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一种基于随机平均的最优时滞控制方法*

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摘要　基于时滞系统的随机平均法与随机动态规划原理,提出一种非线性系统的随机最优时滞控制方法. 先应用时滞随机平均法,将非线性系统的随机最优时滞控制问题变换成非时滞的最优控制问题;再根据随机动态规划原理,建立其动态规划方程;由此确定最优时滞控制律;最后,通过一个例子说明该时滞控制方法的控制效果.

关键词　最优时滞控制,　非线性随机系统,　随机动态规划,　时滞随机平均

引 言

工程结构系统常常经受强随机载荷作用,其非线性随机振动的主动与半主动控制已有很多研究,基于动态规划原理的最优控制方法可以达到很好的控制效果. 然而,反馈控制过程的时滞不可避免,它可能退化最优控制的性能,并导致受控系统的不稳定. 关于确定性系统的时滞控制问题已有一定研究[1-7],而对于随机系统的研究相对很少. 文[8,9]采取控制力的级数展开法研究高斯白噪声激励系统的时滞控制问题,文[10]应用李雅普诺夫指数分析线性时滞控制系统的稳定性. 本文基于时滞系统的随机平均法与随机动态规划原理,研究非线性系统的随机最优时滞控制问题. 先应用时滞随机平均法,将非线性系统的随机最优时滞控制问题变换成非时滞的最优控制问题;再根据随机动态规划原理,建立其动态规划方程;由此确定最优时滞控制律.

1　最优时滞控制问题及其变换

考虑一个受时滞控制的非线性随机结构系统

$$M\ddot{X}+C\dot{X}+\frac{\partial V_p(X)}{\partial X}=FW(t)+BU(X_\tau,\dot{X}_\tau) \tag{1}$$

式中 X 是 n 维位移向量,M 与 C 分别是对称正定的质量阵与阻尼阵,$V_p(X)\geqslant 0$ 是系统势能,$W(t)$ 是 m 维随机过程向量,设为强度 $2D$ 的高斯白噪声,$U(X_\tau,\dot{X}_\tau)$ 是 s 维控制力向量,依赖于过去的状态 $X_\tau=X(t-\tau)$ 和 $\dot{X}_\tau=\dot{X}(t-\tau)$,$\tau$ 是控制时滞,F 与 B 是常数矩阵. 改写式(1)成拟哈密顿方程形式

$$\dot{Q}=\frac{\partial H}{\partial P},\dot{P}=-\frac{\partial H}{\partial Q}-C\frac{\partial H}{\partial P}+FW(t)+BU(Q_\tau,P_\tau) \tag{2}$$

式中 $Q=X,P=M\dot{X},H=P^TM^{-1}P/2+V_p(Q)$ 是哈密顿函数. 系统最优控制的性能指标,对于无限时间间隔各态历经控制情形为

$$J=\lim_{t_f\to\infty}\frac{1}{t_f}\int_0^{t_f}L(Q,P,U(Q_\tau,P_\tau))\mathrm{d}t \tag{3}$$

式中 t_f 是最终时间,$L(Q,P,U)$ 是连续可微的凸函数. 方程(2)和(3)组成一个非线性随机系统的最优时滞控制问题.

设相应于(2)的哈密顿系统可积非共振. 根据时滞系统的随机平均法[11],将系统过去状态用当前状态表示为

$$\begin{aligned}Q_{\tau i}&=Q_i(t-\tau)\doteq Q_i(t)\cos\omega_i\tau-\frac{P_i(t)}{\omega_i}\sin\omega_i\tau\\ P_{\tau i}&=P_i(t-\tau)\doteq Q_i(t)\omega_i\sin\omega_i\tau+P_i(t)\cos\omega_i\tau\end{aligned} \tag{4}$$

