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集中荷载作用下悬索的主共振分析*

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摘要　针对集中荷载作用下两端固定悬索在集中荷载点外激励作用下悬索系统发生的强迫振动，研究了激励频率接近悬索主共振频率时，系统产生的主共振. 采用多尺度法，得到了各阶振型的主共振分叉图和主共振分叉点的解析解. 通过实例计算，得到了悬索的各阶振型的线性频率与集中荷载以及集中荷载的位置关系，还得到了各阶振型的主共振分叉图和各阶振型的主共振点相平面图.

关键词　伽辽金法，　多尺度法，　非线性振动，　分叉条件，　悬索

引 言

悬索的共振现象，是比较经典和工程实际中很关心的问题. 但是，目前工程实际中采用的悬链理论和柔索理论只是用于分析悬索的稳态构型，没有考虑悬索的动力学行为. 伴随着偏微分方程理论的发展，十八世纪 d'Alembert、Euler、伯努利和拉格朗日就张紧弦的振动进行了许多研究，并提出了弦的基本振动理论. 1820 年 Poisson 推导了悬索的运动方程，随后 Rohrs 获得了不可伸压缩悬索小振动的自振频率的近似解和解析解[1]. 在二十的世纪中叶，随着 Pugsley[2]、Saxon 和 Cahn[3] 对均匀无伸张索的面内振动的详细研究，悬索动力学得到了重新发展. Luongo[4] 应用多尺度法研究了具有几何非线性的水平悬索的非线性单频振动. 文献[5]研究了面内模态的主共振问题. Rao 和 Iyengar[6] 以及李和 Perkins[7] 采用 Galerkin 方法，分析了谐波激励下系统的一阶面内模态，讨论了其一阶分叉现象. 张伟[8] 对悬索的参数激励非线性振动问题进行了深入的全局非线性动力学分析. 金栋平、胡海岩[9] 研究了横向流体激励下行进索的非线性运动特征. 对于桥梁斜拉索的研究，许多作者[10] 有过深入研究，而对于工程索道中运用的有集中荷载作用的悬索，目前研究较少.

对于集中荷载作用下的悬索，更重要的是分析悬索上集中荷载对共振频率的影响，即研究集中荷载作用下的悬索以及荷载本身的共振问题. 这也是工程索道中悬索区别于其它用途悬索的关键所在.

本章考虑集中荷载和悬索自重的影响，研究了集中荷载作用下悬索的面内振动的主共振，得到了主共振分叉点的解析解. 通过实例计算，得到了各阶振型的主共振分叉图和主共振点相平面图.

1　基于悬索静态挠曲线的悬索平面非线性动力学方程

文献[11]从弦的非线性方程出发，考虑运动集中荷载和悬索自重及悬索运动速度的影响，对于如图1(a)所示悬索. 在某一时刻取运动悬索中任意截取一微段，如图1(b)所示，长为 ds. 通过对微段的分析，考虑参数激励，取边界条件为

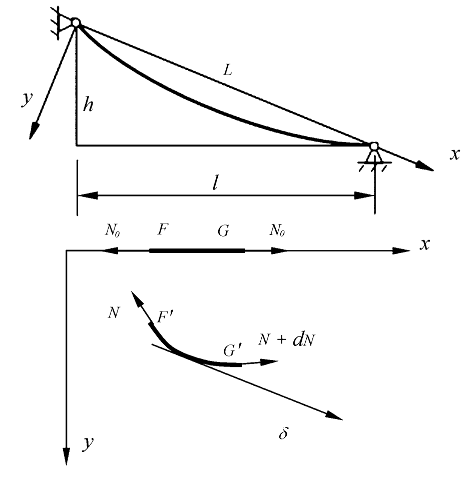

图1　悬索分析模型

Fig. 1　The analytical model of the suspension cable system

$$\begin{cases} u = u_0, & \text{当 } x = 0 \\ u = u_0 + lP(t), & \text{当 } x = l \end{cases} \tag{1}$$

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其中 u 为悬索 F 点沿 x 方向位移,u_0 为悬索沿 x 方向的速度(设为常数).

