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矩形中厚板自由振动问题的哈密顿体系与辛几何解法

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摘要　以矩形中厚板的胡海昌方程为基础,将中厚板自由振动问题导入哈密顿体系,然后利用辛几何中的分离变量和本征函数展开的方法求出了对边简支板自由振动的精确解. 文中采用的辛方法不必事先人为地引入试函数,而是通过完全理性的推导,从而突破了传统半逆解法的限制,使得问题的求解更加合理,易于推广. 计算实例证明了本文推导结果的正确性.

关键词　矩形中厚板,　自由振动,　哈密顿体系,　辛几何

引言

中厚板[1-3]是工程实际中常见的一种重要结构形式,例如高速公路的水泥混凝土路面、机场的停机坪以及各种建筑工程中的楼板等等. 但是,由于数学上的困难,其静力与动力问题的解析求解一直是一个难题.

钟万勰教授将辛几何方法引入弹性力学中[4,5],为理性地求解弹性力学问题开辟了新思路. 之后,许多学者利用辛方法在求解板的静力弯曲问题方面作了大量的工作. 文献[6,7]基于平面弹性与薄板弯曲问题的相似性,通过引入弯矩函数向量,由分离变量及本征向量展开方法得到了辛体系下薄板弯曲问题的解析解. 姚伟岸基于 Reissner 板弯曲问题的 Hellinger - Reissner 变分原理,通过引入对偶变量,导出 Reissner 板弯曲问题的 Hamilton 对偶方程组[8],从而将该问题导入到 Hamilton 体系,并求解出了零本征值的本征解,它们构成圣维南问题的基本解. 鲍四元,邓子辰探讨了 Mindlin 中厚板弯曲的辛求解方法[9]. 鞠伟,岑松,龙驭球基于 Hamilton 解法详细计算了厚板弯曲的典型算例[10]. 付宝连等用功的互等定理求解了厚矩形板的弯曲[11]. 钟阳,李锐等构造了一种形式简洁的 Reissner 板弯曲求解辛体系[12]. 另外,辛几何方法在结构的动力问题中的应用已经成为一个新的研究方向[13-15]. 然而,迄今鲜见有利用辛几何解法研究中厚板自由振动问题的文献.

本文采用胡海昌教授在 Reissner 板理论基础上提出的中厚板微分方程及边界条件[16],将中厚板自由振动问题的控制方程引入 Hamilton 体系,利用辛几何方法推导出了对边简支矩形中厚板自由振动问题的精确解. 在推导过程中,首先导出问题的 Hamilton 对偶方程,然后利用辛几何方法对全状态向量进行分离变量,求出本征值及本征向量后,再按本征函数展开的方法求出了问题的精确解. 由于求解过程直接从中厚板振动的基本方程出发,利用数学方法求出问题的精确解,无须人为选定试函数,从而使问题的求解更加理性化. 求解方法易于推广至其他任意边界条件. 最后给出了数值算例以验证本文推导计算的正确性.

1　矩形中厚板自由振动的哈密顿体系

矩形中厚板的三个广义位移可以用两个函数表示如下[16]:

$$W = F - \frac{D}{C}\nabla^2 F;\quad \phi_x = \frac{\partial F}{\partial x} + \frac{\partial \Psi}{\partial y}\quad \phi_y = \frac{\partial F}{\partial y} - \frac{\partial \Psi}{\partial x} \tag{1}$$

而板的自由振动的基本方程可以写成:

$$D\nabla^2\nabla^2 F - \rho\omega^2\left(F - \frac{D}{C}\nabla^2 F\right) = 0 \tag{2}$$

$$\nabla^2 \Psi - \frac{2C}{D(1-v)\Psi} = 0 \tag{3}$$

其中: $\nabla^2 = \frac{\partial^2}{\partial x^2}$, $C = \frac{5}{6}Gh$ 为剪切刚度, $D =$

