蜂窝夹层板的非线性动力学研究*

孙佳　张伟　陈丽华　姚明辉
（北京工业大学机电学院，北京　100022）

摘要　研究了正六角形蜂窝夹层板的非线性动力学问题. 考虑高阶横向剪切变形和横向阻尼的影响，建立了面内激励和横向外激励联合作用下的四边简支蜂窝夹层板的非线性偏微分运动控制方程. 综合运用 Galerkin 方法和数值方法，模拟不同激励作用下的混沌运动，得到二维相图、二维波形图和频谱图. 研究结果表明：随着激励的增加，系统会重复呈现周期运动、混沌运动、周期运动的变化规律.

关键词　蜂窝夹层板，　高阶剪切效应，　非线性动力学，　混沌

引言

蜂窝结构起源于仿生学，1938 年 Bruyne 博士在英国的剑桥申请了一项专利，用铝泊制成了六角型铝蜂窝芯. 蜂窝结构具有低密度、高比刚度、高比强度、压缩应变大、有良好的能量吸收特性，因此在航空航天领域广泛应用. 五、六十年代，高性能的蜂窝制造技术已经成熟，蜂窝结构被普遍地应用于飞机、航空以及宇航工业. 铝合金蜂窝复合材料被大量用于制造各种飞机部件，提高了飞机的性能，为航空航天技术的发展做出了巨大的贡献，可以说，它是随着航空航天科技的特殊需要而发展起来的一种超轻型的结构[1].

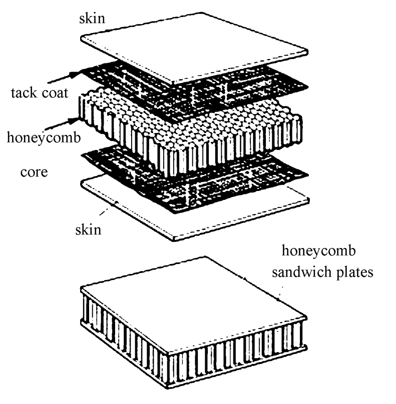

图 1　蜂窝夹层板的结构

Fig. 1　The structure of honeycomb sandwich plates

在航空航天领域里应用的蜂窝结构大多可以简化成蜂窝夹层板结构. 蜂窝夹层板是一种复合层合结构，它由上下较薄的蒙皮层和中间一个较厚的蜂窝芯通过粘结、压制而构成的，如图 1 所示.

对于复合层合结构非线性动力学的研究，早在 1910 年，von Karman 发表了平板大挠度非线性方程，奠定了各向同性薄板非线性分析的基础[2]. 1984 年，Reddy[3] 提出了一种高阶剪切变形板理论，从而可以很好的描述沿厚度方向剪切变形的变化. 同年，他又将此理论推广到包含几何非线性的复合材料层合板中. 早期对蜂窝夹层板的研究成果主要集中在对蜂窝芯体力学性能的理论和实验方面的研究. 1982 年，Gibson 等人[4] 对蜂窝芯层静力学行为做了大量研究，给出了均匀壁厚蜂窝芯杨氏模量、剪切模量与蜂窝芯几何尺寸之间的计算公式. 1991 年，王颖坚[5] 指出 Gibson 等人提出的面内剪力作用下的蜂窝结构变形模式不满足平衡条件，并在 Gibson 的基础上修正了面内剪切模量公式. 1995 年，Lim 等人[6, 7] 研究了不同边界条件对蜂窝夹层板固有频率的影响. 2001 年，Zhang 等人[8, 9] 分别研究了面内激励作用下四边简支矩形板以及面内激励和横向外激励联合作用下四边简支矩形板的非线性动力学. 2005 年，Ye 等人[10] 利用全局摄动法研究了复合材料层合板的非线性动力学. 同年，鲍四元和邓子辰[11] 应用 Hamilton 原理研究了矩形薄板自由振动特性. 2008 年，Hao 等人[12] 研究了面内激励和横向外激励联合作用下四边简支矩形功能梯度板的非线性动力学行为.