式中 Q_i 与 P_i 分别是向量 Q 与 P 的元素,ω_i 是子系统的平均频率. 平均系统的 *Itô* 随机微分方程为

$$\mathrm{d}H_\tau=\left[m_r(H)+\left\langle(B\bar{U}(Q,P))_i\frac{\partial H_r}{\partial P_i}\right\rangle\right]\mathrm{d}t+\sigma_{rk}(H)\mathrm{d}B_k(t),$$
$$r,\ i=1,2,\cdots,n,\quad k=1,2,\cdots,m \tag{5}$$

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式中 H_r 是首次积分,$H=[H_1,H_2,\cdots,H_n]^T$,<. > 是平均算子,$\bar{U}(Q,P)=U(Q_i\cos\omega_i\tau-P_i\sin\omega_i\tau/\omega_i,Q_i\omega_i\sin\omega_i\tau+P_i\cos\omega_i\tau)$ 是变换后的控制力,$B_k(t)$ 是单位维纳过程,$m_r(H)$ 与 $\sigma_{rk}(H)$ 分别是漂移与扩散系数,

$$m_r(H)=\frac{1}{T(H)}\int[(-C_{ij}\frac{\partial H_r}{\partial p_i}\frac{\partial H_r}{\partial p_j}+D_{kl}F_{ik}F_{jl}\frac{\partial^2 H_r}{\partial p_i\partial p_j})/(\frac{\partial H_1}{\partial p_1}\frac{\partial H_2}{\partial p_2}\cdots\frac{\partial H_n}{\partial p_n})]\mathrm{d}q_1\mathrm{d}q_2\cdots\mathrm{d}q_n,$$

$$\sigma_{r\alpha}(H)\sigma_{s\alpha}(H)=\frac{1}{T(H)}\int[2D_{kl}F_{ik}F_{jl}\frac{\partial H_r\partial H_s}{\partial p_i\ \partial p_j})/(\frac{\partial H_1}{\partial p_1}\frac{\partial H_2}{\partial p_2}\cdots\frac{\partial H_n}{\partial p_n})]\mathrm{d}q_1\mathrm{d}q_2\cdots\mathrm{d}q_n,$$

$$T(H)=\int[1/(\frac{\partial H_1}{\partial p_1}\frac{\partial H_2}{\partial p_2}\cdots\frac{\partial H_n}{\partial p_n})]\mathrm{d}q_1\mathrm{d}q_2\cdots\mathrm{d}q_n,$$

$$r,s,i,j=1,2,\cdots,n,\quad k,l,\alpha=1,2,\cdots,m \tag{6}$$

相应于系统(5),性能指标(3)成为

$$\bar{J}=\lim_{t_f\to\infty}\frac{1}{t_f}\int_0^{t_f}L(H(t),\langle\bar{U}(Q(t),P(T))\rangle)\mathrm{d}t \tag{7}$$

基于式(4)与随机平均,最优时滞控制问题[(2)和(3)]被变换成非时滞的最优控制问题[(5)和(7)].

2　最优时滞控制律

根据随机动态规划原理,引入值函数 V,建立最优控制问题(5)和(7)的动态规划方程[12]

$$\min_{\bar{U}}\{L(H,\langle\bar{U}(Q,P)\rangle)+[m_r(H)+\langle B\ \bar{U}(Q,P))_i\frac{\partial H_r}{\partial P_i}\rangle]\frac{\partial V}{\partial H_r}+\frac{1}{2}\sigma_{rk}(H)\sigma_{sk}(H)\frac{\partial^2 V}{\partial H_r\partial H_s}\}=\lambda \tag{8}$$

式中 λ 是常数. 由式(8)左边极小化可得最优控制律. 令性能函数

$$L(H,\langle\bar{U}\rangle)=g(H)+\langle\bar{U}^T(Q,P)R\ \bar{U}(Q,P)\rangle \tag{9}$$

式中 $g(H)\geq 0$,R 是对称正定的常数矩阵,则最优控制力为

$$\bar{U}^*(Q,P)=-\frac{1}{2}R^{-1}B^T(\frac{\partial H_r}{\partial P}\frac{\partial V}{\partial H_r})(Q,P) \tag{10}$$

