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弹性膜结构动力学的各类非传统 Hamilton 型变分原理*

李纬华　罗恩

（中山大学应用力学与工程系，广州　510275）

摘要　根据古典阴阳互补和现代对偶互补的基本思想,通过罗恩早已提出的一条简单而统一的新途径,系统地建立了弹性膜结构动力学的各类非传统 Hamilton 型变分原理. 这种新的非传统 Hamilton 型变分原理能反映这种动力学初值－边值问题的全部特征. 文中首先给出膜结构动力学的广义虚功原理的表式,然后从该式出发,不仅能得到膜结构动力学的虚功原理,而且通过所给出的一系列广义 Legendre 变换,还能系统地成对导出弹性膜结构动力学的 5 类变量($p_\alpha,p_\beta,p_\gamma,v_\alpha,v_\beta,v_\gamma,N_\alpha,N_\beta,S_{\alpha\beta},\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta},u,v,w$)、4 类变量($p_\alpha,p_\beta,p_\gamma,N_\alpha,N_\beta,S_{\alpha\beta},\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta},u,v,w$)、3 类变量($N_\alpha,N_\beta,S_{\alpha\beta},\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta},u,v,w$)和 2 类变量($N_\alpha,N_\beta,S_{\alpha\beta},u,v,w$)非传统 Hamilton 型变分原理的互补泛函、以及相空间($p_\alpha,p_\beta,p_\gamma,v_\alpha,u,v,w$)非传统 Hamilton 型变分原理的泛函与 1 类变量(u,v,w)非传统 Hamilton 型变分原理势能形式的泛函. 同时,通过这条新途径还能清楚地阐明这些原理的内在联系.

关键词　非传统 Hamilton 型变分原理，　膜结构，　几何非线性，　弹性动力学，　对偶互补，　初值－边值问题，　相空间

引言

膜结构是近 30 年来发展起来的一种新颖的结构形式,它是一种重量更轻、受力更合理的张力结构. 膜结构是以性能优良的织物为材料,利用柔性拉索或刚性骨架将膜面绷紧,或者向膜内充气,通过空气压力来支承膜面,使膜材产生一定的预张力,从而形成具有一定刚度、并能够承受各种外荷载的大跨度空间结构体系. 膜结构建筑既有造型独特而优美的外观,又有梦幻般的内部空间,因而极大地丰富了建筑师对建筑空间与造型的想象力. 膜结构被誉为现代建筑的高新科技,是 21 世纪的“绿色建筑结构体系”. 由于膜结构具有优良的建筑与结构性能,建造方便快捷,经济优势突出,因而得到越来越广泛地应用. 随着膜材性能及再利用技术的不断提高和膜结构形式的不断创新,膜结构还将会显示出更为强大的生命力.

虽然已有一些有关膜结构的专著和论文[1-5],但是作者至今还没有见到国内外有关论述弹性膜结构动力学变分原理的文献，所以弹性膜结构动力学的基本理论还存在不能令人满意之处. 由于弹性膜结构动力学的基本理论中最核心的部分—虚功原理和各种动力学变分原理至今还没有系统建立,因此弹性膜结构动力学的基本理论目前还是不完整的.

本文根据文[6]提出的一条简单而统一的新途径,并考虑到膜结构的几何非线性,系统地建立了膜结构动力学的广义虚功原理与虚功原理、以及弹性膜结构动力学的各类非传统 Hamilton 型变分原理和相空间非传统 Hamilton 型变分原理. 这些新的变分原理与传统 Hamilton 型变分原理相比,最根本的区别在于前者能反映这种动力学初值－边值问题的全部特征.

1　弹性膜结构动力学的基本方程及条件

设有各向异性的膜结构,在膜面上建立以主曲率线 α β 和法线 γ 为坐标轴的右手正交曲线坐标系. 由于膜结构只能承受拉力，不能承受压力和弯矩作用[1,7]，因此，对于几何非线性弹性膜结构动力学,根据几何非线性薄壳结构的薄膜理论[8],并考虑到膜结构的受力与变形的特性,其基本方程和条件如下：

1.1　速度位移关系

$$v_\alpha = \partial u/\partial t = \dot{u},\ v_\beta = \partial v/\partial t = \dot{v},\ v_\gamma = \partial w/\partial t = \dot{w} \quad (1)$$

式中,u,v 和 w 分别为膜面上任一点沿,和方向的位移 α、β 和 γ 分别为膜面上任一点沿 v_α,v_β 和 v_γ 方向的速度.

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1.2 动量速度关系

$$p_\alpha=\bar{m}v_\alpha, p_\beta=\bar{m}v_\beta, p_\gamma=\bar{m}v_\gamma \tag{2}$$

相应的动能密度和余动能密度分别为

$$K(v_\alpha,v_\beta,v_\gamma)=(\bar{m}v_\alpha^2+\bar{m}v_\beta^2+\bar{m}v_\gamma^2)AB/2;$$

$$K^*(p_\alpha,p_\beta,p_\gamma)=(p_\alpha^2+p_\beta^2+p_\gamma^2)AB/2\bar{m}$$

式中,$\bar{m}$ 为膜的面密度,p_α,p_β 和 p_γ 分别为膜沿 α、β 和 γ 方向的动量,A 和 B 分别为膜面上任一点沿 α 和 β 方向的拉梅系数.

1.3 几何方程

根据几何非线性薄壳的薄膜理论[8]有

$$\begin{cases}\varepsilon_\alpha=d_{11}+d_{11}^2/2+d_{21}^2/2+d_{31}^2/2\\ \varepsilon_\beta=d_{22}+d_{22}^2/2+d_{12}^2/2+d_{32}^2/2\\ \varepsilon_{\alpha\beta}=d_{12}+d_{21}+d_{11}d_{12}+d_{21}d_{22}+d_{31}d_{32}\end{cases} \tag{3}$$

其中

$$d_{11}=\frac{1}{A}\frac{\partial u}{\partial\alpha}+\frac{v}{AB}\frac{\partial A}{\partial\beta}-k_\alpha w, d_{12}=\frac{1}{B}\frac{\partial u}{\partial\beta}+\frac{v}{AB}\frac{\partial B}{\partial\alpha},$$

$$d_{21}=\frac{1}{A}\frac{\partial v}{\partial\alpha}-\frac{u}{AB}\frac{\partial A}{\partial\beta}, d_{22}=\frac{1}{B}\frac{\partial v}{\partial\beta}+\frac{u}{AB}\frac{\partial B}{\partial\alpha}-k_\beta w,$$

$$d_{31}=\frac{1}{A}\frac{\partial w}{\partial\alpha}+-k_\alpha u, d_{32}=\frac{1}{B}\frac{\partial w}{\partial\beta}+k_\beta v \tag{4}$$

式中,ε_α,ε_β 和 $\varepsilon_{\alpha\beta}$ 均为工程 Green 应变,$k_\alpha=1/R_\alpha$,$k_\beta=1/R_\beta$,k_α 和 k_β 分别为膜面上任一点沿 α 和 β 方向的主曲率,R_α 和 R_β 分别为膜面上任一点沿 α 和 β 方向的主曲率半径,对于膜结构,有 $k_\alpha k_\beta=1/R_\alpha R_\beta<0$.