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$\frac{Eh^3}{12(1-v^2)}$为抗弯刚度,$G=\frac{E}{2(1+V)}$为材料的剪切模量. E,v,h,ρ,ω 分别为材料的弹性模量,泊松比,板的厚度,板单位面积的质量和板的固有频率. 板的内力分别为:

$$M_x=-D\left[\frac{\partial^2 F}{\partial x^2}+v\frac{\partial^2 F}{\partial y^2}+(1+v)\frac{\partial^2 \Psi}{\partial x\partial y}\right] \tag{4}$$

$$M_y=-D\left[\frac{\partial^2 F}{\partial y^2}+v\frac{\partial^2 F}{\partial x^2}-(1+v)\frac{\partial^2 \Psi}{\partial x\partial y}\right] \tag{5}$$

$$M_{xy}=-D(1+v)\left[\frac{\partial^2 \Psi}{\partial x\partial y}-\frac{1}{2}\left(\frac{\partial^2 \Psi}{\partial y^2}-\frac{\partial^2 \Psi}{\partial x^2}\right)\right] \tag{6}$$

$$Q_x=-D\left[\frac{\partial}{\partial x}\nabla^2 F+\frac{D}{C}\frac{\partial \Psi}{\partial y}\right]$$

$$Q_y=-D\left[\frac{\partial}{\partial y}\nabla^2 F+\frac{D}{C}\frac{\partial \Psi}{\partial x}\right] \tag{7}$$

由式(4)和式(5)可以得到

$$M_x+M_y=-D\left[\frac{\partial^2 F}{\partial x^2}+v\frac{\partial^2 F}{\partial y^2}+\frac{\partial^2 F}{\partial y^2}+v\frac{\partial^2 F}{\partial x^2}\right]=-D(1+v)\nabla^2 F \tag{8}$$

令 $M=-\frac{M_x+M_y}{D(1+v)}$,则式(8)可以表示为

$$\nabla^2 F=M \tag{9}$$

将式(9)代入式(2)有

$$D\nabla^2 M-\rho\omega^2\left(F-\frac{D}{C}M\right)=0 \tag{10}$$

通过以上分析可知,只要解出函数 $F(x,y)$ 和 $\Psi(x,y)$,就可以求出矩形中厚板问题的所有内力和位移. 于是问题归结为式(3),(9),(10)的求解.

令

$$\frac{\partial \Psi}{\partial y}=\theta \tag{11}$$

由式(3)可得

$$\frac{\partial \theta}{\partial y}=\frac{2C}{D(1-v)}\Psi-\frac{\partial^2 \Psi}{\partial x^2} \tag{12}$$

令

$$\frac{\partial F}{\partial y}=\alpha \tag{13}$$

$$\frac{\partial M}{\partial y}=\frac{\rho\omega}{D}\beta \tag{14}$$

则(9),(10)两式可写成

$$\frac{\partial \alpha}{\partial y}=M-\frac{\partial^2 F}{\partial x^2} \tag{15}$$

$$\frac{\partial \beta}{\partial y}=F-\frac{D}{C}M-\frac{D}{\rho\omega^2}\frac{\partial^2 M}{\partial x^2} \tag{16}$$

为了将问题导入 Hamilton 体系,将式(11)到(16)写成矩阵形式

$$\partial Z/\partial y=HZ \tag{17}$$

其中

$$H=\begin{bmatrix}0 & F\\ G & 0\end{bmatrix},F=\begin{bmatrix}\frac{\rho\omega^2}{D} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},$$

$$G=\begin{bmatrix}-\frac{D}{\rho\omega^2}\frac{\partial^2}{\partial x^2}-\frac{D}{C} & 1 & 0\\ 1 & -\frac{\partial^2}{\partial x^2} & 0\\ 0 & 0 & \frac{2C}{D(1-v)}-\frac{\partial^2}{\partial x^2}\end{bmatrix},$$

$Z=[M,F,\Psi,\beta,\alpha,\theta]^T$,因为 $H^T=JHJ$(其中 $J=\begin{bmatrix}0 & I_3\\ -I_3 & 0\end{bmatrix}$为辛几何中的度量矩阵,$I_3$ 为三阶单位矩阵),说明 H 是 Hamilton 算子矩阵,因此式(17)即为矩形中厚板自由振动问题在 Hamilton 体系中的一种表示.

按照辛几何方法,可以利用分离变量法求解方程(17),令

$$Z=X(x)Y(y) \tag{18}$$

其中,$X(x)=[M(x),F(x),\Psi(x),\beta(x),\alpha(x),\theta(x)]$. 将(18)式代入(17)式可以得到

$$dY(y)/dy=\mu Y(y),\quad HX(x)=\mu X(x) \tag{19}$$

式中 μ 为待求本征值,$X(x)$ 为本征向量. 方程(19)的第二式是本征值问题,其特征方程为

$$det\begin{bmatrix}-\mu & 0 & 0 & \frac{\rho\omega^2}{D} & 0 & 0\\ 0 & -\mu & 0 & 0 & 1 & 0\\ 0 & 0 & -\mu & 0 & 0 & 1\\ -\frac{D}{\rho\omega^2}\lambda^2-\frac{D}{C} & 1 & 0 & -\mu & 0 & 0\\ 1 & -\lambda^2 & 0 & 0 & -\mu & 0\\ 0 & 0 & \frac{2C}{D(1-v)}-\lambda^2 & 0 & 0 & -\mu\end{bmatrix}=0 \tag{20}$$