它依赖于当前系统状态(Q,P). 其中值函数 V 由下式解得

$$\frac{1}{2}\sigma_{rk}(H)\sigma_{sk}(H)\frac{\partial^2 V}{\partial H_r\partial H_s}-\frac{1}{4}(\frac{\partial V}{\partial H_r})^T\times\langle(\frac{\partial H_r}{\partial P})^T BR^{-1}B^T(\frac{\partial H_r}{\partial P})\rangle\frac{\partial V}{\partial H_r}+m_r(H)\frac{\partial V}{\partial H_r}+g(H)-\lambda=0 \tag{11}$$

利用式(4),将系统当前状态用过去状态表示为

$$Q_i=Q_i(t)\doteq Q_{\tau i}\cos\omega_i\tau+\frac{P_{\tau i}}{\omega_i}\sin\omega_i\tau,$$

$$P_i=P_i(t)\doteq-Q_{\tau i}\omega_i\sin\omega_i\tau+P_{\tau i}\cos\omega_i\tau \tag{12}$$

于是,可得到最优时滞控制力

$$\begin{aligned}U^*(X(t-\tau),\dot{X}(t-\tau))&=U^*(Q_\tau,P_\tau)=\\U^*(Q_i\cos\omega_i\tau-P_i\sin\omega_i\tau/\omega_i,&Q_i\omega_i\sin\omega_i\tau+P_i\cos\omega_i\tau)=\bar{U}^*(Q,P)=\bar{U}^*(Q_{\tau i}\cos\omega_i\tau+\\P_{\tau i}\sin\omega_i\tau/\omega_i,&-Q_{\tau i}\omega_i\sin\omega_i\tau+P_{\tau i}\cos\omega_i\tau)=\\-\frac{1}{2}R^{-1}B^T(\frac{\partial H_r}{\partial P}\frac{\partial V}{\partial H_r})&(X_{\tau i}\cos\omega_i\tau+(M\dot{X}_\tau)_i\sin\omega_i\tau/\omega_i,-X_{\tau i}\omega_i\sin\omega_i\tau+(M\dot{X}_\tau)_i\cos\omega_i\tau)\end{aligned} \tag{13}$$

它完全依赖于系统过去状态,可由主动控制设备,例如 AMD 实施.

半主动的 MR 阻尼器[13]具有很好的结构控制应用前景. 其控制力 U 可分成被动部分 U_{ps} 和依赖于外电压的半主动部分 U_{sa},即 $U=U_{ps}+U_{sa}$. 按照 Bingham 模型,它们为[14]

$$U_{ps}=-C_dB^T\dot{X},U_{sa}=-F_d\mathrm{sgn}(B^T\dot{X}) \tag{14}$$

式中 $C_d=\mathrm{diag}(C_{d1},C_{d2},\cdots,C_{ds})$ 是阻尼系数矩阵,$F_d=\mathrm{diag}(F_{d1},F_{d2},\cdots,F_{ds})$ 是可控力幅值矩阵. 将被动部分 U_{ps} 并入系统,比较(14)的第二式与(13),可得半主动部分 U_{sa}

$$U_{sa}^*(X(t-\tau),\dot{X}(t-\tau))=[u_{sa1}^*(X_\tau,\dot{X}_\tau),u_{sa2}^*(X_\tau,\dot{X}_\tau),\cdots,u_{sas}^*(X_\tau,\dot{X}_\tau)],$$

$$u_{saj}^*(X_\tau,\dot{X}_\tau)=-\frac{1}{2}(F_{dj}^*+|F_{dj}^*|)\mathrm{sgn}\{(B^TM^{-1}P)_j\} \tag{15}$$