从已有的文献来看，研究蜂窝夹层板的非线性动力学相对较少，一方面是因为建立蜂窝夹层板的

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运动方程是一个难点,另外一方面是蜂窝夹层板的运动方程非常复杂,是互相耦合的非线性偏微分方程组,这样就给后面的理论分析带来许多困难. 由于航空航天中发生的振动都是大挠度、大变形的非线性振动,因此不能应用线性理论描述结构的大变形. 非线性的大振幅振动可以引起飞机机翼损坏,卫星星体的大挠度振动可能引起卫星系统的内能耗散,最后引起卫星姿态失稳而翻滚. 所以,用非线性理论研究蜂窝夹层板结构,具有重要的理论和实际意义.

以面内激励和横向外激励联合作用下的四边简支蜂窝夹层板为研究对象. 考虑高阶横向剪切变形和横向阻尼对系统的影响,综合运用 Galerkin 方法和数值方法研究了蜂窝夹层板的非线性动力学行为.

文献[8 - 10]和文献[12]中所研究的内容是四边简支均质板、复合材料板以及功能梯度板在某种特定共振关系情况下板类结构的非线性动力学. 学者们首先建立板类结构的运动控制方程,运用 Galerkin 方法对其非线性偏微运动控制方程进行截断,得到多自由度常微分方程,再利用多尺度方法得到平均方程,在平均方程基础之上应用全局摄动法和数值模拟进行混沌动力学的研究. 而在本文中,研究蜂窝夹层板的非线性动力学所采用的本构关系是 Gibson 公式[4],我们对四边简支蜂窝夹层板离散后的常微分方程进行研究,在实际工程应用背景的基础之上选取参数进行数值模拟系统的混沌动力学,因此,本文的研究成果为蜂窝夹层板在航空航天领域的应用提供重要的理论依据.

1　蜂窝夹层板的动力学模型

蜂窝夹层板的力学模型如图 2 所示,此模型为四边简支条件下蜂窝夹层板同时受到 x 方向的面内载荷与横向面外载荷联合作用,夹层板在振动过程中考虑横向阻尼的影响. 夹层板的长、宽、高分别为 a、b、H,直角坐标 Oxy 位于层合板的中性面内,z 轴向下,设板内任一点沿 x、y 和 z 方向的位移分别为 u、v 和 w,沿着 x 方向作用的面内载荷为 $p = p_0 - p_1\cos\Omega_2 t$,横向载荷为 $f = F(x,y)\cos\Omega_1 t$. 蜂窝夹层板分为三层,上下蒙皮是完全相同的各向同性材料,蒙皮层厚度为 h_f. 中间由正六角形蜂窝芯层隔开,蜂窝芯轴向为坐标 z 方向,蜂窝芯厚度为 h_c. 由 Gibson 经典理论[4]可知,正六边形蜂窝芯层为面内各向同性材料. 假设蒙皮与蜂窝芯结合紧密,不会脱开,并且粘结层很薄,其本身不发生变形,即各层板之间变形连续. 这里采用 Reddy 的三阶剪切变形理论[3]建立蜂窝夹层板的非线性动力学方程.

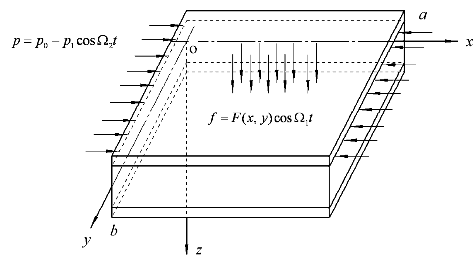

图 2　受横向与 X 方向的面内载荷联合作用下的蜂窝夹层板模型

Fig. 2　The model of the plates subjected to in-plane excitation and transverse excitation

由 Reddy 的三阶剪切变形板理论可知,板的位移场为

$$u(x,y,z,t) = u_0(x,y,t) + z\phi_x(x,y,t) - z^3\frac{4}{3H^2}(\phi_x + \frac{\partial w_0}{\partial x}) \tag{1a}$$

$$v(x,y,z,t) = v_0(x,y,t) + z\phi_y(x,y,t) - z^3\frac{4}{3H^2}(\phi_y + \frac{\partial w_0}{\partial y}) \tag{1b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{1c}$$

其中 u_0、v_0、w_0 为中性面($z=0$)上的位移,ϕ_x、ϕ_y 为中性面法线相对于 x、y 轴的转角.