考虑到膜结构的变形特点,可略去(3)式的二次项中如(4)式所示的划线项[9],从而(3)式简化为

$$\begin{cases}\varepsilon_\alpha=d_{11}+\bar{d}_{11}^2/2+\bar{d}_{21}^2/2+\bar{d}_{31}^2/2\\ \varepsilon_\beta=d_{22}+\bar{d}_{22}^2/2+\bar{d}_{12}^2/2+\bar{d}_{32}^2/2\\ \varepsilon_{\alpha\beta}=d_{12}+d_{21}+\bar{d}_{11}\bar{d}_{12}+\bar{d}_{21}\bar{d}_{22}+\bar{d}_{31}\bar{d}_{32}\end{cases} \tag{5}$$

其中,$\bar{d}_{11}=\frac{1}{A}\frac{\partial u}{\partial\alpha}$, $\bar{d}_{12}=\frac{1}{B}\frac{\partial u}{\partial\beta}$, $\bar{d}_{21}=\frac{1}{A}\frac{\partial v}{\partial\alpha}$, $\bar{d}_{22}=\frac{1}{B}\frac{\partial v}{\partial\beta}$,$\bar{d}_{31}=\frac{1}{A}\frac{\partial w}{\partial\alpha}$,$\bar{d}_{32}=\frac{1}{B}\frac{\partial w}{\partial\beta}$.

1.4 运动方程

$$\begin{cases}\{B[(N_{\alpha0}+N_\alpha)+(N_{\alpha0}+N_\alpha)\bar{d}_{11}+S_{\alpha\beta}\bar{d}_{12}]\}_{,\alpha}+\\ \quad\{A[S_{\alpha\beta}+S_{\alpha\beta}\bar{d}_{11}+(N_{\beta0}+N_\beta)\bar{d}_{12}]\}_{,\beta}-\\ \quad(N_{\beta0}+N_\beta)B_{,\alpha}+S_{\alpha\beta}A_{,\beta}+ABf_\alpha=AB\dot{p}_\alpha\\ \{A[(N_{\beta0}+N_\beta)+(N_{\beta0}+N_\beta)\bar{d}_{22}+S_{\alpha\beta}\bar{d}_{21}]\}_{,\beta}+\\ \quad\{B[S_{\alpha\beta}+S_{\alpha\beta}\bar{d}_{22}+(N_{\alpha0}+N_\alpha)\bar{d}_{21}]\}_{,\alpha}+\\ \quad S_{\alpha\beta}B_{,\alpha}-(N_{\alpha0}+N_\alpha)A_{,\beta}+ABf_\beta=AB\dot{p}_\beta\\ \{B[(N_{\alpha0}+N_\alpha)\bar{d}_{31}+S_{\alpha\beta}\bar{d}_{32}]\}_{,\alpha}+\{A[(N_{\beta0}+\\ \quad N_\beta)\bar{d}_{32}+S_{\alpha\beta}\bar{d}_{31}]\}_{,\beta}+ABk_\alpha(N_{\alpha0}+N_\alpha)+\\ \quad AKk_\beta(N_{\beta0}+N_\beta)+ABf_\gamma=AB\dot{p}_\gamma\end{cases} \tag{6}$$

或者

$$\begin{cases}\{B[(N_{\alpha0}+N_\alpha)+(N_{\alpha0}+N_\alpha)\bar{d}_{11}+S_{\alpha\beta}\bar{d}_{12}]\}_{,\alpha}+\\ \quad\{A[S_{\alpha\beta}+S_{\alpha\beta}\bar{d}_{11}+(N_{\beta0}+N_\beta)\bar{d}_{12}]\}_{,\beta}-\\ \quad(N_{\beta0}+N_\beta)B_{,\alpha}+S_{\alpha\beta}A_{,\beta}+ABf_\alpha=AB\bar{m}\ddot{u}\\ \{A[(N_{\beta0}+N_\beta)+(N_{\beta0}+N_\beta)\bar{d}_{22}+S_{\alpha\beta}\bar{d}_{21}]\}_{,\beta}+\\ \quad\{B[S_{\alpha\beta}+S_{\alpha\beta}\bar{d}_{22}+(N_{\alpha0}+N_\alpha)\bar{d}_{21}]\}_{,\alpha}+\\ \quad S_{\alpha\beta}B_{,\alpha}-(N_{\alpha0}+N_\alpha)A_{,\beta}+ABf_\beta=AB\bar{m}\ddot{v}\\ \{B[(N_{\alpha0}+N_\alpha)\bar{d}_{31}+S_{\alpha\beta}\bar{d}_{32}]\}_{,\alpha}+\{A[(N_{\beta0}+\\ \quad N_\beta)\bar{d}_{32}+S_{\alpha\beta}\bar{d}_{31}]\}_{,\beta}+ABk_\alpha(N_{\alpha0}+N_\alpha)+\\ \quad AKk_\beta(N_{\beta0}+N_\beta)+ABf_\gamma=AB\bar{m}\ddot{w}\end{cases} \tag{7}$$

式中,$N_{\alpha0}$ 和 $N_{\beta0}$ 分别为膜面沿 α 和 β 方向的初始预拉力,N_α 和 N_β 分别为受荷后膜面沿 α 和 β 方向拉力的增加量,$S_{\alpha\beta}$ 为受荷后膜的剪力,f_α,f_β 和 f_γ 分别为膜面沿,α,β 和 γ 方向的面荷载密度.

1.5 物理方程

$$\begin{aligned}&N_\alpha=D_{11}\varepsilon_\alpha+D_{12}\varepsilon_\beta+D_{12}\varepsilon_{\alpha\beta},\\ &N_\beta=D_{12}\varepsilon_\alpha+D_{22}\varepsilon_\beta+D_{23}\varepsilon_{\alpha\beta},\\ &S_{\alpha\beta}=D_{13}\varepsilon_\alpha+D_{23}\varepsilon_\beta+D_{33}\varepsilon_{\alpha\beta}\end{aligned} \tag{8}$$

或

$$\begin{aligned}&\varepsilon_\alpha=s_{11}N_\alpha+s_{12}N_\beta+s_{13}S_{\alpha\beta},\\ &\varepsilon_\beta=s_{12}N_\alpha+s_{22}N_\beta+s_{23}S_{\alpha\beta},\\ &\varepsilon_{\alpha\beta}=s_{13}N_\alpha+s_{23}N_\beta+s_{33}S_{\alpha\beta}\end{aligned} \tag{9}$$

相应的应变能密度和余应变能密度分别为

$$\Phi(\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta})=AB(D_{11}\varepsilon_\alpha^2+D_{22}\varepsilon_\beta^2+D_{33}\varepsilon_{\alpha\beta}^2+2D_{12}\varepsilon_\alpha\varepsilon_\beta+2D_{13}\varepsilon_\alpha\varepsilon_{\alpha\beta}+2D_{23}\varepsilon_\beta\varepsilon_{\alpha\beta})/2$$

$$\Psi(N_\alpha,N_\beta,S_{\alpha\beta})=AB(s_{11}N_\alpha^2+s_{22}N_\beta^2+s_{33}N_{\alpha\beta}^2+2s_{12}N_\alpha N_\beta+2s_{13}N_\alpha N_{\alpha\beta}+2s_{23}N_\beta N_{\alpha\beta})/2$$

式中,D_{ij} 和 s_{ij} 分别为膜材的刚度系数和柔度系数.