展开行列式即得特征方程

$$\left[\lambda^2+\mu^2-\frac{2C}{D(1+v)}\right]\times\left[(\lambda^2+\mu^2)^2+\frac{\rho\omega^2}{D}(\lambda^2+\mu^2)-\frac{\rho\omega^2}{D}\right]=0 \tag{21}$$

即 $\lambda=\pm i\alpha_1$ 或 $\lambda=\pm i\alpha_2$ 或 $\lambda=\pm i\alpha_3$,其中

$$\alpha_1=\sqrt{\frac{\omega}{2C}\sqrt{\frac{\rho}{D}(4C^2+D\rho\omega^2)}+\mu^2+\frac{\rho\omega^2}{D}}$$

$$\alpha_2=\sqrt{\frac{\omega}{2C}\sqrt{\frac{\rho}{D}(4C^2+D\rho\omega^2)}-\mu^2-\frac{\rho\omega^2}{D}}$$

$$\alpha_3=\sqrt{\frac{2C}{D(1-v)}-\mu^2} \tag{22}$$

于是方程的通解为

$$\begin{aligned}M&=A_1\cos(\alpha_1x)+B_1\sin(\alpha_1x)+C_1\mathrm{ch}(\alpha_2x)+D_1\mathrm{sh}(\alpha_2x)+E_1\mathrm{ch}(\alpha_3x)+F_1\mathrm{sh}(\alpha_3x),\\F&=A_2\cos(\alpha_1x)+B_2\sin(\alpha_1x)+C_2\mathrm{ch}(\alpha_2x)+D_2\mathrm{sh}(\alpha_2x)+E_2\mathrm{ch}(\alpha_3x)+F_2\mathrm{sh}(\alpha_3x),\\\Psi&=A_3\sin(\alpha_1x)+B_3\cos(\alpha_1x)+C_3\mathrm{sh}(\alpha_2x)+D_3\mathrm{ch}(\alpha_2x)+E_3\mathrm{sh}(\alpha_3x)+F_3\mathrm{ch}(\alpha_3x),\\\beta&=A_4\cos(\alpha_1x)+B_4\sin(\alpha_1x)+C_4\mathrm{ch}(\alpha_2x)+D_4\mathrm{sh}(\alpha_2x)+E_4\mathrm{ch}(\alpha_3x)+F_4\mathrm{sh}(\alpha_3x),\\\alpha&=A_5\cos(\alpha_1x)+B_5\sin(\alpha_1x)+C_5\mathrm{ch}(\alpha_2x)+D_5\mathrm{sh}(\alpha_2x)+E_5\mathrm{ch}(\alpha_3x)+F_5\mathrm{sh}(\alpha_3x),\\\theta&=A_6\sin(\alpha_1x)+B_6\cos(\alpha_1x)+C_6\mathrm{sh}(\alpha_2x)+D_6\mathrm{ch}(\alpha_2x)+E_6\mathrm{sh}(\alpha_3x)+F_6\mathrm{ch}(\alpha_3x)\end{aligned} \tag{23}$$

由式(23)可见,通解可分为关于 y 轴对称和反对称两部分. 关于 y 轴对称的振动为

$$\begin{aligned}M&=A_1\cos(\alpha_1x)+C_1\mathrm{ch}(\alpha_2x)+E_1\mathrm{ch}(\alpha_3x),\\F&=A_2\cos(\alpha_1x)+C_2\mathrm{ch}(\alpha_2x)+E_2\mathrm{ch}(\alpha_3x),\\\Psi&=A_3\sin(\alpha_1x)+C_3\mathrm{sh}(\alpha_2x)+E_3\mathrm{sh}(\alpha_3x),\\\beta&=A_4\cos(\alpha_1x)+C_4\mathrm{ch}(\alpha_2x)+E_4\mathrm{ch}(\alpha_3x),\\\alpha&=A_5\cos(\alpha_1x)+C_5\mathrm{ch}(\alpha_2x)+E_5\mathrm{ch}(\alpha_3x),\\\theta&=A_6\sin(\alpha_1x)+C_6\mathrm{sh}(\alpha_2x)+E_6\mathrm{sh}(\alpha_3x)\end{aligned} \tag{24}$$