应变与位移的关系即几何关系采用非线性形式

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,$$
$$\varepsilon_{zz} = \frac{\partial w}{\partial z}, \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y},$$
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \tag{2}$$

于是应变可以写成

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} + z\begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} - c_1 z^3\begin{Bmatrix}\varepsilon_{xx}^{(3)}\\ \varepsilon_{yy}^{(3)}\\ \gamma_{xy}^{(3)}\end{Bmatrix} \tag{3a}$$

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = \begin{Bmatrix}\gamma_{yz}^{(0)}\\ \gamma_{xz}^{(0)}\end{Bmatrix} - c_2 z^2\begin{Bmatrix}\gamma_{yz}^{(2)}\\ \gamma_{xz}^{(2)}\end{Bmatrix} \tag{3b}$$

其中 $c_1 = \frac{4}{3}H^2$, $c_2 = 3c_1$,

$$\begin{Bmatrix}\gamma_{yz}^{(0)}\\ \gamma_{xz}^{(0)}\end{Bmatrix} = \begin{Bmatrix}\phi_y + \frac{\partial w_0}{\partial y}\\ \phi_x + \frac{\partial w_0}{\partial x}\end{Bmatrix}, \begin{Bmatrix}\gamma_{yz}^{(2)}\\ \gamma_{xz}^{(2)}\end{Bmatrix} = \begin{Bmatrix}\phi_y + \frac{\partial w_0}{\partial y}\\ \phi_x + \frac{\partial w_0}{\partial x}\end{Bmatrix},$$

$$\begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}+\dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2\\ \dfrac{\partial v_0}{\partial y}+\dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}+\dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y}\end{Bmatrix},$$

$$\begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial \phi_x}{\partial x}\\ \dfrac{\partial \phi_y}{\partial y}\\ \dfrac{\partial \phi_x}{\partial y}+\dfrac{\partial \phi_y}{\partial x}\end{Bmatrix},$$

$$\begin{Bmatrix}\varepsilon_{xx}^{(3)}\\ \varepsilon_{yy}^{(3)}\\ \gamma_{xy}^{(3)}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial \phi_x}{\partial x}+\dfrac{\partial^2 w_0}{\partial x^2}\\ \dfrac{\partial \phi_y}{\partial y}+\dfrac{\partial^2 w_0}{\partial y^2}\\ \dfrac{\partial \phi_x}{\partial y}+\dfrac{\partial \phi_y}{\partial x}+2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix} \tag{4}$$

对于蜂窝夹层板蜂窝芯层采用 Gibson 经典理论[4]给出的本构关系和王颖坚[5]给出的修正后剪切模量计算公式

$$\frac{E_{xxc}^*}{E_s}=\frac{E_{yyc}^*}{E_s}=\frac{4\sqrt{3}}{3}\left(\frac{d}{l}\right)^3,\ \frac{G_{xzc}^*}{G_s}=\frac{G_{yzc}^*}{G_s}=\frac{\sqrt{3}}{3}\frac{d}{l},$$
$$\frac{G_{xyc}^*}{E_s}=\frac{\sqrt{3}}{2}\left(\frac{d}{l}\right)^3,\ \frac{\rho^*}{\rho_s}=\frac{2}{\sqrt{3}}\frac{d}{l} \tag{5}$$

其中 E_{xxc}^*、E_{yyc}^*、G_{xzc}^*、G_{yzc}^*、G_{xyc}^*、ρ^* 为蜂窝芯的模量、密度,E_s、G_s、ρ_s 为蜂窝芯构成材料的模量、密度,d 为正六角形蜂窝壁厚,l 为正六角形边长.

对于各向同性材料,剪切模量、杨氏模量的关系如下

$$G=\frac{E}{2(1+\nu)} \tag{6}$$

由此可知蜂窝芯的泊松比为 $\nu=1/3$.

根据 Hamilton 原理,把方程表示成广义位移形式

$$A_{11}\frac{\partial u_0}{\partial x^2}+A_{66}\frac{\partial^2 u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x\partial y}+A_{11}\frac{\partial w_0}{\partial x}\times$$
$$\frac{\partial^2 w_0}{\partial x^2}+A_{66}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}=$$
$$I_0\ddot{u}_0+J_1\ddot{\phi}_x-c_1I_3\frac{\partial\ddot{w}_0}{\partial x} \tag{7a}$$