其中

$$F_{dj}^*=\frac{1}{2}(R^{-1}B^T\frac{\partial H_r}{\partial P}\frac{\partial V}{\partial H_r})_j(Q,P)\mathrm{sgn}\{(B^TM^{-1}P)_j\},$$

$$Q_r=X_{\tau r}\cos\omega_r\tau+\frac{(M\dot{X}_\tau)_r}{\omega_r}\sin\omega_r\tau,$$

$$P_r=-X_{\tau r}\omega_r\sin\omega_r\tau+(M\dot{X}_\tau)_r\cos\omega_r\tau \tag{16}$$

U_{sa}(15)完全依赖于系统过去状态,是一个最优半主动时滞控制力.适当选取 $g(H)$ 与正定对角的 R,使 $\partial V/\partial H_1=\partial V/\partial H_2=\cdots=\partial V/\partial H_n\geqslant 0$,则最优半主动时滞控制力(15)成为

$$u_{saj}^*(X_\tau,\dot{X}_\tau)=-F_{dj}^*\operatorname{sgn}\{(B^TM^{-1}P)_j\}=-\frac{1}{2R_{jj}}(B^TM^{-1}P)_j\frac{\partial V}{\partial H_1}(X_{\tau r}\cos\omega_r\tau+(M\dot{X}_\tau)_r\sin\omega_r\tau/\omega_r,-X_{\tau r}\omega_r\sin\omega_r\tau+(M\dot{X}_\tau)_r\cos\omega_r\tau)\tag{17}$$

它是一个最优主动时滞控制力(13),此时半主动的 MR 阻尼器按(17)可执行最优主动时滞控制律.

3 应用例子

作为一个例子,考虑承受时滞控制力与基础激励的滞回柱[15]

$$\ddot{X}+2\zeta_0\dot{X}+(\alpha-k_1)X+(1-\alpha)Z=W_1(t)+k_2XW_2(t)+u(X_\tau,\dot{X}_\tau)\tag{18}$$

式中 X 是无量纲位移,ζ_0 是线性阻尼系数,α 是滞回力比例系数,k_1 与 k_2 是常系数,$W_1(t)$ 与 $W_2(t)$ 分别是横向与纵向基础激励,设为强度 $2D_1$ 与 $2D_2$ 的高斯白噪声,$u(X_\tau,\dot{X}_\tau)$ 是时滞控制力,依赖于过去的位移 $X_\tau=X(t-\tau)$ 与速度 $\dot{X}_\tau=\dot{X}(t-\tau)$. Z 是 Bouc-Wen 滞回力,由下式确定

$$\dot{Z}=A\dot{X}-\beta\dot{X}|Z|^n-\gamma|\dot{X}|Z|Z|^{n-1}\tag{19}$$

式中 A、β、γ 与 n 是滞回参数.

应用时滞随机平均法推导平均系统的 *Itô* 方程

$$dH=[m(H)+\langle\bar{u}(Q,P)\frac{\partial H}{\partial P}\rangle]dt+\sigma(H)dB_e(t)\tag{20}$$

式中 $B_e(t)$ 是单位维纳过程,$\bar{u}(Q,P)=u(X_\tau,\dot{X}_\tau)$

$$H=\dot{x}^2/2+V_p(x),$$

$$\sigma^2(H)=\frac{2}{T(H)}\int_{-a}^{a}(2D_1+2k_2^2D_2x^2)\sqrt{2H-2V_p(x)}dx,$$

$$m(H)=\frac{1}{T(H)}[-A_r-4\zeta_0\int_{-a}^{a}\sqrt{2H-2V_p(x)}dx+2k_2^2D_2\int_{-a}^{a}\frac{x^2dx}{\sqrt{2H-2V_p(x)}}]+D_1,$$

$$T(H)=2\int_{-a}^{a}\frac{dx}{\sqrt{2H-2V_p(x)}},\ \omega=\frac{2\pi}{T(H)},$$

$$Q=X\doteq X_\tau\cos\omega\tau+\dot{X}_\tau\sin\omega\tau/\omega,$$

$$P=\dot{X}\doteq-X_\tau\omega\sin\omega\tau+\dot{X}_\tau\cos\omega\tau\tag{21}$$

$V_p(x)$是等价的系统势能,A_r 是滞回环面积,a 是位移幅值,与 H 的关系为 $H=V_p(\pm a)$,其表达式见文[15].