其中各常数并不全独立. 将式(24)代回式(19)的第二式,可以得到各常数之间存在如下关系

$$A_2=RA_1,A_3=A_6=0,A_4=\frac{D\mu}{\rho\omega^2}A_1,A_5=\mu RA_1$$

$$C_2=SC_1,C_3=C_6=0,C_4=\frac{D\mu}{\rho\omega^2}C_1,C_5=\mu SC_1$$

$$E_1=E_2=E_4=E_5=0,E_3=E_2/\mu \tag{25}$$

其中 $R=\frac{D}{2C}-\frac{1}{2C\omega}\sqrt{\frac{D}{\rho}(4C^2+D\rho\omega^2)}$, $S=\frac{D}{2C}+\frac{1}{2C\omega}\sqrt{\frac{D}{\rho}(4C^2+D\rho\omega^2)}$. 同理容易讨论反对称振动的情形.

2 对边简支矩形中厚板自由振动的辛本征解

x 方向对边简支板的边界条件为

$$x=\pm a/2:\quad W=0;\phi_y=0;M_x=0 \tag{26}$$

将式(24),(25)代入式(26)得到

$$\begin{cases}(CR-D)\cos(a\alpha_1/2)A_1+(CS-D)\mathrm{ch}(a\alpha_2/2)C_1=0\\R\mu^2\cos(a\alpha_1/2)A_1+S\mu^2\mathrm{ch}(a\alpha_2/2)C_1-\alpha_3\mathrm{ch}(a\alpha_3/2)E_6=0\\[v-(1-v)R\alpha_1^2]\cos(a\alpha_1/2)A_1+[v+(1-v)S\alpha_2^2]\times\\\quad\mathrm{ch}(a\alpha_2/2)C_1+(1-v)\alpha_3\mathrm{ch}(a\alpha_3/2)E_6=0\end{cases} \tag{27}$$

令式(27)的系数行列式为零,经化简即得到对边简支矩形中厚板对称振动的本征值超越方程

$$\cos(\frac{a\alpha_1}{2})\mathrm{ch}(\frac{a\alpha_2}{2})\mathrm{ch}(\frac{a\alpha_3}{2})=0 \tag{28}$$

由式(28)得到 $\alpha_1=\frac{m\pi}{a}$或 $\alpha_2=\frac{m\pi}{a}i$ 或 $\alpha_3=\frac{m\pi}{a}i$($m=1,3,5,\cdots$). 将它们代入式(22),于是得到本征值为

$$\mu_{\pm m}^{(1)}=\pm\sqrt{(\frac{m\pi}{a})^2-\frac{\omega}{2C}\sqrt{\frac{\rho}{D}(4C^2+D\rho\omega^2)}-\frac{\rho\omega^2}{D}}\quad(\alpha_1=\pm\frac{m\pi}{a}i) \tag{29}$$

或

$$\mu_{\pm m}^{(2)}=\pm\sqrt{(\frac{m\pi}{a})^2+\frac{\omega}{2C}\sqrt{\frac{\rho}{D}(4C^2+D\rho\omega^2)}-\frac{\rho\omega^2}{D}}\quad(\alpha_2=\pm\frac{m\pi}{a}i) \tag{30}$$

或

$$\mu_{\pm m}^{(3)}=\pm\sqrt{(\frac{m\pi}{a})^2+\frac{2C}{D(1-v)}},(\alpha_3=\pm\frac{m\pi}{a}i) \tag{31}$$

求出了本征值,即可写出对应的本征向量. 如 $\mu_m^{(1)}$ 对应的本征向量为

$$X_m^{(1)}(x)=[Q_1\cos(\alpha_1x),\cos(\alpha_1x),0,T_1\cos(\alpha_1x),\mu_m^{(1)}\cos(\alpha_1x),0]^T \tag{32}$$

其中

$$Q_1=\frac{2C\omega\sqrt{\rho}}{D\omega\sqrt{\rho}-\sqrt{D(4C^2+D\rho\omega^2)}},$$

$$T_1=\frac{2C\mu\sqrt{D}}{\rho\omega^2\sqrt{D}-\omega\sqrt{\rho(4C^2+D\rho\omega^2)}}.$$