$$A_{66}\frac{\partial v_0}{\partial x^2}+A_{22}\frac{\partial^2 v_0}{\partial y^2}+(A_{21}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}+A_{66}\frac{\partial w_0}{\partial y}\times$$
$$\frac{\partial^2 w_0}{\partial x^2}+A_{22}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}+(A_{21}+A_{66})\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial y}=$$
$$I_0\ddot{v}_0+J_1\ddot{\phi}_y-c_1I_3\frac{\partial\ddot{w}_0}{\partial y} \tag{7b}$$

$$A_{11}\frac{\partial u_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}+A_{21}\frac{\partial u_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}+2A_{66}\frac{\partial u_0}{\partial y}\frac{\partial^2 w_0}{\partial x\partial y}+A_{11}\frac{\partial^2 u_0}{\partial x^2}\times$$
$$\frac{\partial w_0}{\partial x}+(A_{21}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}\frac{\partial w_0}{\partial y}+A_{66}\frac{\partial^2 v_0}{\partial x^2}\frac{\partial w_0}{\partial y}+(A_{12}+$$
$$A_{66})\frac{\partial^2 v_0}{\partial x\partial y}\frac{\partial w_0}{\partial x}+A_{22}\frac{\partial^2 v_0}{\partial y^2}\frac{\partial w_0}{\partial y}+\frac{3}{2}A_{11}\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2 w_0}{\partial x^2}+$$
$$(\frac{1}{2}A_{21}+A_{66})\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2 w_0}{\partial y^2}+(A_{12}+A_{21}+4A_{66})\frac{\partial w_0}{\partial x}\times$$
$$\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x\partial y}+\frac{3}{2}A_{22}\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2 w_0}{\partial y^2}+(\frac{1}{2}A_{21}+A_{66})\left(\frac{\partial w_0}{\partial y}\right)^2\times$$
$$\frac{\partial^2 w_0}{\partial x^2}+(A_{55}-2c_2D_{55}+c_2^2F_{55}-(p_0-p_1\cos\Omega_2 t))\frac{\partial^2 w_0}{\partial x^2}+$$
$$(A_{44}-2c_2D_{44}+c_2^2F_{44})\frac{\partial^2 w_0}{\partial y^2}-c_1^2H_{11}\frac{\partial^4 w_0}{\partial x^4}-c_1^2(H_{21}+4H_{66}+$$
$$H_{12})\frac{\partial^4 w_0}{\partial x^2\partial y^2}+(A_{55}-2c_2D_{55}+c_2^2F_{55})\frac{\partial\phi_x}{\partial x}-c_1^2H_{22}\frac{\partial^4 w_0}{\partial y^4}+$$
$$c_1(F_{11}-c_1H_{11})\frac{\partial^3\phi_x}{\partial x^3}+c_1(F_{21}+2F_{66}-c_1H_{21}-2c_1H_{66})\times$$
$$\frac{\partial^3\phi_x}{\partial x\partial y^2}+(A_{44}-2c_2D_{44}+c_2^2F_{44})\frac{\partial\phi_y}{\partial y}+c_1(F_{12}2F_{66}-$$
$$c_1H_{12}-2c_1H_{66})\frac{\partial^3\phi_y}{\partial x^2\partial y}+c_1(F_{22}-c_1H_{22})\frac{\partial^3\phi_y}{\partial y^3}+$$
$$F\cos\Omega_1 t-\gamma\dot{w}_0=I_0\ddot{w}_0-c_1^2I_6\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)+$$
$$c_1I_3\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)+c_1J_4\left(\frac{\partial\ddot{\phi}_x}{\partial x}+\frac{\partial\ddot{\phi}_y}{\partial y}\right) \tag{7c}$$

$$(D_{11}-2F_{11}c_1+H_{11}c_1^2)\frac{\partial^2\phi_x}{\partial x^2}+(D_{66}-2F_{66}c_1+$$
$$H_{66}c_1^2)\frac{\partial^2\phi_x}{\partial y^2}-c_1(F_{11}-H_{11}c_1)\frac{\partial^3 w_0}{\partial x^3}-(F_{55}c_2^2-2D_{55}c_2+$$
$$A_{55})\frac{\partial w_0}{\partial x}+(D_{12}+D_{66}+H_{66}c_1^2-2F_{66}c_1+H_{12}c_1^2-$$
$$2F_{12}c_1)\frac{\partial^2\phi_y}{\partial y\partial x}-c_1(2F_{66}+F_{12}-2H_{66}c_1-H_{12}c_1)\frac{\partial^3 w_0}{\partial y^2\partial x}+$$
$$(2D_{55}c_2-A_{55}-F_{55}c_2^2)\phi_x=J_1\ddot{u}_0+K_2\ddot{\phi}_x-c_1J_4\frac{\partial\ddot{w}_0}{\partial x} \tag{7d}$$