对于正交各向异性的膜材,有[1]

$$D_{11}=\frac{E_1}{1-v_1v_2}, D_{22}=\frac{E_2}{1-v_1v_2},$$

$$D_{12}=D_{21}=\frac{E_2v_1}{1-v_1v_2}=\frac{E_1v_2}{1-v_1v_2},$$

$$D_{33}=G_{12}, D_{13}=D_{23}=0$$

其中,E_1 和 E_2 分别为膜材 α 和 β 方向的弹性模量,v_1 和 v_2 分别为膜材 α 方向引起的 β 方向和 β 方向引起的 α 方向的 Poisson 比,G_{12} 为膜材的剪切模量.

1.6 位移边界条件

$$\begin{cases}\text{在 }\partial\Omega_{u_n}\text{ 上}: u_n=u\cos\theta+v\sin\theta=\bar{u}_n;\\ \text{在 }\partial\Omega_{u_s}\text{ 上}: u_s=-u\sin\theta+v\cos\theta=\bar{u}_s;\\ \text{在 }\partial\Omega_{u_w}\text{ 上}: w=\bar{w}\end{cases} \tag{10}$$

式中，$\bar{u}_n$，$\bar{u}_s$ 和 $\bar{w}$ 为边界上的已知位移函数，n 是边界的外法线方向，s 是边界的切线方向，θ 是法线 n 与 α 轴的夹角.

1.7 初始条件

$$\begin{cases} u_0(\alpha,\beta) = u(\alpha,\beta,0) = \tilde{u}_0(\alpha,\beta), \\ v_0(\alpha,\beta) = v(\alpha,\beta,0) = \tilde{v}_0(\alpha,\beta), \\ w_0(\alpha,\beta) = w(\alpha,\beta,0) = \tilde{w}_0(\alpha,\beta) \end{cases} \tag{11}$$

$$\begin{cases} p_{\alpha 0}(\alpha,\beta) = p_\alpha(\alpha,\beta,0) = \tilde{p}_{\alpha 0}(\alpha,\beta), \\ p_{\beta 0}(\alpha,\beta) = p_\beta(\alpha,\beta,0) = \tilde{p}_{\beta 0}(\alpha,\beta), \\ p_{\gamma 0}(\alpha,\beta) = p_\gamma(\alpha,\beta,0) = \tilde{p}_{\gamma 0}(\alpha,\beta) \end{cases} \tag{12}$$

式中，$\tilde{u}_0$，$\tilde{v}_0$ 和 $\tilde{w}_0$ 为已知初始位移函数，$p_{\alpha 0}$，$\tilde{p}_{\beta 0}$ 和 $\tilde{p}_{\gamma 0}$ 为已知初始动量函数.

2 广义虚功原理和虚功原理

可以证明对于互不相关的任意函数 p_α，p_β，p_γ，u，v，w，N_α，N_β，$S_{\alpha\beta}$ 下列积分关系式恒成立.

$$\begin{aligned} &\int_0^{t_1}\iint_\Omega [p_\alpha \dot{u} + p_\beta \dot{v} + p_\gamma \dot{w} - W_{FD}(N_\alpha, N_\beta, S_{\alpha\beta}, u, v, w) - \\ &B_{FD}(N_\alpha, N_\beta, S_{\alpha\beta}, u, v, w)] AB \mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}t + \\ &\int_0^{t_1}\iint_\Omega W_{DE}(p_\alpha, p_\beta, p_\gamma, N_\alpha, N_\beta, S_{\alpha\beta}, u, v, w) \mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}t + \\ &\int_0^{t_1}\int_{\partial\Omega}(N_n u_n + S_{ns} u_s + Q_n w) \mathrm{d}s \mathrm{d}t - \iint_\Omega (u_1 p_{\alpha 1} + \\ &v_1 p_{\beta 1} + w_1 p_{\gamma 1} - u_0 p_{\alpha 0} - v_0 p_{\beta 0} - w_0 p_{\gamma 0}) AB \mathrm{d}\alpha \mathrm{d}\beta = \\ &\Pi_1 + \Pi_2 + \Pi_3 - \Pi_4 = 0 \end{aligned} \tag{13}$$

式中，Π_1，Π_2，Π_3，Π_4 依次表示第 1，2，3，4 积分项. W_{FD}，B_{FD} 和 W_{DE} 及边界 $\partial\Omega_{u_n}$ 上的 N_n，$\partial\Omega_{u_s}$ 上的 S_{ns} 和 $\partial\Omega_w$ 上的 Q_n 分别为：

$$\begin{aligned} W_{FD} = &(N_{\alpha 0} + N_\alpha)(d_{11} + \bar{d}_{11}^2/2 + \bar{d}_{21}^2/2 + \bar{d}_{31}^2/2) + \\ &(N_{\beta 0} + N_\beta)(d_{22} + \bar{d}_{22}^2/2 + \bar{d}_{12}^2/2 + \bar{d}_{32}^2/2) + \\ &S_{\alpha\beta}(d_{12} + d_{21} + \bar{d}_{11}\bar{d}_{12} + \bar{d}_{21}\bar{d}_{22} + \bar{d}_{31}\bar{d}_{32}) \end{aligned}$$

$$\begin{aligned} B_{FD} = &(N_{\alpha 0} + N_\alpha)(\bar{d}_{11}^2/2 + \bar{d}_{21}^2/2 + \bar{d}_{31}^2/2) + \\ &(N_{\beta 0} + N_\beta)(\bar{d}_{22}^2/2 + \bar{d}_{12}^2/2 + \bar{d}_{32}^2/2) + \\ &S_{\alpha\beta}(\bar{d}_{11}\bar{d}_{12} + \bar{d}_{21}\bar{d}_{22} + \bar{d}_{31}\bar{d}_{32}) \end{aligned}$$