μ_m^2 对应的本征向量为

$$X_m^{(2)}(x)=[Q_2\mathrm{ch}(\alpha_2x),\mathrm{ch}(\alpha_2x),0,T_2\mathrm{ch}(\alpha_2x),\mu_m^{(2)}\mathrm{ch}(\alpha_2x),0]^T \tag{33}$$

其中

$$Q_2 = \frac{2C\omega\sqrt{\rho}}{D\omega\sqrt{\rho} + \sqrt{D(4C^2 + D\rho\omega^2)}},$$

$$T_2 = \frac{2C\mu\sqrt{D}}{\rho\omega^2\sqrt{D} + \omega\sqrt{\rho(4C^2 + D\rho\omega^2)}}.$$

μ_m^3 对应的本征向量为

$$X_m^{(3)}(x) = [0,0,\mathrm{sh}(\alpha_3 x),0,0,\mu_m^{(3)}\mathrm{sh}(\alpha_3 x)]^T \tag{34}$$

将各本征向量中的 μ_m 替换为 $-\mu_m$,即得到 $-\mu_m$ 对应的本征向量 $X_{-m}^{(1)}(x),X_{-m}^{(2)}(x),X_{-m}^{(3)}(x)$. Hamilton 矩阵的本征向量之间存在着共轭辛正交关系,不难验证,与 $X_m^{(i)}$ 共轭的本征向量一定是 $X_{-m}^{(i)}$,即有 $\int_{-a/2}^{a/2} X_m^{(i)T} J X_{-m}^{(i)} \mathrm{d}x \neq 0 (i = 1,2,3)$,其余本征向量之间均为辛正交关系.

有了本征值、本征向量及共轭辛正交的性质,就可以写出对边简支板对称振动的解为

$$Z = \sum_{m=1,3,\cdots}^{\infty} (f_m^{(1)} e^{\mu_m^{(1)} y} X_m^{(1)} + f_{-m}^{(1)} e^{\mu_{-m}^{(1)} y} X_{-m}^{(1)} + f_m^{(2)} e^{\mu_m^{(2)} y} X_m^{(2)} + f_{-m}^{(2)} e^{-\mu_m^{(2)} y} X_{-m}^{(2)} + f_m^{(3)} e^{\mu_m^{(3)} y} X_m^{(3)} + f_{-m}^{(3)} e^{\mu_{-m}^{(3)} y} X_{-m}^{(3)}) \tag{35}$$

其中 $f_m^{(k)} (k = 1,2,3; m = \pm 1, \pm 3, \pm 5, \cdots)$ 为待定常数,它们由板在 y 方向的边界条件确定.

3　矩形中厚板典型振动问题的精确解与数值算例

为简化篇幅,本文得出对边简支另两边固支板固有频率的精确解. 其余边界条件下板频率的精确解完全可以类似求出.

y 方向固支的板的边界条件为

$$y = 0, b: \quad W = 0; \phi_x = 0; \phi_y = 0 \tag{36}$$

由式(1)、(35)及式(36)可以得到关于待定常数 $f_m^{(k)}$ 的联立方程组,令其系数方阵的行列式为零,经过化简,即得到板对称振动的频率方程为

$$\beta_1\beta_3[1 + \delta_b m^2\beta^2(1 + \beta_2^2)]\mathrm{th}(\frac{m\pi\beta}{2}\beta_1) + \beta_2\beta_3[1 + \delta_b m^2\beta^2(1 - \beta_1^2)]\tan(\frac{m\pi\beta}{2}\beta_2) - \delta_b m^2\beta^2(\beta_1^2 + \beta_2^2)\mathrm{th}(\frac{m\pi\beta}{2}\beta_3) = 0 \tag{37}$$

其中

$$\beta_1 = \sqrt{\frac{\sqrt{k_\omega^2\delta_b^2 + 4k_\omega} - k_\omega\delta_b}{2m^2\beta^2} + 1},$$

$$\beta_2 = \sqrt{\frac{\sqrt{k_\omega^2\delta_b^2 + 4k_\omega} + k_\omega\delta_b}{2m^2\beta^2} + 1}$$

$$\beta_3 = \sqrt{\frac{2}{\delta_b m^2\beta^2(1 - v)} + 1},$$

$$k_\omega = \frac{\rho b^4\omega^2}{D\pi^2}, \delta_b = \frac{D\pi^2}{Cb^2}, \beta = \frac{b}{a} \tag{38}$$