$$(D_{66}-2F_{66}c_1+H_{66}c_1^2)\frac{\partial^2\phi_y}{\partial x^2}-c_1(F_{21}+2F_{66}-H_{21}c_1-$$
$$2H_{66}c_1)\frac{\partial^3 w_0}{\partial y\partial x^2}-c_1(F_{22}-H_{22}c_1)\frac{\partial^3 w_0}{\partial y^3}+(H_{21}c_1^2+D_{66}+$$
$$D_{21}-2F_{21}c_1+H_{66}c_1^2-2F_{66}c_1)\frac{\partial^2\phi_x}{\partial y\partial x}+(H_{22}c_1^2+$$

$$D_{22}\text{-}2F_{22}c_1)\frac{\partial^2\phi_y}{\partial y^2}\text{-}(F_{44}c_2^2\text{-}2D_{44}c_2+A_{44})\frac{\partial w_0}{\partial y}+$$

$$(2D_{44}c_2\text{-}F_{44}c_2^2\text{-}A_{44})\phi_y=J_1\ddot{v}_0+K_2\ddot{\phi}_y\text{-}c_1J_4\frac{\partial\ddot{w}_0}{\partial y} \tag{7e}$$

其中 γ 为横向振动阻尼系数,并且

$$(A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij})=\sum_{k=1}^{3}\int_{z_k}^{z_{k+1}}Q_{ij}^k(1,z,z^2,z^3,z^4,z^6)\mathrm{d}z,(i,j=1,2,6) \tag{8a}$$

$$(A_{ij},D_{ij},F_{ij})=\sum_{k=1}^{3}\int_{z_k}^{z_{k+1}}Q_{ij}^k(1,z^2,z^4)\mathrm{d}z,(i,j=4,5) \tag{8b}$$

$$I_i=\sum_{k=1}^{3}\int_{z_k}^{z_{k+1}}\rho_k(z)^i\mathrm{d}z,(i=0,1,2,\cdots,6) \tag{8c}$$

$$J_i=I_i-c_1I_{i+2},K_2=I_2-2c_1I_4+c_1^2I_6 \tag{8d}$$

其中 Q_{ij}^k为第 k 层的刚度系数

$$Q_{11}=\frac{E_1}{1-v_{12}v_{21}},Q_{12}=\frac{v_{12}E_2}{1-v_{12}v_{21}},Q_{22}=\frac{E_2}{1-v_{12}v_{21}},$$

$$Q_{66}=G_{12},Q_{44}=G_{23},Q_{55}=G_{13} \tag{9}$$

$A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij}$ 均为蜂窝夹层板各层刚度矩阵在厚度方向上积分加和得到,可以很好的反应整个夹层板的刚度性质.

2　Galerkin 离散

蜂窝夹层板的四边简支边界条件为

$$u_0(x,0,t)=0,\phi_x(x,0,t)=0,$$

$$u_0(x,b,t)=0,\phi_x(x,b,t)=0 \tag{10a}$$

$$v_0(0,y,t)=0,\phi_y(0,y,t)=0,$$

$$v_0(a,y,t)=0,\phi_y(a,y,t)=0 \tag{10b}$$

$$w_0(x,0,t)=0,w_0(x,b,t)=0,$$

$$w_0(0,y,t)=0,w_0(a,y,t)=0 \tag{10c}$$

$$N_{xx}(0,y,t)=0,N_{xx}(a,y,t)=0,$$

$$N_{yy}(x,0,t)=0,N_{yy}(x,b,t)=0 \tag{10d}$$

$$M_{xx}(0,y,t)=0,M_{xx}(a,y,t)=0,$$

$$M_{yy}(x,0,t)=0,M_{yy}(x,b,t)=0 \tag{10e}$$

根据(10)式可设模态函数为

$$u_0=u_1\cos\frac{\pi x}{a}\sin\frac{\pi y}{b},v_0=v_1\sin\frac{\pi x}{a}\cos\frac{\pi y}{b},$$

$$w_0=w_1\sin\frac{\pi x}{a}\sin\frac{\pi y}{b},\phi_x=\phi_1\cos\frac{\pi x}{a}\sin\frac{\pi y}{b},$$

$$\phi_y=\phi_3\sin\frac{\pi x}{a}\cos\frac{\pi y}{b},F=F_1\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \tag{11}$$