$$\begin{aligned} W_{DE} = &u\{AB\dot{p}_\alpha - [B((N_{\alpha 0} + N_\alpha) + (N_{\alpha 0} + N_\alpha)\bar{d}_{11} + \\ &S_{\alpha\beta}\bar{d}_{12})]_{,\alpha} - [A(S_{\alpha\beta} + S_{\alpha\beta}\bar{d}_{11} + (N_{\beta 0} + \\ &N_\beta)\bar{d}_{12})]\}_{,\beta} + (N_{\beta 0} + N_\beta)B_{,\alpha} - S_{\alpha\beta}A_{,\beta}\} + \\ &v\{AB\dot{p}_\beta - [A((N_{\beta 0} + N_\beta) + (N_{\beta 0} + N_\beta)\bar{d}_{22} + \\ &S_{\alpha\beta}\bar{d}_{21})]\}_{,\beta} - [B(S_{\alpha\beta} + S_{\alpha\beta}\bar{d}_{22} + (N_{\alpha 0} + \\ &N_\alpha)\bar{d}_{21})]_{,\alpha} - S_{\alpha\beta}B_{,\alpha} + (N_{\alpha 0} + N_\alpha)A_{,\beta}\} + \\ &w\{AB\dot{p}_\gamma - [B((N_{\alpha 0} + N_\alpha)\bar{d}_{31} + S_{\alpha\beta}\bar{d}_{32})]_{,\alpha} - \\ &[A((N_{\beta 0} + N_\beta)\bar{d}_{32} + S_{\alpha\beta}\bar{d}_{31})]_{,\beta} - ABk_\alpha(N_{\alpha 0} + \\ &N_\alpha) - AKk_\beta(N_{\beta 0} + N_\beta)\} \end{aligned}$$

$$\begin{aligned} N_n = &[(N_{\alpha 0} + N_\alpha) + (N_{\alpha 0} + N_\alpha)\bar{d}_{11} + S_{\alpha\beta}\bar{d}_{12}]\cos^2\theta + \\ &\{[S_{\alpha\beta} + S_{\alpha\beta}\bar{d}_{11} + (N_{\beta 0} + N_\beta)\bar{d}_{12}] + [S_{\alpha\beta} + \\ &S_{\alpha\beta}\bar{d}_{22} + (N_{\alpha 0} + N_\alpha)\bar{d}_{21}]\}\cos\theta\sin\theta + [(N_{\beta 0} + \\ &N_\beta) + (N_{\beta 0} + N_\beta)\bar{d}_{22} + S_{\alpha\beta}\bar{d}_{21}]\sin^2\theta \end{aligned}$$

$$\begin{aligned} S_{ns} = &\{[(N_{\beta 0} + N_\beta) + (N_{\beta 0} + N_\beta)\bar{d}_{22} + S_{\alpha\beta}\bar{d}_{21}] - \\ &[(N_{\alpha 0} + N_\alpha) + (N_{\alpha 0} + N_\alpha)\bar{d}_{11} + S_{\alpha\beta}\bar{d}_{12}]\}\cos\theta\sin\theta + \\ &[S_{\alpha\beta} + S_{\alpha\beta}\bar{d}_{22} + (N_{\alpha 0} + N_\alpha)\bar{d}_{21}]\cos^2\theta - [S_{\alpha\beta} + \\ &S_{\alpha\beta}\bar{d}_{11} + (N_{\beta 0} + N_\beta)\bar{d}_{12}]\sin^2\theta \end{aligned}$$

$$\begin{aligned} Q_n = &[(N_{\alpha 0} + N_\alpha)\bar{d}_{31} + S_{\alpha\beta}\bar{d}_{32}]\cos\theta + [(N_{\beta 0} + \\ &N_\beta)\bar{d}_{32} + S_{\alpha\beta}\bar{d}_{31}]\sin\theta \end{aligned}$$

(13)式是本文给出的一个重要关系式，在力学上可认为是膜结构动力学广义虚功原理的表式. 从该式出发，不仅能系统地建立膜结构动力学的虚功原理和弹性膜结构动力学的各类非传统 Hamilton 型变分原理，而且能清楚地阐明这些原理之间的内在联系.

对于几何非线性动力学系统，由于几何算子的非线性，使得几何结构与平衡结构或阴结构与阳结构之间的对称性出现破缺. (13)式中的 B_{FD} 就是为了恢复其对称性而引入的破缺函数[10].

当 N_α，N_β，$S_{\alpha\beta}$，p_α，p_β，p_γ 满足方程(6)和条件(12)；u，v，w 满足方程(1)，(5)和条件(10)，(11)时，由(13)式可得

$$\begin{aligned} &\int_0^{t_1}\iint_\Omega (f_\alpha u + f_\beta v + f_\gamma w) AB \mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}t + \int_0^{t_1}\int_{\partial\Omega}(N_n u_n + \\ &S_{ns} u_s + Q_n w) \mathrm{d}s \mathrm{d}t - \iint_\Omega (u_1 p_{\alpha 1} + v_1 p_{\beta 1} + w_1 p_{\gamma 1} - \tilde{u}_0 \tilde{p}_{\alpha 0} - \\ &\tilde{v}_0 \tilde{p}_{\beta 0} - \tilde{w}_0 \tilde{p}_{\gamma 0}) AB \mathrm{d}\alpha \mathrm{d}\beta = \int_0^{t_1}\iint_\Omega [W_{FS}(N_\alpha, N_\beta, S_{\alpha\beta}, \\ &\varepsilon_\alpha, \varepsilon_\beta, \varepsilon_{\alpha\beta}) + N_{\alpha 0}\varepsilon_\alpha + N_{\beta 0}\varepsilon_\beta + \underline{(N_{\alpha 0} + N_\alpha)\varepsilon_{b\alpha} +} \\ &\underline{(N_{\beta 0} + N_\beta)\varepsilon_{b\beta} + S_{\alpha\beta}\varepsilon_{b\alpha\beta}} - p_\alpha v_\alpha - p_\beta v_\beta - p_\gamma v_\gamma] AB \mathrm{d}\alpha \mathrm{d}\beta \mathrm{d}t \end{aligned} \tag{14}$$

式中，$W_{FS} = N_\alpha \varepsilon_\alpha + N_\beta \varepsilon_\beta + S_{\alpha\beta}$，$\varepsilon_{\alpha\beta} = \bar{d}_{11}^2/2 + \bar{d}_{21}^2/2 + \bar{d}_{31}^2/2$，$\varepsilon_{b\beta} = \bar{d}_{22}^2/2 + \bar{d}_{12}^2/2 + \bar{d}_{32}^2/2$，$\varepsilon_{b\alpha\beta} = \bar{d}_{11}\bar{d}_{12} + \bar{d}_{21}\bar{d}_{22} + \bar{d}_{31}\bar{d}_{32}$，而划线项可称为破缺的内力虚功.

(14)式可以看成是膜结构动力学虚功原理的表式，它反映了广义动力可能状态与广义运动可能状态之间的最一般关系，或者说，它反映了阴变量 u，v，w，v_α，v_β，v_γ，$(\varepsilon_\alpha + \varepsilon_{b\alpha})$，$(\varepsilon_\beta + \varepsilon_{b\beta})$，$(\varepsilon_{\alpha\beta} + \varepsilon_{b\alpha\beta})$ 和阳变量 f_α，f_β，f_γ，p_α，p_β，p_γ，$(N_{\alpha 0} + N_\alpha)$，$(N_{\beta 0} + N_\beta)$，$S_{\alpha\beta}$ 这两组对偶变量之间的最一般关系.

从(14)式可以看出,几何非线性动力学的虚功原理与线性动力学的虚功原理最显著的区别就在于,前者增加了破缺的内力虚功项. 因此,对于几何非线性动力学,破缺函数对于虚功原理的成立起着关键作用.