得到了集中荷载作用下悬索的面内运动非线性控制方程.

$$v_{tt}-c_2^2v_{xx}+2\mu v_t=c_1^2Pv_{xx}+\frac{c_1^2}{2l}v_{xx}\int_0^l v_x^2\mathrm{d}x+g(x,t) \quad (2)$$

式中,v 为悬索 F 点沿 y 方向位移,μ 为阻尼系数,$g(x,t)$ 为外激励函数. c_1 和 c_2 为悬索纵向弹性波和横向弹性波速度,定义为

$$c_1^2=\frac{E}{\rho},c_2^2=\frac{N_0}{\rho A}=\frac{\sigma_0}{\rho} \quad (3)$$

其中,E 为悬索弹性模量,N_0 为悬索预张力,σ_0 为悬索预张应力,ρ 为悬索密度,A 为悬索初始横截面积. 设

$$v=v_s+\boldsymbol{v} \quad (4)$$

其中,v_s 为悬索在自重作用下沿 y 方向的静态挠度. $\boldsymbol{v}$ 为悬索自重以外的其它外力因素引起的悬索沿 y 方向的挠度.

将(4)式代入(2)式整理后,得

$$\boldsymbol{v}_{tt}-(c_2^2+\frac{c_1^2}{2l}\int_0^l(v_s)_x^2\mathrm{d}x)\boldsymbol{v}_{xx}+2\mu\boldsymbol{v}_t=g(x,t)+c_2^2(v_s)_{xx}+c_1^2P(v_s+\boldsymbol{v})_{xx}+\frac{c_1^2}{2l}\boldsymbol{v}_{xx}\int_0^l(2(v_s)_x\boldsymbol{v}_x+\boldsymbol{v}_x^2)\mathrm{d}x+\frac{c_1^2}{2l}(v_s)_{xx}\int_0^l((v_s)_x^2+2(v_s)_x\boldsymbol{v}_x+\boldsymbol{v}_x^2)\mathrm{d}x \quad (5)$$

对于静止悬索,由文献[11],有

$$\mathrm{d}s_1-\mathrm{d}x=[1+v_s^2]^{\frac{1}{2}}\mathrm{d}x-\mathrm{d}x\approx\frac{1}{2}(v_s)_x^2 \quad (6)$$

所以

$$c_{20}^2=c_c^2+\frac{c_1^2}{2l}\int_0^l(v_s)_x^2\mathrm{d}x=\frac{N_0}{\rho A}+\frac{E}{\rho l}\int_0^l(\frac{\mathrm{d}s_1-\mathrm{d}x}{\mathrm{d}x})\mathrm{d}x=\frac{T_1}{\rho A} \quad (7)$$

式中,s_1 为悬索静态曲线,T_1 为悬索静态曲线平均拉力. 所以若只研究面内运动,不考虑悬索的面外运动,(5)式化为

$$\boldsymbol{v}_{tt}-c_{20}^2\boldsymbol{v}_{xx}+2\mu\boldsymbol{v}_t=g(x,t)+c_2^2(v_s)_{xx}+c_1^2P(v_s+\boldsymbol{v})_{xx}+\frac{c_1^2}{2l}\boldsymbol{v}_{xx}\int_0^l(2(v_s)_x\boldsymbol{v}_x+\boldsymbol{v}_x^2)\mathrm{d}x+\frac{c_1^2}{2l}(v_s)_{xx}\int_0^l((v_s)_x^2+2(v_s)_x\boldsymbol{v}_x+\boldsymbol{v}_x^2)\mathrm{d}x \quad (8)$$

式(8)即基于悬索静态挠曲线的悬索平面非线性动力学方程.

2　伽辽金方法分析

对于集中荷载作用下的两端固定悬索,如果集中荷载作用点有外激励 F 作用,则外激励函数可以写为

$$g(x,t)=\frac{1}{\rho A}(\delta(x-x_d)(Mg\times\cos\beta-Mv_{dtt}+F)+Q_y) \quad (9)$$

其中 M 为集中荷载质量,Q_y 为悬索沿 x 轴单位长度重量(设均匀分布),δ 为笛瑞克函数,F 为集中荷载作用点处外激励函数. x_d 为集中荷载点坐标,v_{dtt} 为集中荷载点加速度.