把(11)式代入方程(7)中,并在方程(7)两边分别乘以相对应的模态函数后在整个板内积分,利用三角函数的正交性可得

$$a_{14}\ddot{u}_1+a_{11}u_1+a_{12}v_1+a_{13}w_1^2=0, \tag{12a}$$

$$b_{14}\ddot{v}_1+b_{11}u_1+b_{12}v_1+b_{13}w_1^2=0, \tag{12b}$$

$$c_{14}\ddot{w}_1+c_{15}\ddot{\phi}_3+c_{13}w_1+c_{11}\phi_1+c_{12}\phi_3=0, \tag{12c}$$

$$d_{14}\ddot{w}_1+d_{15}\ddot{\phi}_3+d_{13}w_1+d_{12}\phi_1+d_{11}\phi_3=0, \tag{12d}$$

$$e_{110}\ddot{w}_1+e_{111}\ddot{\phi}_1+e_{112}\ddot{\phi}_3+e_{19}\gamma\dot{w}_1+e_{11}w_1+e_{18}(p_0\text{-}p_1\cos\Omega_2t)w_1+e_{15}\phi_1+e_{16}\phi_3+e_{12}w_1^3+e_{13}u_1w_1+e_{14}v_1w_1=e_{17}F_1\cos\Omega_1t \tag{12e}$$

忽略方程(12a)-(12d)的惯性项,把前四式代入方程(12e),引入无量纲变量 $\bar{w}_1=w_1/H$,对横向位移 w_1 无量纲化,得到单自由度非线性动力学方程如下

$$\ddot{w}_1+c\dot{w}_1-(\alpha_{11}+\alpha_{13}p_0)w_1+\alpha_{13}p_1\cos\Omega_2tw_1-\alpha_{12}w_1^3=\alpha_{15}F_1\cos\Omega_1t \tag{13}$$

3　数值模拟

利用二阶变步长 Runge - Kutta 法对方程(13)进行数值模拟. 我们选用的蜂窝夹层板长 $a=10\text{m}$,宽 $b=20\text{m}$,上下蒙皮层均采用 $\rho=2.7\times10^3\text{kg/m}^2$,$E=2.1\times10^{11}\text{Pa}$,$v=1/3$ 的铝板,蒙皮层厚度 $h_f=0.0015\text{m}$,蜂窝芯材质同样为铝芯,本构关系通过 Gibson 公式计算得出,正六角形蜂窝芯的壁厚边长比 $\mathrm{d}/l=1/10$,蜂窝芯厚度 $h_c=0.012\text{m}$. 对应方程(12)中的系数分别为 $\alpha_{11}=-0.1880$,$\alpha_{12}=-58.69$,$\alpha_{13}=0.00001250$,$\alpha_{15}=5.630$.

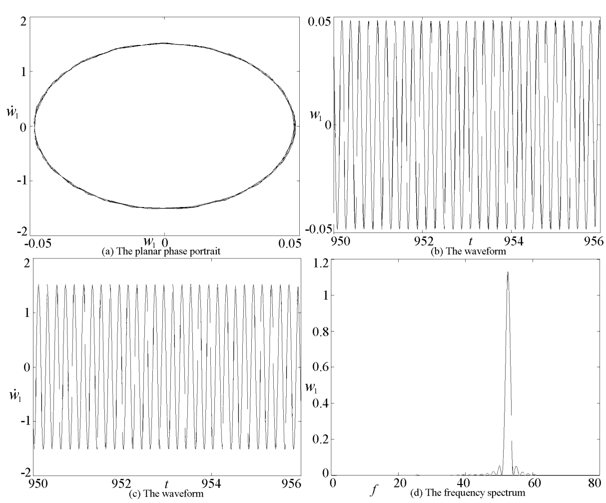

图 3　单倍周期运动

Fig. 3　Period - 1 solution of the honeycomb sandwich plates

对于上述系统,阻尼取 $c=0.063\text{kg/s}$,参数激励 p 和横向外激励 f 的频率取 $\Omega_1=\Omega_2=31.4\text{Hz}$,只改变参数激励 p 和横向外激励 f 的幅值. 当参数激励为