对于系统(20)与性能指标(7)的最优控制,根据随机动态规划原理,建立动态规划方程,由此可得最优时滞控制力.对于 $L(H,\bar{u})=g(H)+\langle r\bar{u}^2(Q,P)\rangle$,有

$$u^*(X_\tau,\dot{X}_\tau)=-\frac{1}{2r}(-X_\tau\omega\sin\omega\tau+\dot{X}_\tau\cos\omega\tau)\frac{d}{dH}V(H(X_\tau\cos\omega\tau+\dot{X}_\tau\sin\omega\tau/\omega,-X_\tau\omega\sin\omega\tau+\dot{X}_\tau\cos\omega\tau))\tag{22}$$

式中 r 是正的权参数.设 $g(H)=s_0+s_1H+s_2H^2+s_3H^3$,$dV/dH$ 由下式解得

$$\frac{1}{2}\sigma^2(H)\frac{d^2V}{dH^2}+m(H)\frac{dV}{dH}-\frac{1}{4r}G(H)(\frac{dV}{dH})^2+g(H)-\lambda=0\tag{23}$$

$$G(H)=\frac{2}{T(H)}\int_{-a}^{a}\sqrt{2H-2V_p(x)}dx\tag{24}$$

当控制力由 MR 阻尼器产生时,最优半主动时滞控制力为

$$u_{sa}^*(X_\tau,\dot{X}_\tau)=-\frac{1}{2}(F_d^*+|F_d^*|)\times\operatorname{sgn}\{-X_\tau\omega\sin\omega\tau+\dot{X}_\tau\cos\omega\tau\},$$

$$F_d^*=\frac{1}{2r}|-X_\tau\omega\sin\omega\tau+\dot{X}_\tau\cos\omega\tau|\frac{d}{dH}V\times(H(X_\tau\cos\omega\tau+\dot{X}_\tau\sin\omega\tau/\omega,-X_\tau\omega\sin\omega\tau+\dot{X}_\tau\cos\omega\tau))\tag{25}$$

选取 r 与 $g(H)$ 使 $dV/dH\geqslant 0$,则

$$u_{sa}^*(X_\tau,\dot{X}_\tau)=u^*(X_\tau,\dot{X}_\tau)\tag{26}$$

即半主动的 MR 阻尼器可执行最优主动时滞控制律.

为估算控制功效,将最优时滞控制力(22)代入式(20),得到受控系统的平均 Itô 方程

$$dH=[m(H)-\frac{1}{2r}G(H)\frac{dV}{dH}]dt+\sigma(H)dB_e(t)\tag{27}$$

建立相应的 FPK 方程,求得平稳概率密度

$$p(H)=C_0\exp\{-\int_0^H[(-2m(y)+\frac{G(y)}{r}\frac{dV}{dy}+\frac{d\sigma^2(y)}{dy})/\sigma^2(y)]dy\}\tag{28}$$

式中 C_0 是归一化常数. 利用式(28)可估算受控滞回柱的均方响应与均方控制力，即

$$E[X_c^2] = \int_0^\infty \frac{p(H)}{T(H)} \mathrm{d}H \int_{-a}^{a} \frac{2x^2 \mathrm{d}x}{\sqrt{2H-2V_p(x)}} \qquad (29)$$

$$E[u^{*2}] = \int_0^\infty \frac{G(H)}{4r^2} \left(\frac{\mathrm{d}V}{\mathrm{d}H}\right)^2 p(H)\,\mathrm{d}H \qquad (30)$$

未控滞回柱的均方响应 $E[X_u^2]$ 可类似地求得. 用如下指标评价控制功效

$$K = \frac{\sqrt{E[X_u^2]} - \sqrt{E[X_c^2]}}{\sqrt{E[X_u^2]}} \times 100\% \qquad (31)$$

$$\mu = \frac{K}{\sqrt{E[u^{*2}]}/\sqrt{2D_1}} \qquad (32)$$

指标 K 是系统受控前后均方根位移之差的百分比，反映控制效果；指标 μ 是系统位移百分比降低与规范化均方根控制力之比，反映控制效率. 显然，K 与 μ 值越高，控制方法越好.