如果只研究悬索的面内振动,将方程(9)代入方程(8),有

$$\boldsymbol{v}_{tt}-c_{20}^2\boldsymbol{v}_{xx}+2\mu\boldsymbol{v}_t=\frac{1}{\rho A}\delta(x-x_d)(Mg\times\cos\beta-Mv_{Mtt}+F)+\frac{Q_y}{\rho A}+c_2^2(v_s)_{xx}+\frac{c_1^2}{2l}\boldsymbol{v}_{xx}\int_0^l(2(v_s)_x\boldsymbol{v}_x+\boldsymbol{v}_x^2)\mathrm{d}x+\frac{c_1^2}{2l}(v_s)_{xx}\int_0^l((v_s)_x^2+2(v_s)_x\boldsymbol{v}_x+\boldsymbol{v}_x^2)\mathrm{d}x \quad (10)$$

对于没有集中荷载作用的静止悬索,$\boldsymbol{v}=0$,(10)式化为

$$\frac{Q_y}{\rho A}+c_2^2\boldsymbol{v}_{xx}+\frac{c_1^2}{2l}(v_s)_{xx}\int_0^l(v_s)_x^2\mathrm{d}x=0 \quad (11)$$

将(11)式代入(10)式,有

$$\boldsymbol{v}_{tt}-c_{20}^2\boldsymbol{v}_{xx}+2\mu\boldsymbol{v}_t=\frac{1}{\rho A}\delta(x-x_d)(Mg\times\cos\beta-Mv_{Mtt}+F)+\frac{c_1^2}{2l}\boldsymbol{v}_{xx}\int_0^l(2(v_s)_x\boldsymbol{v}_x+\boldsymbol{v}_x^2)\mathrm{d}x+\frac{c_1^2}{2l}(v_s)_{xx}\int_0^l(2(v_s)_x\boldsymbol{v}_x+\boldsymbol{v}_x^2)\mathrm{d}x \quad (12)$$

采用伽辽金方法对(12)式进行求解. 设

$$\boldsymbol{v}=l\sum_{n=1}^{\infty}\varepsilon^{\frac{1}{2}}\zeta_n(t)\sin\frac{n\pi k}{l} \quad (13)$$

式中 $\varepsilon^{\frac{1}{2}}$ 为一个量级为运动幅值的无量纲小量. 将方程(13)代入方程(12),并利用模态的正交性,我们得到

$$(1+\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})\ddot{\zeta}_n(t)+c_{20}^2(\frac{n\pi}{l})^2\zeta_n(t)=-\varepsilon[2\mu_n\dot{\zeta}_n(t)+\Gamma\sum_{m=1}^{\infty}\kappa_m\zeta_m(t)\zeta_n(t)+\Psi n^2\zeta_n(t)\sum_{m=1}^{\infty}(\frac{m\pi}{l})^2\zeta_n^2(t)+\frac{2M}{\varepsilon\rho Al}(\sum_{m=1,m\neq n}^{\infty}\ddot{\zeta}_m(t)\sin\frac{m\pi x_d}{l})\sin\frac{n\pi x_d}{l}-\frac{2(F+Mg\cos\beta)}{\varepsilon^{\frac{3}{2}}\rho Al^2}\times\sin\frac{n\pi x_d}{l}-\frac{2}{\varepsilon l^2}(c_1^2\int_0^l(v_s)_{xx}\sin\frac{n\pi x_d}{l}\mathrm{d}x\int_0^l((v_s)_x\times$$

$$(\sum_{m=1}^{\infty}\frac{m\pi}{l}\zeta_n(t)\cos\frac{m\pi x}{l}))\mathrm{d}x-\frac{c_1^2}{2l^2\varepsilon^{\frac{1}{2}}}\int_0^l(v_s)_{xx}\times$$

$$\sin\frac{n\pi x}{l}\mathrm{d}x\pi^2\sum_{m=1}^{\infty}m^2\zeta_m^2(t)] \tag{14}$$