$p_0=7.7$Pa,$p_1=3.5$Pa 横向外激励 $F_1=8.5$Pa,系统出现单倍周期运动,如图3所示,其中(a)为 w_1 和 $\dot{w}_1$ 的相图,(b)和(c)为波形图,(e)为频谱图. 随着参数激励和横向外激励变大,当 $p_0=2002$Pa,$p_1=910$Pa,$F_1=2210$Pa 时,系统出现二倍周期运动,如图4所示. 当 $p_0=2156$Pa,$p_1=980$Pa,$F_1=2380$Pa 时,系统由二倍周期变成概周期运动,如图5所示.

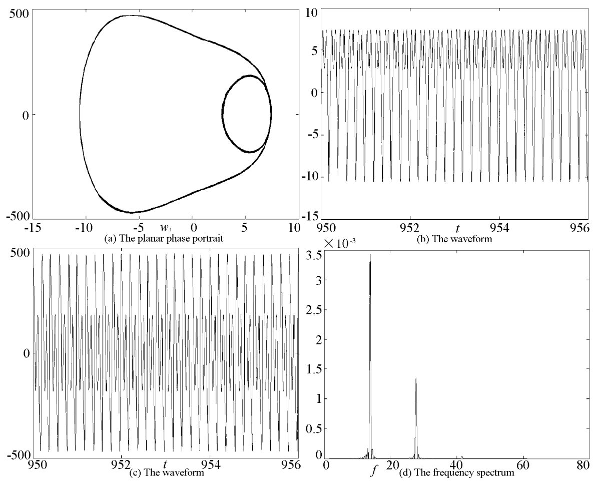

图4 二倍周期运动

Fig. 4 Period - 2 solution of the honeycomb sandwich plates

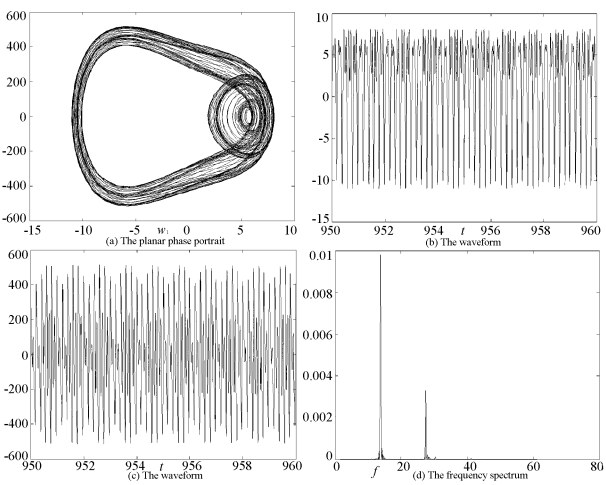

图5 概周期运动

Fig. 5 Quasiperiodic motion of the honeycomb sandwich plates

继续加大参数激励 p 和横向外激励 f,当 $p_0=2387$Pa,$p_1=1085$Pa,$F_1=2635$Pa 时,系统出现混沌运动,如图6所示. 当 $p_0=6160$Pa,$p_1=2800$Pa,$F_1=6800$Pa 时,系统由混沌运动变成多倍周期运动,如图7所示. 最后继续加大激励时,当 $p_0=7469$Pa,$p_1=3395$Pa,$F_1=8245$Pa 时,系统变回二倍周期运动,但是振幅明显比 $p_0=2156$Pa 时增大,如图8所示. 从以上结果能明显的看出,改变参数激励 p 和横向外激励 f 的幅值可以显著的改变系统的振动状态.