从上述可知,几何非线性动力学中的对称性破缺和破缺函数的存在,是其重要的特点. 这是线性动力学所没有的.

3 各类非传统 Hamilton 型变分原理

由于篇幅所限, 本文仅给出 5 类变量与 1 类变量非传统 Hamilton 型变分原理和相空间非传统 Hamilton 型变分原理.

3.1 5 类变量的广义变分原理

当$(p_\alpha,p_\beta,p_\gamma)$与$(v_\alpha,v_\beta,v_\gamma)$是互不相关的任意函数时,有下列关系式

$$(p_\alpha v_\alpha+p_\beta v_\beta+p_\gamma v_\gamma)AB=K+K^*-L(v_\alpha,v_\beta,v_\gamma,p_\alpha,p_\beta,p_\gamma) \tag{15}$$

式中,$L=AB[(\bar{m}v_\alpha-p_\alpha)^2+(\bar{m}v_\beta-p_\beta)^2+(\bar{m}v_\gamma-p_\gamma)^2]/2\bar{m}$.

只有当$(p_\alpha,p_\beta,p_\gamma)$与$(v_\alpha,v_\beta,v_\gamma)$满足(2)式时,才有

$$(p_\alpha v_\alpha+p_\beta v_\beta+p_\gamma v_\gamma)AB=K+K^* \tag{16}$$

于是, (13)式的第 1 项 Π_1 中的被积函数$(p_\alpha\dot{u}+p_\beta\dot{v}+p_\gamma\dot{w})AB$ 可变换为

$$(p_\alpha\dot{u}+p_\beta\dot{v}+p_\gamma\dot{w})AB=K-[p_\alpha(v_\alpha-\dot{u})+p_\beta(v_\beta-\dot{v})+p_\gamma(v_\gamma-\dot{w})]AB+K^*-L \tag{17}$$

当$(N_\alpha,N_\beta,S_{\alpha\beta})$与$(\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta})$是互不相关的任意函数时,有下列关系式

$$W_{FS}^{\cdot}AB=\Phi+\Psi+C(\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta},N_\alpha,N_\beta,S_{\alpha\beta}) \tag{18}$$

式中

$$C=AB\{[N_\alpha-(D_{11}\varepsilon_\alpha+D_{12}\varepsilon_\beta+D_{13}\varepsilon_{\alpha\beta})][\varepsilon_\alpha-(s_{11}N_\alpha+s_{12}N_\beta+s_{13}S_{\alpha\beta})]+[N_\beta-(D_{12}\varepsilon_\alpha+D_{22}\varepsilon_\beta+D_{23}\varepsilon_{\alpha\beta})][\varepsilon_\beta-(s_{12}N_\alpha+s_{22}N_\beta+s_{23}S_{\alpha\beta})]+[S_{\alpha\beta}-(D_{13}\varepsilon_\alpha+D_{23}\varepsilon_\beta+D_{33}\varepsilon_{\alpha\beta})][\varepsilon_{\alpha\beta}-(s_{13}N_\alpha+s_{23}N_\beta+s_{33}S_{\alpha\beta})]\}/2$$

只有当$(N_\alpha,N_\beta,S_{\alpha\beta})$与$(\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta})$满足(8)和(9)式时,才有

$$W_{FS}^{\cdot}AB=\Phi+\Psi \tag{19}$$

于是, (13)式的第 1 项中的被积函数$-[W_{FD}+B_{FD}]AB$ 可变换为

$$-[W_{FD}+B_{FD}]AB=-\Phi+(W_{FS}-W_{FD})AB-\Psi-C-B_{FD}^{\cdot}AB \tag{20}$$

上述(15)和(18)式是本文给出的广义 Legendre 变换式.

而(13)式中的 $\Pi_2+\Pi_3-\Pi_4$ 可变换为

$$\Pi_2+\Pi_3-\Pi_4=\int_0^{t_1}\iint_\Omega[W_{DE}-(f_\alpha u+f_\beta v+f_\gamma w)AB]\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}t+\Gamma_{IB}+\mathring{\Gamma}+\int_0^{t_1}\iint_\Omega(f_\alpha u+f_\beta v+f_\gamma w)AB\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}t+\Pi_{IB}+\mathring{\Pi} \tag{21}$$

式中

$$\Pi_{IB}=\int_0^{t_1}\Big[\int_{\partial\Omega_{u_n}}N_n(u_n-\bar{u}_n)\mathrm{d}s+\int_{\partial\Omega_{u_s}}S_{ns}(u_s-\bar{u}_s)\mathrm{d}s+\int_{\partial\Omega_w}Q_n(w-\bar{w})\mathrm{d}s\Big]\mathrm{d}t+\iint_\Omega[\tilde{p}_{\alpha0}u_1-(\tilde{u}_0-u_0)p_{\alpha0}+\tilde{p}_{\beta0}v_1-(\tilde{v}_0-v_0)p_{\beta0}+\tilde{p}_{\gamma0}w_1-(\tilde{w}_0-w_0)p_{\gamma0}]AB\mathrm{d}\alpha\mathrm{d}\beta$$

$$\Gamma_B=\int_0^{t_1}\Big[\int_{\partial\Omega_{u_n}}N_n\bar{u}_n\mathrm{d}s+\int_{\partial\Omega_{u_s}}S_{ns}\bar{u}_s\mathrm{d}s+\int_{\partial\Omega_w}Q_n\bar{w}\mathrm{d}s\Big]\mathrm{d}t+\iint_\Omega(\tilde{u}_0p_{\alpha0}-\tilde{p}_{\alpha0}u_1+\tilde{v}_0p_{\beta0}-\tilde{p}_{\beta0}v_1+\tilde{w}_0p_{\gamma0}-\tilde{p}_{\gamma0}w_1)AB\mathrm{d}\alpha\mathrm{d}\beta$$

$$\mathring{\Pi}=-\iint_\Omega[(\mathring{p}_{\alpha1}+\mathring{p}_{\alpha0})u_1+(\mathring{p}_{\beta1}+\mathring{p}_{\beta0})v_1+(\mathring{p}_{\gamma1}+\mathring{p}_{\gamma0})w_1]AB\mathrm{d}\alpha\mathrm{d}\beta$$

$$\mathring{\Gamma}=-\iint_\Omega(\mathring{u}_1p_{\alpha1}-\mathring{p}_{\alpha0}u_1+\mathring{v}_1p_{\beta1}-\mathring{p}_{\beta0}v_1+\mathring{w}_1p_{\gamma1}-\mathring{p}_{\gamma0}w_1)AB\mathrm{d}\alpha\mathrm{d}\beta$$

式中带顶标$^\circ$ 的量称为限制变分量[11].