3 主共振多尺度法求解

如果在集中荷载点有外激励作用，系统发生强迫振动. 激励频率接近悬索主共振频率时，系统产生主共振. 若悬索两端固定，$P=0$，采用多尺度法求解. 设

$$\frac{2F}{\rho Al^2\sqrt{\varepsilon}}\sin\frac{n\pi x_d}{l}=\frac{2F_0}{\rho Al^2\sqrt{\varepsilon}}\sin\frac{n\pi x_d}{l}\sin\Omega t=$$
$$2\varepsilon k_n\sin\Omega t \tag{15}$$

式中，$\Omega=\omega_s+\varepsilon\sigma$. 若 s 固定，(14)式化为

$$(1+\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})\ddot{\zeta}_n(t)+\omega_{0n}^2\zeta_n(t)=-\varepsilon(2\mu_n\dot{\zeta}_n(t)+$$
$$\Gamma\sum_{m=1}^{\infty}\kappa_m\zeta_m(t)\zeta_n(t)+\Psi n^2\zeta_n(t)\sum_{m=1}^{\infty}(\frac{m\pi}{l})^2\zeta_m^2(t)+$$
$$\Phi\sum_{m=1,m\neq n}^{\infty}\ddot{\zeta}_m(t)\sin\frac{m\pi x_d}{l})\sin\frac{n\pi x_d}{l}+\Phi_1\sin\frac{n\pi x_d}{l}+$$
$$\frac{2}{l^2}c_1^2\kappa_{n1}\sum_{n=1}^{\infty}\kappa_m\zeta_m(t)+\frac{c_1^2}{2l^2}\kappa_{n1}\pi^2\sum_{m=1}^{\infty}m^2\zeta_m^2(t))+$$
$$2\varepsilon k_n\sin\Omega t \tag{16}$$

式中，$\Gamma=c_1^2(\frac{n\pi}{l})^2$，$\Psi=\frac{1}{4}c_1^2\pi^2$，$\Omega=\omega_s+\varepsilon\sigma$，

$$k_n=\varepsilon^{-\frac{3}{2}}\frac{F_0}{\rho Al^2}\sin\frac{n\pi x_d}{l},\ \Phi=\frac{2M}{\varepsilon\rho Al},$$
$$\Phi_1=-\varepsilon^{-\frac{3}{2}}\frac{2(Mg\cos\beta)}{\rho Al^2},\ \omega_{0n}=c_{20}(\frac{n\pi}{l}),$$
$$\Gamma=c_1^2(\frac{n\pi}{l})^2,\ \kappa_m=\frac{m\pi}{l}\varepsilon^{-\frac{1}{2}}\int_0^l((v_s)_x(\cos\frac{m\pi x}{l}))\mathrm{d}x,$$
$$\kappa_{n1}=-\varepsilon^{-\frac{1}{2}}\int_0^l(v_s)_{xx}\sin\frac{n\pi x}{l}\mathrm{d}x \tag{17}$$

设

$$\zeta_n=\zeta_{n0}(T_0,T_1)+\varepsilon\zeta_{n1}(T_0,T_1)+\cdots \tag{18}$$

式中 $T_0=t$，$T_1=\varepsilon t$. 将(18)式代入方程(16)，有

$$(1+\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})(D_0^2+2\varepsilon D_0D_1+\varepsilon^2D_1^2)(\zeta_{n0}+\varepsilon\zeta_{n1})+$$
$$\omega_{0n}^2(\zeta_{n0}+\varepsilon\zeta_{n1})+\varepsilon[2\mu_n(D_0\zeta_{n0}+\varepsilon(D_0\zeta_{n1}+D_1\zeta_{n0})+$$
$$\varepsilon^2D_1\zeta_{n1})+\Gamma[\sum_{m=1}^{\infty}\kappa_m(\zeta_{m0}+\varepsilon\zeta_{m1})](\zeta_{n0}+\varepsilon\zeta_{n1})+\Psi n^2(\zeta_{n0}+$$
$$\varepsilon\zeta_{n1})[\sum_{m=1}^{\infty}(\frac{m\pi}{l})^2(\zeta_{m0}+\varepsilon\zeta_{m1})^2+\Phi(\sum_{m=1,m\neq n}^{\infty}(D_0^2+$$
$$2\varepsilon D_0D_1+\varepsilon^2D_1^2)(\zeta_{m0}+\varepsilon\zeta_{m1})\sin\frac{m\pi x_d}{l})\sin\frac{n\pi x_d}{l}+$$
$$\frac{2}{l^2}c_1^2\kappa_{n1}\sum_{n=1}^{\infty}\kappa_m(\zeta_{m0}+\varepsilon\zeta_{m1})+\Phi_1\sin\frac{n\pi x_d}{l}+$$
$$\frac{c_1^2}{2l^2}\kappa_{n1}\pi^2\sum_{m=1}^{\infty}m^2(\zeta_{m0}+\varepsilon\zeta_{m1})^2]-2\varepsilon k_n\sin\Omega t=0 \tag{19}$$

