}=SV_{q_1q_1}+(T-Q)V_{q_2q_1}-RV_{q_2q_2}\end{aligned} \tag{9a}$$

$$\begin{aligned}&\frac{\partial W}{\partial p_1}p_2+\frac{\partial Q}{\partial p_1}V_{q_1}+\frac{\partial R}{\partial p_1}V_{q_2}-\frac{\partial Q}{\partial q_1}p_1-\frac{\partial S}{\partial q_1}p_2-\\&\quad\frac{\partial U}{\partial q_1}V_{q_2}=UV_{q_2q_1}\end{aligned} \tag{9b}$$

$$\begin{aligned}&\frac{\partial W}{\partial p_2}p_2+\frac{\partial Q}{\partial p_2}V_{q_1}+\frac{\partial R}{\partial p_2}V_{q_2}-\frac{\partial R}{\partial q_1}p_1-\frac{\partial T}{\partial q_1}p_2+\\&\quad\frac{\partial U}{\partial q_1}V_{q_2}=-W-UV_{q_1q_1}\end{aligned} \tag{9c}$$

$$\begin{aligned}&-\frac{\partial W}{\partial p_1}p_1+\frac{\partial S}{\partial p_1}V_{q_1}+\frac{\partial T}{\partial p_1}V_{q_2}-\frac{\partial Q}{\partial q_2}p_1-\frac{\partial S}{\partial q_2}p_2-\\&\quad\frac{\partial U}{\partial q_2}V_{q_2}=W+UV_{q_2q_2}\end{aligned} \tag{9d}$$

$$\begin{aligned}&-\frac{\partial W}{\partial p_2}p_1+\frac{\partial S}{\partial p_2}V_{q_1}+\frac{\partial T}{\partial p_2}V_{q_2}-\frac{\partial R}{\partial q_2}p_1-\frac{\partial T}{\partial q_2}p_2+\\&\quad\frac{\partial U}{\partial q_2}V_{q_2}=-UV_{q_1q_2}\end{aligned} \tag{9e}$$

$$\begin{aligned}&\frac{\partial Q}{\partial p_2}p_1+\frac{\partial S}{\partial p_2}p_2+\frac{\partial U}{\partial p_2}V_{q_2}-\frac{\partial R}{\partial p_1}p_1-\frac{\partial T}{\partial p_1}p_2+\\&\quad\frac{\partial U}{\partial p_1}V_{q_2}=-S+R\end{aligned} \tag{9f}$$

由式(9a)～(9f)可解得未知函数 W,Q,R,S,T 和 U，并将解代入式(6)可得守恒量 I. 理论上说，式(9a)～(9f)的解有多组，但由于式(3a)，(3b)的复杂性，未知函数 W,Q,R,S,T 和 U 不易求得. 本文只考虑一组特殊的解，取

$$W=-p_2,Q=0,R=S=q_2,T=-2q_1,U=0 \tag{10}$$

则微扰 *Kepler* 系统的守恒量为

$$I = p_2(p_1q_2 - p_2q_1) + \frac{Kq_1}{(q_1^2+q_2^2)^{1/2}} + \frac{A}{2}q_2^2 - \frac{2Bq_1}{q_2^2} \tag{11}$$

下面用 *Noether* 方法确定守恒量，引进相空间的群的无限小变换

$$t^* = t + \varepsilon\xi_0(t,q,p) \quad q_s^* = q_s + \varepsilon\xi_s(t,q,p)$$

$$p_s^* = p_s + \varepsilon\eta_s(t,q,p)(s=1,2) \tag{12}$$

这里 ε 是无限小参数,ξ_0,ξ_s 和 η_s 是无限小变换生成元. 其无限小生成元向量为

$$X^{(0)} = \xi_0\frac{\partial}{\partial t} + \sum_{s=1}^{2}\xi_s\frac{\partial}{\partial q_s} + \sum_{s=1}^{2}\eta_s\frac{\partial}{\partial p_s}. \tag{13}$$

式(13)的一次扩展为

$$X^{(1)} = X^{(0)} + \sum_{s=1}^{2}(\dot\xi_s - \dot q_s\dot\xi_0)\frac{\partial}{\partial\dot q_s} + \sum_{s=1}^{2}(\dot\eta_s - \dot p_s\dot\xi_0)\frac{\partial}{\partial\dot p_s} \tag{14}$$

如果无限小生成元满足 *Noether* 恒等式[1]

$$\sum_{s=1}^{2}p_s\dot\xi_s - \frac{\partial H}{\partial t}\xi_0 - \sum_{s=1}^{2}\frac{\partial H}{\partial q_s}\xi_s - H\dot\xi_0 = -\dot G \tag{15}$$

其中 G 是规范函数，根据广义 *Noether* 定理[1],系统具有守恒量

$$I = \sum_{s=1}^{2}p_s\xi_s - H\xi_0 + G \tag{16}$$

由式(15)可得一组特殊解

$$\xi_0 = q_1, \xi_1 = q_1p_1 + q_2p_2, \xi_2 = -q_1p_2 + q_2p_1,$$

$$G = -\frac{q_1p_1^2}{2} - q_2p_1p_2 + \frac{q_1p_2^2}{2} + \frac{Aq_2^2}{2} + Aq_1^2 - \frac{Bq_1}{q_2^2} \tag{17}$$

将式(1) 和 (17) 代入式(16),我们得到守恒量

$$I = p_2(p_1q_2 - p_2q_1) + \frac{Kq_1}{(q_1^2+q_2^2)^{1/2}} + \frac{A}{2}q_2^2 - \frac{2Bq_1}{q_2^2} \tag{18}$$

显然,用两种方法得到的守恒量是一致的.