将(17), (20)及(21)式代入(13)式,整理后可得

$$\Pi_5+\Gamma_5=0 \tag{22}$$

而泛函 Π_5 和 Γ_5 分别为

$$\Pi_5=\int_0^{t_1}\iint_\Omega[K/AB-p_\alpha(v_\alpha-\dot{u})-p_\beta(v_\beta-\dot{v})-p_\gamma(v_\gamma-\dot{w})-\Phi/AB+W_{FS}-W_{FD}+f_\alpha u+f_\beta v+f_\gamma w]AB\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}t+\Pi_{IB}+\mathring{\Pi} \tag{23}$$

$$\Gamma_5=\int_0^{t_1}\iint_\Omega[K^*-L-\Psi-C-B_{FD}^{\cdot}AB+W_{DE}-(f_\alpha u+f_\beta v+f_\gamma w)AB]\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}t+\Gamma_{IB}+\mathring{\Gamma} \tag{24}$$

定理 1 当且仅当 $p_\alpha,p_\beta,p_\gamma,v_\alpha,v_\beta,v_\gamma,N_\alpha,N_\beta,S_{\alpha\beta},\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta},u,v,w$ 是混合问题(1),(2),(5),(6),(8),(10),(11)和(12)式的解,则必定满足下列变分式

$$\delta\Pi_5=0 \text{ 或 } \delta\Gamma_5=0 \tag{25}$$

证明 将泛函 Π_5 对自变函数 $p_\alpha,p_\beta,p_\gamma,v_\alpha,v_\beta,v_\gamma,N_\alpha,N_\beta,S_{\alpha\beta},\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta},u,v,w$ 变分,整理后可得

$$\delta\Pi_5 = \int_0^{t_1}\iint_\Omega\{(\overline{m}v_\alpha - p_\alpha)\delta v_\alpha + (\overline{m}v_\beta - p_\beta)\delta v_\beta + (\overline{m}v_\gamma - p_\gamma)\delta v_\gamma - (v_\alpha - \dot{u})\delta p_\alpha - (v_\beta - \dot{v})\delta p_\beta - (v_\gamma - \dot{w})\delta p_\gamma - (D_{11}\varepsilon_\alpha + D_{12}\varepsilon_\beta + D_{13}\varepsilon_{\alpha\beta} - N_\alpha)\delta\varepsilon_\alpha - (D_{12}\varepsilon_\alpha + D_{22}\varepsilon_\beta + D_{23}\varepsilon_{\alpha\beta} - N_\beta)\delta\varepsilon_\beta - (D_{13}\varepsilon_\alpha + D_{23}\varepsilon_\beta + D_{33}\varepsilon_{\alpha\beta} - S_{\alpha\beta})\delta\varepsilon_{\alpha\beta} + [\varepsilon_\alpha - (d_{11} + \overline{d}_{11}^2/2 + \overline{d}_{21}^2/2 + \overline{d}_{31}^2/2)]\delta N_\alpha + [\varepsilon_\beta - (d_{22} + \overline{d}_{22}^2/2 + \overline{d}_{12}^2/2 + \overline{d}_{32}^2/2)]\delta N_\beta + [\varepsilon_{\alpha\beta} - (d_{12} + d_{21} + \overline{d}_{11}\overline{d}_{12} + \overline{d}_{21}\overline{d}_{22} + \overline{d}_{31}\overline{d}_{32})]\delta S_{\alpha\beta}\}AB\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}t + \int_0^{t_1}\iint_\Omega\{\{[B(N_{\alpha0} + N_\alpha) + (N_{\alpha0} + N_\alpha)\overline{d}_{11} + S_{\alpha\beta}\overline{d}_{12}]_{,\alpha} + [A(S_{\alpha\beta} + S_{\alpha\beta}\overline{d}_{11} + (N_{\beta0} + N_\beta)\overline{d}_{12})]_{,\beta} - (N_{\beta0} + N_\beta)B_{,\alpha} + S_{\alpha\beta}A_{,\beta} + ABf_\alpha - AB\dot{p}_\alpha\}\delta u + \{[A((N_{\beta0} + N_\beta) + (N_{\beta0} + N_\beta)\overline{d}_{22} + S_{\alpha\beta}\overline{d}_{21})]_{,\beta} + [B(S_{\alpha\beta} + S_{\alpha\beta}\overline{d}_{22} + (N_{\alpha0} + N_\alpha)\overline{d}_{21})]_{,\alpha} + S_{\alpha\beta}B_{,\alpha} - (N_{\alpha0} + N_\alpha)A_{,\beta} + ABf_\beta - AB\dot{p}_\beta\}\delta v + \{[B((N_{\alpha0} + N_\alpha)\overline{d}_{31} + S_{\alpha\beta}\overline{d}_{32})]_{,\alpha} + [A((N_{\beta0} + N_\beta)\overline{d}_{32} + S_{\alpha\beta}\overline{d}_{31})]_{,\beta} + ABk_\alpha(N_{\alpha0} + N_\alpha) + ABk_\beta(N_{\beta0} + N_\beta) + ABf_\gamma - AB\dot{p}_\gamma\}\delta w\}\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}t + \int_0^{t_1}[\int_{\partial\Omega_{u_n}}(u_n - \overline{u}_n)\delta N_n\mathrm{d}s + \int_{\partial\Omega_{u_s}}(u_s - \overline{u}_s)\delta S_{ns}\mathrm{d}s + \int_{\partial\Omega_w}(w - \overline{w})\delta Q_n\mathrm{d}s]\mathrm{d}t - \iint_\Omega[(p_{\alpha0} - \tilde{p}_{\alpha0})\delta u_1 + (p_{\beta0} - \tilde{p}_{\beta0})\delta v_1 + (p_{\gamma0} - \tilde{p}_{\gamma0})\delta w_1 - (u_0 - \tilde{u}_0)\delta p_{\alpha0} - (v_0 - \tilde{v}_0)\delta p_{\beta0} - (w_0 - \tilde{w}_0)\delta p_{\gamma0}]AB\mathrm{d}\alpha\mathrm{d}\beta \tag{26}$$

充分性　若 $p_\alpha,p_\beta,p_\gamma,v_\alpha,v_\beta,v_\gamma,N_\alpha,N_\beta,S_{\alpha\beta},\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta},u,v,w$ 是混合问题(1),(2),(5),(6),(8),(10),(11)和(12)式的解,则(26)式就变成为 $\delta\Pi_5=0$,即 $\delta\Pi_5=0$ 成立.

必要性　若 $\delta\Pi_5=0$,由于 $\delta p_\alpha,\delta p_\beta,\delta p_\gamma,\delta v_\alpha,\delta v_\beta,\delta v_\gamma,\delta N_\alpha,\delta N_\beta,\delta S_{\alpha\beta},\delta\varepsilon_\alpha,\delta\varepsilon_\beta,\delta\varepsilon_{\alpha\beta},\delta u,\delta v,\delta w$ 的任意性,并根据变分法的有关引理,故由此可得(1),(2),(5),(6),(8),(10),(11)和(12)式,即 $p_\alpha,p_\beta,p_\gamma,v_\alpha,v_\beta,v_\gamma,N_\alpha,N_\beta,S_{\alpha\beta},\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta},u,v,w$ 是上述混合问题的解.

Π_5 和 Γ_5 分别是弹性膜结构动力学的 5 类变量非传统 Hamilton 型广义变分原理的势能形式和余能形式的泛函,对于任意无关的函数 $p_\alpha,p_\beta,p_\gamma,v_\alpha,v_\beta,v_\gamma,N_\alpha,N_\beta,S_{\alpha\beta},\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta},u,v,w$,它们之间存在互补关系(22).