式中 $D_0=\partial/\partial T_0$，$D_1=\partial/\partial T_1$. 令 ε 相同幂次的系数相等，所以

ε^0 阶为

$$(1+\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})D_0^2\zeta_{n0}+\omega_{0n}^2\zeta_{n0}=0 \tag{20}$$

ε^1 阶为

$$(1+\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})(2D_0D_1\zeta_{n0}+D_0^2\zeta_{n1})+\omega_{0n}^2\zeta_{n1}+$$
$$[2\mu_n(D_0\zeta_{n0})+\Gamma[\sum_{m=1}^{\infty}\kappa_m\zeta_{m0}]\zeta_{n0}+\Psi n^2(\zeta_{n0})\times$$
$$[\sum_{m=1}^{\infty}(\frac{m\pi}{l})^2(\zeta_{m0})^2+\Phi(\sum_{m=1,m\neq n}^{\infty}(D_0^2\zeta_{m0})\sin\frac{m\pi x_d}{l})\times$$
$$\sin\frac{n\pi x_d}{l}+\frac{2}{l^2}c_1^2\kappa_{n1}\sum_{n=1}^{\infty}\kappa_m(\zeta_{m0})+\Phi_1\sin\frac{n\pi x_d}{l}+$$
$$\frac{c_1^2}{2l^2}\kappa_{m1}\pi^2\sum_{m=1}^{\infty}m^2(\zeta_{m0})^2]-2\varepsilon k_n\sin\Omega t=0 \tag{21}$$

对于方程(20)，设

$$\zeta_{n0}=A_n(T_1)\exp(i\omega_nT_0)+cc \tag{22}$$

式中，$\omega_n=\omega_{0n}\sqrt{\frac{1}{(1+\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})}}$. cc 表示其左边项的共轭复数. 将(22)式代入(21)式，有

$$(1+\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})(D_0^2\zeta_{n1})+\omega_{0n}^2\zeta_{n1}=-\{(1+$$
$$\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})2i\omega_n\dot{A}_n(T_1)\exp(i\omega_nT_0)+2\mu_ni\omega_nA_n(T_1)+$$
$$n^2pA_n(T_1)\exp(i\omega_nT_0)+\Gamma[\sum_{m=1}^{\infty}\kappa_mA_m(T_1)\exp(i\omega_nT_0)]\times$$
$$A_n(T_1)\exp(i\omega_nT_0)+\Psi n^2A_n(T_1)\exp(i\omega_nT_0)[\sum_{m=1}^{\infty}(\frac{m\pi}{l})^2\times$$
$$(A_m(T_1)\exp(i\omega_nT_0)+\bar{A}_m(T_1)\exp(-i\omega_mT_0))^2+$$
$$\Phi(\sum_{m=1,m\neq n}^{\infty}(-\omega_m^2A_m(T_1)\exp(i\omega_mT_0))\sin\frac{m\pi x_d}{l})\sin\frac{n\pi x_d}{l}+$$
$$\frac{2}{l^2}c_1^2\kappa_{n1}\sum_{n=1}^{\infty}\kappa_mA_m(T_1)\exp(i\omega_mT_0)+\Phi_1\sin\frac{n\pi x_d}{l}+\frac{c_1^2}{2l^2}\kappa_{m1}\times$$
$$\pi^2\sum_{m=1}^{\infty}m^2(A_m(T_1)\exp(i\omega_mT_0))^2-2k_n\sin\Omega t+cc\} \tag{23}$$

由(23)式可以发现，如果

$$\{(1+\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})2i\omega_nA_n'+2\mu_ni\omega_nA_n+2\Psi n^2\times$$
$$A_n(\frac{\pi}{l})^2\sum_{m=1}^{\infty}m^2(A_m\bar{A}_m)+\Psi n^4(\frac{\pi}{l})^2\bar{A}_nA_n^2+$$

$$\frac{2}{l^2}c_1^2\kappa_{n1}\kappa_n A_n\} + \delta_{ns}k_n i\exp(i\sigma T_1) = 0 \qquad (24)$$

则 ζ_{n1} 中不存在永年项.