众所周知,Kepler 系统有三个守恒量:能量(即 Hamilton 函数)、角动量和 Runge-Lenz 矢量. 由于微扰 Kepler 系统的势能函数中存在微扰项,因此,角动量不再守恒. 除 Hamilton 函数 H 仍守恒外,由式(18)表示的另一守恒量 I 具有与 Runge-Lenz 矢量相同的量纲,但其表达式与 Runge-Lenz 矢量不同,因此可以称守恒量 I 为“类 Runge-Lenz 矢量守恒量”. 另外, 比较式(18)与式(1)可知,守恒量 I 不能表示成 Hamilton 函数 H 的函数,守恒量 I 与 Hamilton 函数 H 是相互独立的.

2　微扰 Kepler 系统的 Noether 对称性、Lie 对称性和 Mei 对称性

根据 Hamilton 系统的 Noether 对称性理论,式(12)表示的无限小变换的生成元 η_s 可由下式得到[1]

$$\eta_s = \frac{\partial p_s}{\partial t}\xi_0 + \sum_{s=1}^{2}\frac{\partial p_s}{\partial q_k}\xi_k + \sum_{s=1}^{2}\frac{\partial p_s}{\partial\dot q_k}(\dot\xi_k - \dot q_k\dot\xi_0) \quad (s=1,2) \tag{19}$$

将式(17)代入式(19), 可得

$$\eta_1 = \dot\xi_1 - \dot q_1\dot\xi_0 = q_1\dot p_1 + q_2\dot p_2 + p_2^2$$

$$\eta_2 = \dot\xi_2 - \dot q_2\dot\xi_0 = -q_1\dot p_2 + q_2\dot p_1 - p_1p_2 \tag{20}$$

因此,相空间的无限小变换(12)是 Noether 对称变换.

根据 Hamilton 系统的 Lie 对称性理论,式(2a)-(2d) 保持不变的充要条件是[1]

$$\dot\xi_s - \dot\xi_0 g_s = X^{(0)}(g_s)$$

$$\dot\eta_s - \dot\xi_0 h_s = X^{(0)}(h_s)(s=1,2) \tag{21}$$

这里 g_s, h_s 由式($2a$)-($2d$)给出. 可以证明式(17)、(20)表示的生成元 ξ_0,ξ_s,η_s 满足式(21),这表明无限小变换(12) 是系统的 Lie 对称变换.

根据 Hamilton 系统的 Mei 对称性理论[4-6],若在无限小变换(12)下,$H(t,q,p)$ 变成 $H^* = H^*(t^*,q^*,p^*)$,方程(2a)-(2d)的形式保持不变,即

$$\frac{dq_1}{dt} = \frac{\partial H^*}{\partial p_1}, \frac{dq_2}{dt} = \frac{\partial H^*}{\partial p_2}, \frac{dp_1}{dt} = -\frac{\partial H^*}{\partial q_1}, \frac{dp_2}{dt} = -\frac{\partial H^*}{\partial q_2}. \tag{22}$$

则这种不变性为系统在相空间的 *Mei* 对称性. 也即对于系统($2a$)-($2d$),若无限小生成元 ξ_0,ξ_s,η_s 满足

$$\frac{\partial}{\partial p_s}(X^{(0)}(H)) = 0, -\frac{\partial}{\partial q_s}(X^{(0)}(H)) = 0. \quad (s=1,2) \tag{23}$$

则相应的不变性是 Mei 对称不变性,相应的变换为系统的 Mei 对称变换.

可以证明式(17)、(20)表示的生成元 ξ_0,ξ_s,η_s 满足式(23), 这表明无限小变换(12)也是系统的 Mei 对称变换.

3　结 语

本文用直接积分法和 Noether 法研究微扰

Kepler 系统的守恒量,而且两种方法得到了相同的守恒量(不同于 Hamilton 函数). 所得守恒量 I 具有与 Runge-Lenz 矢量相同的量纲,但其表达式与 Runge-Lenz 矢量不同,因此可以称守恒量 I 为"类 Runge-Lenz 矢量守恒量". 文中还讨论了守恒量的 Noether 对称性、Lie 对称性与 Mei 对称性,结果表明:与守恒量相应的无限小变换同时是 Noether 对称变换、Lie 对称变换、Mei 对称变换. 因此,本文的结论对于研究微扰 Kepler 系统具有理论和实际意义.

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THE CONSERVED QUANTITY AND SYMMETRY OF PERTURBED KEPLER SYSTEM

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Abstract The conserved quantity of the perturbed Kepler system was studied by using the direct integral method and Noether method. One conserved quantity , different from the Hamiltonian, was obtained by two different methods . The dimension of the conserved quantity is identical with the Runge-Lenz vector's , so we can name this conserved quantity "analogous Runge-Lenz vector conserved quantity" . Furthermore, the Noether symmetry, the Lie symmetry and the Mei symmetry of the conserved quantity were also discussed. The research indicates that the infinitesimal transformations of the conserved quantity are all Noether symmetry, Lie symmetry and Mei symmetry.

Key words perturbed Kepler system, direct integral method, Noether method, conserved quantity, Noether symmetry, Lie symmetry, Mei symmetry

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