3.2　2 类变量的广义变分原理-相空间广义变分原理

当$(p_\alpha,p_\beta,p_\gamma)$与$(v_\alpha,v_\beta,v_\gamma)$满足(2)式;$(\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta})$与$(u,v,w)$满足(5)式时,泛函 Π_5 就变成为

$$\Pi_2 = \int_0^{t_1}\iint_\Omega[p_\alpha\dot{u} + p_\beta\dot{v} + p_\gamma\dot{w} - H(p_\alpha,p_\beta,p_\gamma,u,v,w)]AB\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}t + \Pi_{IB} + \mathring{\Pi} \tag{27}$$

式中,Hamilton 函数 H 为

$$H = [K^* + \Phi(u,v,w)]/AB + N_{\alpha0}(d_{11} + \overline{d}_{11}^2/2 + \overline{d}_{21}^2/2 + \overline{d}_{31}^2/2) + N_{\beta0}(d_{22} + \overline{d}_{22}^2/2 + \overline{d}_{12}^2/2 + \overline{d}_{32}^2/2) - f_\alpha u - f_\beta v - f_\gamma w$$

由 $\delta\Pi_2=0$ 可推导出 Hamilton 正则方程

$$\dot{u} = \partial H/\partial p_\alpha = H_{p_\alpha},\quad \dot{v} = \partial H/\partial p_\beta = H_{p_\beta},\quad \dot{w} = \partial H/\partial p_\gamma = H_{p_\gamma} \tag{28}$$

$$\dot{p}_\alpha = -\partial H/\partial u = -H_u,\quad \dot{p}_\beta = -\partial H/\partial v = -H_v,\quad \dot{p}_\gamma = -\partial H/\partial w = -H_w \tag{29}$$

或

$$\dot{u} = p_\alpha/\overline{m},\quad \dot{v} = p_\beta/\overline{m},\quad \dot{w} = p_\gamma/\overline{m} \tag{30}$$

$$\begin{cases} \dot{p}_\alpha = \{\{B[(N_{\alpha0} + \hat{N}_\alpha) + (N_{\alpha0} + \hat{N}_\alpha)\overline{d}_{11} + \hat{S}_{\alpha\beta}\overline{d}_{12}]\}_{,\alpha} + \{A[\hat{S}_{\alpha\beta} + \hat{S}_{\alpha\beta}\overline{d}_{11} + (N_{\beta0} + \hat{N}_\beta)\overline{d}_{12}]\}_{,\beta} - (N_{\beta0} + \hat{N}_\beta)B_{,\alpha} + \hat{S}_{\alpha\beta}A_{,\beta} + ABf_\alpha\}/AB \\ \dot{p}_\beta = \{\{A[(N_{\beta0} + \hat{N}_\beta) + (N_{\beta0} + \hat{N}_\beta)\overline{d}_{22} + \hat{S}_{\alpha\beta}\overline{d}_{21}]\}_{,\beta} + \{B[\hat{S}_{\alpha\beta} + \hat{S}_{\alpha\beta}\overline{d}_{22} + (N_{\alpha0} + \hat{N}_\alpha)\overline{d}_{21}]\}_{,\alpha} + \hat{S}_{\alpha\beta}B_{,\alpha} - (N_{\alpha0} + \hat{N}_\alpha)B_{,\beta} + ABf_\alpha\}/AB \\ \dot{p}_\gamma = \{\{B[(N_{\alpha0} + \hat{N}_\alpha)\overline{d}_{31} + \hat{S}_{\alpha\beta}\overline{d}_{32}]\}_{,\alpha} + \{A[(N_{\beta0} + \hat{N}_\beta)\overline{d}_{32} + \hat{S}_{\alpha\beta}\overline{d}_{31}]\}_{,\beta} + ABk_\alpha(N_{\alpha0} + \hat{N}_\alpha) + ABk_\beta(N_{\beta0} + \hat{N}_\beta) + ABf_\gamma\}/AB \end{cases} \tag{31}$$

及边界条件(10)与初始条件(11)和(12). 其中,$\hat{N}_\alpha,\hat{N}_\beta$ 和 $\hat{S}_{\alpha\beta}$分别为

$$\hat{N}_\alpha = D_{11}(d_{11} + \overline{d}_{11}^2/2 + \overline{d}_{21}^2/2 + \overline{d}_{31}^2/2) + D_{12}(d_{22} + \overline{d}_{22}^2/2 + \overline{d}_{12}^2/2 + \overline{d}_{32}^2/2) + D_{13}(d_{12} + d_{21} + \overline{d}_{11}\overline{d}_{12} + \overline{d}_{21}\overline{d}_{22} + \overline{d}_{31}\overline{d}_{32})$$

$$\hat{N}_\beta = D_{12}(d_{11} + \overline{d}_{11}^2/2 + \overline{d}_{21}^2/2 + \overline{d}_{31}^2/2) + D_{22}(d_{22} + \overline{d}_{22}^2/2 + \overline{d}_{12}^2/2 + \overline{d}_{32}^2/2) + D_{23}(d_{12} + d_{21} + \overline{d}_{11}\overline{d}_{12} + \overline{d}_{21}\overline{d}_{22} + \overline{d}_{31}\overline{d}_{32})$$

$$\hat{S}_{\alpha\beta} = D_{13}(d_{11} + \overline{d}_{11}^2/2 + \overline{d}_{21}^2/2 + \overline{d}_{31}^2/2) + D_{23}(d_{22} + \overline{d}_{22}^2/2 + \overline{d}_{12}^2/2 + \overline{d}_{32}^2/2) + D_{33}(d_{12} + d_{21} + \overline{d}_{11}\overline{d}_{12} + \overline{d}_{21}\overline{d}_{22} + \overline{d}_{31}\overline{d}_{32})$$

定理 2 当且仅当 $p_\alpha,p_\beta,p_\gamma,u,v,w$ 是混合问题(30),(31),(10),(11)和(12)式的解,则必定满足变分式 $\delta\Pi_2=0$. Π_2 是弹性膜结构动力学相空间非传统 Hamilton 型变分原理的泛函.

为了揭示 Hamilton 正则方程的数学结构,就要

打破传统概念的限制，引进新概念. 为此，将(28)和(29)式写成矩阵形式

$$[\dot{u},\dot{v},\dot{w},\dot{p}_\alpha,\dot{p}_\beta,\dot{p}_\gamma]^T = J[H_u,H_v,H_w,H_{p_\alpha},H_{p_\beta},H_{p_\gamma}]^T \tag{32}$$

式中，$J=\begin{bmatrix} 0 & I_3 \\ -I_3 & 0 \end{bmatrix}$，$I_3$ 为三阶单位阵，方阵 J 是辛几何的度量矩阵，它是辛矩阵.