引进复数的极坐标形式:

$$A_n = \frac{1}{2}a_n\exp(i\alpha_n) \qquad (25)$$

将(25)式代入(24)式,并将实部和虚部分开,有

$$(1+\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})\omega_n a_n' + \mu_n\omega_n a_n + \delta_{ns}k_n\cos(\upsilon_n) = 0 \qquad (26)$$

$$-(1+\frac{2M}{\rho Al}\sin^2\frac{n\pi x_d}{l})\omega_n(a_n' a_n) + \frac{1}{4}\Psi n^2 a_n(\frac{\pi}{l})^2\times\sum_{m=1}^{\infty}(m^2a_n^2) + \frac{1}{8}\Psi n^4(\frac{\pi}{l})^2a_n^3 + \frac{1}{l^2}c_1^2\kappa_{n1}\kappa_n a_n - \delta_{ns}k_n\sin(\upsilon_n) = 0 \qquad (27)$$

式中

$$v_n = \sigma T_1 - \alpha_n, \varepsilon^{-\frac{3}{2}}(\frac{1}{l^2}F) = k_n, \varepsilon\mu_n = \mu \qquad (28)$$

对于稳态解,a_n 和 υ_n 都是常数,(26)式可以改写为:

$$\mu_n\omega_n a_n + \delta_{ns}k_n\cos(\upsilon_n) = 0 \qquad (29)$$

从(29)式可以看出:若 $n\neq s, a_n = 0$. 所以,对于所有 $a_n\neq 0, n = s$. 此时,(26)、(27)式化为

$$(1+\frac{2M}{\rho Al}\sin^2\frac{s\pi x_d}{l})\omega_s a_s' + \mu_s\omega_s a_s + k_s\cos(\upsilon_s) = 0 \qquad (30)$$

$$-(1+\frac{2M}{\rho Al}\sin^2\frac{s\pi x_d}{l})\omega_s(a_s' a_s) + \frac{1}{4}\Psi s^4 a_s^3(\frac{\pi}{l})^2 + \frac{1}{8}\Psi s^4(\frac{\pi}{l})^2a_s^3 + \frac{1}{l^2}c_1^2\kappa_{s1}\kappa_s a_s - k_s\sin(\upsilon_s) = 0 \qquad (31)$$

上式中,a_n 是 ε 的函数.

对于稳态平面运动,(30)、(31)式化为

$$\mu_s\omega_s a_s + k_s\cos(\upsilon_s) = 0 \qquad (32)$$

$$-(1+\frac{2M}{\rho Al}\sin^2\frac{s\pi x_d}{l})\omega_s(\sigma a_s) + \frac{1}{4}\Psi s^4(\frac{\pi}{l})^2a_s^3 + \frac{1}{8}\Psi s^4(\frac{\pi}{l})^2a_s^3 + \frac{1}{l^2}c_1^2\kappa_{s1}\kappa_s a_s - k_s\sin(\upsilon_s) = 0 \qquad (33)$$

利用(32)式,可以求得 ν_s. 若无阻尼,$\mu_s = 0$,解得

$$\nu_s = \pm\frac{\pi}{2} \qquad (34)$$

令

$$\varepsilon^{-1/2}b_s = a_s, \qquad (35)$$

式(33)化为

$$3k_1b_s^3 + k_2\Delta b_s + k_3b_s - \frac{F_0}{\rho Al^2}\sin\frac{s\pi x_d}{l}\sin(\upsilon_s) = 0 \qquad (36)$$

式中,b_s,Δ,H 为与 ε 无关的独立参数. 其中

$$\Delta = \Omega - \omega_s, k_1 = \frac{1}{8}\Psi s^4(\frac{\pi}{l})^2,$$

$$k_2 = -(1+\frac{2M}{\rho Al}\sin^2\frac{s\pi x_d}{l})\omega_s,$$

$$k_3 = \frac{\varepsilon}{l^2}c_1^2k_{s1}k_s = -\frac{1}{l^2}\frac{s\pi}{l}c_1^2\int_0^l(v_s)_{xx}\times\sin\frac{s\pi x}{l}\mathrm{d}x\int_0^l((v_s)_x\cos\frac{s\pi x}{l})\mathrm{d}x \qquad (37)$$

由(36)式通过数值求解即可得到 $b_s-\Delta$ 图形.