经过与对称振动类似的推导,可以得到板反对称振动的频率方程为

$$\beta_1\beta_3[1 + \delta_b m^2\beta^2(1 + \beta_2^2)]\coth(\frac{m\pi\beta}{2}\beta_1) - \beta_2\beta_3[1 + \delta_b m^2\beta^2(1 - \beta_1^2)]\cot(\frac{m\pi\beta}{2}\beta_2) - \delta_b m^2\beta^2(\beta_1^2 + \beta_2^2)\coth(\frac{m\pi\beta}{2}\beta_3) = 0 \tag{39}$$

算例:计算了不同边长比和弯剪刚度比情况下泊松比的对边简支另两边固支矩形中厚板的最低固有频率系数. 计算结果如表 1 所示,表中括号里列出了已有文献[17]用半逆法计算得到的结果以便对比. 显然,本文所给的解与参考文献完全一致,从而证明本文解法与推导是正确的.

表 1　对边简支另两边固支矩形中厚板的最低固有频率系数

Table 1　The lowest natural frequency parameters of moderately thick rectangular plates with two opposite edges simply supported and the others clamped

δ_b \ β	0.2	0.4	0.6	0.8	1.0	1.5	2.0
0	(5.240) 5.24005	(5.562) 5.56251	(6.162) 6.16210	(7.132) 7.13233	(8.604) 8.60445	(15.19) 15.6861	(30.76) 30.7651
0.01	(4.987) 4.98720	(5.287) 5.28707	(5.846) 5.84618	(6.753) 6.75305	(8.131) 8.13065	(14.74) 14.7379	(28.62) 28.6244
0.05	(4.179) 4.17854	(4.415) 4.41512	(4.861) 4.86143	(5.593) 5.59286	(6.710) 6.70985	(12.02) 12.0169	(22.70) 22.7048
0.10	(3.472) 3.47220	(3.662) 3.66195	(4.025) 4.02492	(4.626) 4.62644	(5.549) 5.54927	(9.879) 9.87950	(18.23) 18.2344
0.20	(2.593) 2.59302	(2.773) 2.73318	(3.007) 3.00685	(3.466) 3.46635	(4.172) 4.17199	(7.390) 7.38997	(13.21) 13.2088
0.40	(1.721) 1.72060	(1.819) 1.81852	(2.013) 2.01326	(2.341) 2.34143	(2.839) 2.83890	(4.991) 4.99132	(8.585) 8.58540
0.60	(1.288) 1.28769	(1.366) 1.36595	(1.522) 1.52212	(1.783) 1.78325	(2.173) 2.17255	(3.792) 3.79179	(6.376) 6.37605
0.80	(1.029) 1.02913	(1.096) 1.09551	(1.228) 1.22762	(1.446) 1.44619	(1.767) 1.76706	(3.064) 3.06362	(5.075) 5.07494
1.00	(0.857) 0.857241	(0.915) 0.91544	(1.031) 1.03064	(1.219) 1.21919	(1.492) 1.49238	(2.572) 2.57246	(4.216) 4.21612

4　结论

将矩形中厚板自由振动问题的控制方程导入

Hamilton体系表示成正则方程，建立了问题的Hamilton求解体系，在辛空间中利用分离变量与本征展开方法推导出了矩形中厚板典型问题的精确振动解. 由于求解过程不需人为选定振幅函数，而是直接从中厚板的基本方程出发，推导出完全满足控制方程和边界条件的解析解，从而使问题的求解更理性化和合理化. 求解过程遵循一套统一的方法论，易于推广至任意边界条件，因而有望得到更多振动问题的精确解，这将构成本文的后续工作.

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ON HAMILTON SYSTEM AND NEW SYMPLECTIC APPROACH FOR FREE VIBRATION OF MODERATELY THICK RECTANGULAR PLATES

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Abstract Based on Hu's equations of moderately thick rectangular plates, the free vibration problem for the plates were transferred into Hamilton system. Then, the whole state variables were separated. Using the method of eigenfunction expansion in the symplectic geometry, the exact free vibration solutions of the plates with two opposite sides simply supported were obtained. Since only the basic elasticity equations of the plates were used, the method eliminates the need to pre – determine any trial functions and is hence more reasonable than the conventional semi – inverse methods. Numerical results were presented to demonstrate the validity and accuracy of the approach.

Key words moderately thick rectangular plate, free vibration, Hamilton system, symplectic geometry method

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