(32)式揭示了 Hamilton 正则方程和相应的相空间非传统 Hamilton 变分原理都具有自然辛结构. 这个自然辛结构在 Hamilton 力学中起着决定性的作用，并揭示出力学的辛几何结构，它使 Hamilton 力学显得更加简洁、对称和完美. 正是这个最根本的原因，使得对应于辛几何的 Hamilton 力学体系的算法要比对应于 Riemann 几何的 Lagrange 力学体系的算法和对应于 Euclid 几何的 Newton 力学体系的算法，具有更加优越的性能.

3.3　1 类变量的广义变分原理

当$(p_\alpha,p_\beta,p_\gamma)$，$(v_\alpha,v_\beta,v_\gamma)$，$(u,v,w)$满足(1)和(2)式，$(\varepsilon_\alpha,\varepsilon_\beta,\varepsilon_{\alpha\beta})$与$(u,v,w)$满足(5)式时，泛函 Π_5 就变成为

$$\begin{aligned}\Pi_1 = &\int_0^{t_1}\iint_\Omega\{[K(\dot{u},\dot{v},\dot{w})-\Phi(u,v,w)]/AB-N_{\alpha0}(d_{11}+\\&\bar{d}_{11}^2/2+\bar{d}_{21}^2/2+\bar{d}_{31}^2/2)-N_{\beta0}(d_{22}+\bar{d}_{22}^2/2+\bar{d}_{12}^2/2+\\&\bar{d}_{32}^2/2)+f_\alpha u+f_\beta v+f_\gamma w\}AB\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}t+\Pi_{IB}-\\&\iint_\Omega[(\bar{m}\mathring{u}_1+\bar{m}\mathring{u}_0)u_1+(\bar{m}\mathring{v}_1+\bar{m}\mathring{v}_0)v_1+\\&(\bar{m}\mathring{w}_1+\bar{m}\mathring{w}_0)w_1]AB\mathrm{d}\alpha\mathrm{d}\beta\end{aligned} \tag{33}$$

定理 3　当且仅当是混合问题(10)，(11)，(12)及下式

$$\left\{\begin{aligned}&\{B[(N_{\alpha0}+\hat{N}_\alpha)+(N_{\alpha0}+\hat{N}_\alpha)\bar{d}_{11}+\hat{S}_{\alpha\beta}\bar{d}_{12}]\}_{,\alpha}+\\&\quad\{A[\hat{S}_{\alpha\beta}+\hat{S}_{\alpha\beta}\bar{d}_{11}+(N_{\beta0}+\hat{N}_\beta)\bar{d}_{12}]\}_{,\beta}-\\&\quad(N_{\beta0}+\hat{N}_\beta)B_{,\alpha}+\hat{S}_{\alpha\beta}A_{,\beta}+ABf_\alpha=AB\bar{m}\ddot{u}\\&\{A[(N_{\beta0}+\hat{N}_\beta)+(N_{\beta0}+\hat{N}_\beta)\bar{d}_{22}+\hat{S}_{\alpha\beta}\bar{d}_{21}]\}_{,\beta}+\\&\quad\{B[\hat{S}_{\alpha\beta}+\hat{S}_{\alpha\beta}\bar{d}_{22}+(N_{\alpha0}+\hat{N}_\alpha)\bar{d}_{21}]\}_{,\alpha}+\\&\quad\hat{S}_{\alpha\beta}B_{,\alpha}-(N_{\alpha0}+\hat{N}_\alpha)B_{,\beta}+ABf_\alpha=AB\bar{m}\ddot{v}\\&\{B[(N_{\alpha0}+\hat{N}_\alpha)\bar{d}_{31}+\hat{S}_{\alpha\beta}\bar{d}_{32}]\}_{,\alpha}+\{A[(N_{\beta0}+\\&\quad\hat{N}_\beta)\bar{d}_{32}+\hat{S}_{\alpha\beta}\bar{d}_{31}]\}_{,\beta}+ABk_\alpha(N_{\alpha0}+\hat{N}_\alpha)+\\&\quad ABk_\beta(N_{\beta0}+\hat{N}_\beta)+ABf_\gamma=AB\bar{m}\ddot{w}\end{aligned}\right. \tag{34}$$

的解，则必定满足变分式 $\delta\Pi_1=0$. Π_1 是弹性膜结构动力学的 1 类变量非传统 Hamilton 型广义变分原理的势能形式的泛函.

4　结语

本文所建立的虚功原理和各类非传统 Hamilton 型变分原理及相空间非传统 Hamilton 型变分原理，是对几何非线性弹性膜结构动力学基本理论的发展与完善. 这些新的变分原理能反映这种动力学初值－边值问题的全部特征. 因此，所建立的这些新的变分原理，无论在有关动力学理论研究方面，还是在建立各种近似解法和工程实用理论方面都有重要价值.

因篇幅所限，有关这些新的变分原理的应用研究，将另文阐述.

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UNCONVENTIONAL HAMILTON-TYPE VARIATIONAL PRINCIPLES FOR NONLINEAR ELASTODYNAMICS OF MEMBRANE STRUCTURES *

Li Weihua　Luo En

(*Department of Applied Mechanics and Engineering*, *Sun Yat-sen University*, *Guangzhou* 510275, *China*)

Abstract According to the basic idea of classical yin-yang complementarity and modern dual-complementarity, in a new, simple and unified way proposed by Luo, the unconventional Hamilton-type variational principles for geometrically nonlinear elastodynamics of membrane structures can be established systematically. The unconventional Hamilton-type variational principle can fully characterize the initial-boundary-value problem of geometrically nonlinear elastodynamics. An important integral relation is given, which can be considered as the generalized principle of virtual work for dynamics of membrane structures in mechanics. Based on this relation, it is possible not only to obtain the principle of virtual work for dynamics of membrane structures, but also to derive systematically the complementary functionals for five-field ($p_\alpha, p_\beta, p_\gamma, v_\alpha, v_\beta, v_\gamma, N_\alpha, N_\beta, S_{\alpha\beta}, \varepsilon_\alpha, \varepsilon_\beta, \varepsilon_{\alpha\beta}, u, v, w$), four-field ($p_\alpha, p_\beta, p_\gamma, N_\alpha, N_\beta, S_{\alpha\beta}, \varepsilon_\alpha, \varepsilon_\beta, \varepsilon_{\alpha\beta}, u, v, w$), three-field ($N_\alpha, N_\beta, S_{\alpha\beta}, \varepsilon_\alpha, \varepsilon_\beta, \varepsilon_{\alpha\beta}, u, v, w$) and two-field ($N_\alpha, N_\beta, S_{\alpha\beta}, u, v, w$) unconventional Hamilton-type variational principles. And the functional for the unconventional Hamilton-type variational principle in phase space($p_\alpha, p_\beta, p_\gamma, u, v, w$) and the potential energy functional for one-field (u, v, w) unconventional Hamilton-type variational principle for geometrically nonlinear elastodynamics of membrane structures are obtained by the generalized Legendre transformation. Furthermore, with this approach, the intrinsic relationship among various principles can be explained clearly.

Key words unconventional Hamilton-type variational principle, geometric nonlinearity, elastodynamics, membrane structures, dual-complementary relation, initia1-boundary-value problem, phase space

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