算例　设有水平悬挂悬索,长 $l = 100$m,悬索直径 $d = 28$mm,单位长度重量2.855kg,弹性模量 $E = 230\times10^9$Pa,悬索上集中荷载 $M = 100$kg,悬索初始张力 $T = 200\times10^3$N,在集中荷载点(中点)外激励函数为 $F = F_0\sin\Omega t = 1000\sin\Omega t$.

1)通过数值计算,由(37)式求得各阶振型的主共振分叉图(1,2,3,4阶),如图2所示.

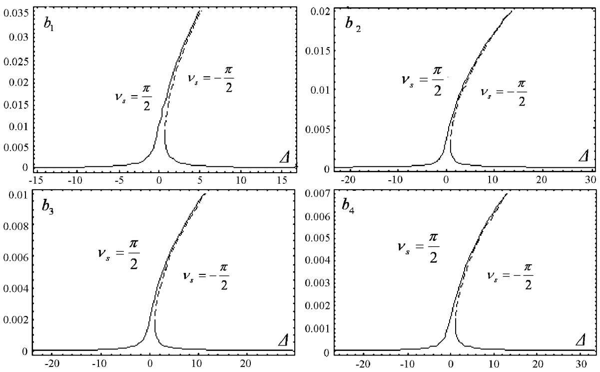

图2　主共振分叉图

Fig. 2　The primary resonance bifurcation graphics

2)通过符号-数值计算,得到各阶振型的主共振点相平面图,如图3所示.

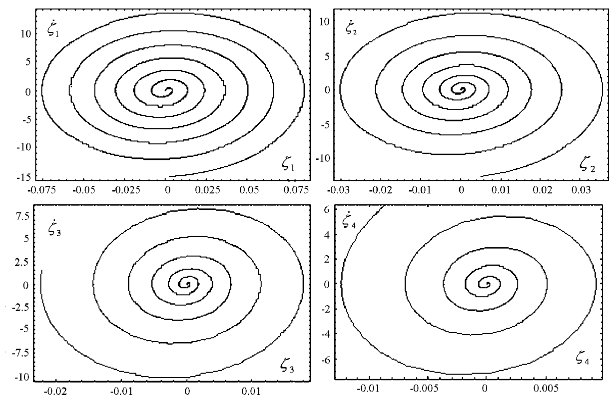

图3　主共振点相平面图

Fig. 3　The phase-plane diagram of the primary resonance bifurcation point

从算例看出:集中荷载作用下的悬索当激励频率

接近悬索各阶振型主共振频率时,系统产生主共振.

4 结论

针对集中荷载作用下两端固定悬索在集中荷载点外激励作用下悬索系统发生的强迫振动,本文研究了激励频率接近悬索主共振频率时,系统产生的主共振.

1)采用多尺度法,得到了各阶振型的主共振分叉图和主共振分叉点的解析解;

2)通过实例计算,得到了悬索的各阶振型的线性频率与集中荷载以及集中荷载的位置关系,得到了各阶振型的主共振分叉图和各阶振型的主共振点相平面图.

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PRIMARY RESONANCE ANALYSIS ON SUSPENSION CABLE SYSTEM WITH A CONCENTRATED LOAD *

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Abstract The forced vibration of a suspension cable system under the external excitation of the point, on which the concentrated load acts, was investigated for suspension cable with both ends built – in and the concentrated load acting on. The analytic solution to primary resonance bifurcation was obtained by adopting the multiple – scale method. Through computing instances, the position and graph of bifurcation of primary resonance for modes of all ranks were obtained.

Key words Galerkin method, multiple-scale method, nonlinear vibration, bifurcation condition, strings

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