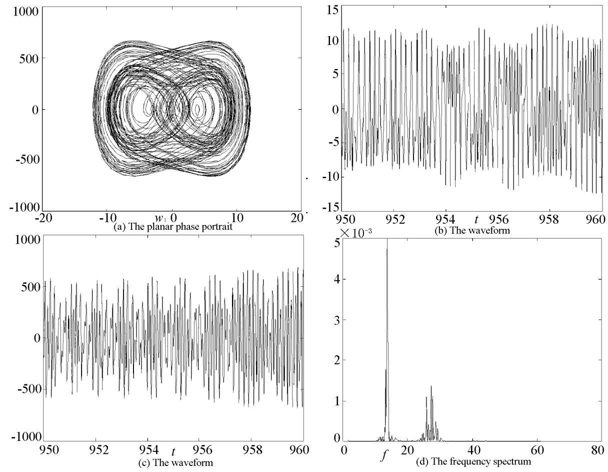

图6 混沌运动

Fig. 6 Chaotic motion of the honeycomb sandwich plates

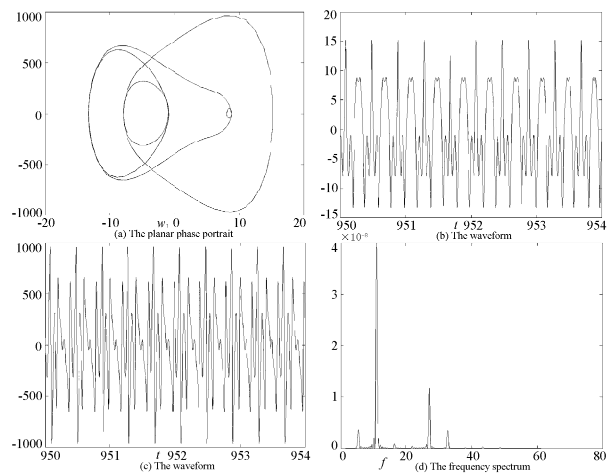

图7 多倍周期运动

Fig. 7 Period - n solution of the honeycomb sandwich plates

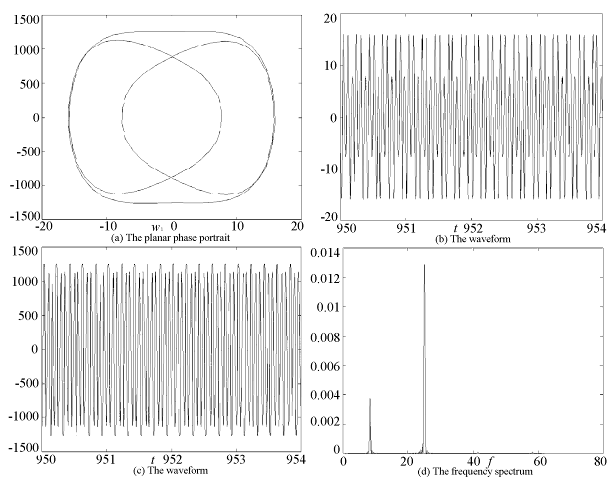

图8 二倍周期运动

Fig. 8 Period - 2 solution of the honeycomb sandwich plates

4 结论

利用 Reddy 的高阶剪切理论和 Hamilton 原理建立了面内激励和横向外激励联合作用下的四边

简支蜂窝夹层板的非线性偏微运动控制方程,综合运用 Galerkin 方法和数值方法,模拟实际参数下不同激励作用的混沌运动,得到二维相图、二维波形图和频谱图.结果表明随着参数激励和横向外激励依次增大,蜂窝夹层板的振幅不断变大,运动形式出现周期运动→概周期运动→混沌运动→周期运动的变化规律.数值模拟表明,参数激励和横向外激励是影响系统混沌运动的主要因素.

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NONLINEAR DYNAMICS OF THE HONEYCOMB SANDWICH PLATES*

Sun Jia　Zhang Wei　Chen Lihua　Yao Minghui

(*College of Mechanical Engineering*, *University of Technology*, *Beijing*　100022)

Abstract　The nonlinear dynamics of simply supported honeycomb sandwich rectangular plates was investigated. Considering the effects of higher-order transverse shear deformation and transversal damping, we established the mathematical model of the honeycomb sandwich plates under the in-plane and transversal excitations. The Galerkin method and numerical simulation were used to analyze the nonlinear responses of different amplitude excitations. The planar phase portrait, waveform and frequency spectrum were plotted. The results of numerical simulation indicate that there exists the following procedure of motion for the honeycomb sandwich rectangular plates under the in-plane and transversal excitations: from periodic motion to chaotic motion and then to periodic motion.

Key words　honeycomb sandwich plates,　third-order shear deformation theory,　nonlinear oscillations,　chaotic motion

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