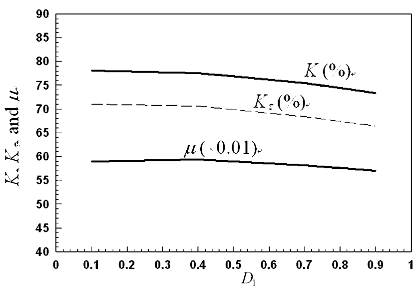

图 1　不同外激强度(D_1)下的控制效果（K-无时滞；K_τ-时滞）与效率(μ)

Fig. 1　Control effectiveness (K-non-delayed; K_τ-delayed) and efficiency (μ) for different excitation intensity (D_1)

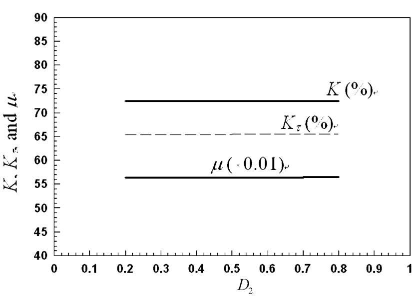

图 1　不同参激强度(D_2)下的控制效果（K-无时滞；K_τ-时滞）与效率(μ)

Fig. 1　Control effectiveness (K-non-delayed; K_τ-delayed) and efficiency (μ) for different excitation intensity (D_2)

选取结构参数 $\xi = 0.025$，$k_1 = 0.04$，$k_2 = 0.1$，$\alpha = \beta = \gamma = 0.5$，$A = n = 1$ 与控制参数 $r = 1$，$s_1 = s_3 = 0$，$s_2 = 1$，$dV(0)/dH = 3.5$ 及时滞 $\tau = 0.05$，进行数值计算，结果如图 1 和 2 所示，它们分别展示了控制效果与效率随外参激强度的变化. 可见，本文的最优时滞控制方法可达到很好的控制功效.

4　结束语

反馈控制过程的时滞不可避免，时滞可能退化最优控制的性能，并导致受控系统的不稳定，如何有效地补偿时滞效应，改善时滞控制效果是控制领域有待研究的一个重要问题. 本文提出了一种基于随机平均的最优时滞控制方法，并通过例子说明其较好的控制效果. 如何更好地提高随机最优时滞控制效果，使方法具有更广的适用性，仍有待于进一步的研究.

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AN OPTIMAL TIME-DELAY CONTROL METHOD BASED ON THE STOCHASTIC AVERAGING *

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Abstract A stochastic optimal time-delay control method for stochastically excited nonlinear systems was proposed based on the stochastic averaging method for time-delay systems and the stochastic dynamical programming principle. The stochastic optimal time-delay control problem of nonlinear systems was converted into another non-time-delay optimal control problem by using the stochastic averaging method for time-delay systems. The dynamical programming equation for the converted control problem was established based on the stochastic dynamical programming principle. The optimal control law was obtained from this equation, and the optimal active time-delay control law was obtained by the transformation of the present and past system states. The optimal semi-active time-delay control law was also obtained according to the Bingham model of MR dampers. An example of stochastically excited and controlled hysteretic column was given to illustrate the application and effectiveness of the proposed stochastic optimal time-delay control method.

Key words optimal time-delay control, nonlinear stochastic system, stochastic dynamical programming, stochastic time-delay averaging

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