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弹性薄板绕轴转动时刚 - 柔耦合动力学非线性分析*

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摘要　从连续介质力学中关于弹性薄板的变形理论出发,讨论绕轴作大范围运动的弹性薄板的动力学性质. 由于在无大范围运动的情况下,弹性薄板的变形对系统的动力学性质影响很小而被忽略,而其一旦与大范围运动耦合,对系统的动力学性质产生明显的影响. 根据弹性薄板的应变 - 位移几何非线性关系,建立了作大范围运动弹性薄板的几何非线性动力学方程,然后利用 Garlerkin 模态截断方法建立了该系统的离散动力学方程,仿真计算验证了理论分析的正确性,从而表明了系统的横向振动是稳定的.

关键词　高速转动,　薄板,　刚 - 柔耦合,　几何非线性

引言

1987 年 kane[1] 基于模态假设法对固结在作大范围运动刚体上的悬臂梁进行精确的动力分析,首次提出了"动力刚化"的概念,但对薄板的讨论资料较少. 柔性多体系统[2] 中的组成构件大多数是梁或板式构件,而梁在许多文献中讨论较多,并形成许多的流派:1)平面梁 [3];2)空间 Euler 梁[4];3)空间 Timoshenko 梁[5];4)空间 Timoshenko 梁的分叉和不稳定性[6];5)空间弯曲 Timoshenko 梁[7];6)连续介质力学原理[8] 等,由于弹性板的变形比较复杂,对作大范围运动弹性板动力学性质的讨论较少一些,只有几种建模理论:1)Kichhoff - Love 模型[9];2)Mindlin - Reissner 模型等. 特别是对作大范围运动弹性板耦合动力学性质几乎没有相关文献涉及.

本文将从连续介质力学中关于弹性薄板的变形理论出发,寻找由于在结构动力学中对无大范围运动弹性薄板的动力学性质影响很小而被忽略的变形量,而其一旦与大范围运动相耦合,将对系统的变形运动影响很显著,并将讨论其对系统动力学性质的影响,从而建立作大范围运动弹性薄板的较精确的刚 - 柔耦合动力学模型;根据弹性薄板的应变 - 位移几何非线性关系,建立作大范围运动弹性薄板的几何非线性动力学模型,并对其性质进行定性分析.

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1　建模理论

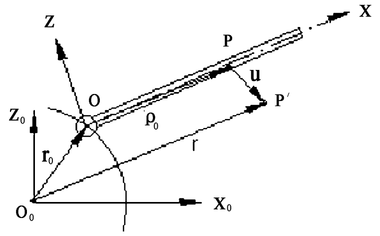

图 1　绕轴转动弹性薄板的运动学描述

Fig. 1　The description of motion of elastic thin plate axial rotation

如图 1 所示,弹性薄板与刚体铰接并以角 ω 速度绕 O 点作大范围运动. $\underline{e}^0(x_0-O_0-z_0)$ 为惯性坐标系,$\underline{e}^b(x-O-z)$ 为浮动坐标系,其中两坐标系的 y 轴垂直于纸面. r_0 为惯性坐标系原点到浮动坐标系原点的矢径,ρ_0 为板内非中面上任意一点 P 到浮动坐标系原点的矢径,u 为其变形位移场,r 为 P 点变形后的 P' 点相对惯性坐标系的矢径. 为了计算方便将铰接 O 点的速度和加速度给定. 其坐标形式为

$$\underline{v}_0=[v_{0x},v_{0y},v_{0z}]^T,\ \underline{a}_0=[a_{0x},a_{0y},a_{0z}]^T$$

$$r=r_0+\rho_0+u \tag{1}$$

由此可得板上任一点在惯性坐标系下的一阶导数速度为

$$\frac{{}^0dr}{dt}=\frac{{}^bdr}{dt}+\omega\times r \tag{2}$$

二阶导数加速度为

$$\frac{{}^{0}d^2 r}{dt^2}=\frac{{}^{b}d}{dt}\left(\frac{{}^{b}dr}{dt}+\omega\times r\right)+\omega\times\left(\frac{{}^{b}dr}{dt}+\omega\times r\right) \quad (3)$$

利用式(1)、(2),将上式在浮动坐标系中展开,可得

$$\frac{{}^{0}d^2 r}{dt^2}=\ddot{\underset{-}{u}}+2\underset{-}{\tilde{\omega}}\dot{\underset{-}{u}}+(\dot{\underset{-}{\tilde{\omega}}}-\underset{-}{\hat{\omega}})\underset{-}{u}+(\dot{\underset{-}{\tilde{\omega}}}-\underset{-}{\hat{\omega}})\rho_0+\dot{\underset{-}{v}}_0+\underset{-}{\tilde{\omega}}\underset{-}{v}_0 \quad (4)$$

其中

$$\underset{-}{\tilde{\omega}}=\begin{bmatrix}0&0&\omega\\0&0&0\\-\omega&0&0\end{bmatrix}$$

$$\underset{-}{\hat{\omega}}=\begin{bmatrix}\omega^2&0&0\\0&0&0\\0&0&\omega^2\end{bmatrix}$$

则板的动能变分可表示为

$$\delta T=\int_m[\ddot{\underset{-}{u}}+2\underset{-}{\tilde{\omega}}\dot{\underset{-}{u}}+(\dot{\underset{-}{\tilde{\omega}}}-\underset{-}{\hat{\omega}})\underset{-}{u}+(\dot{\underset{-}{\tilde{\omega}}}-\underset{-}{\hat{\omega}})\rho_0+\dot{\underset{-}{v}}_0+\underset{-}{\tilde{\omega}}\underset{-}{v}_0]\delta\underset{-}{u}\mathrm{d}m \quad (5)$$

$\mathrm{d}m$ 为板的微单元质量.

根据 Love－Kichhoff 假设和 Von－Karman 变形理论[10],弹性薄板应变能的变分为

$$\delta\Pi=\int_A(\underset{-}{N}+\underset{-}{N}_f)\delta\underset{-}{w}\mathrm{d}A \quad (6)$$

$\mathrm{d}A$ 为板的微单元质量.

若不考虑大挠度引起的几何非线性,则 $\underset{-}{N}_f$ 通常被忽略.

其中

$$\underset{-}{N}=\begin{bmatrix}\dfrac{\partial N_x}{\partial x}+\dfrac{\partial N_{xy}}{\partial y}\\ \dfrac{\partial N_y}{\partial y}+\dfrac{\partial N_{yx}}{\partial x}\\ \dfrac{\partial^2 M_x}{\partial x^2}+2\dfrac{\partial^2 M_{xy}}{\partial x\partial y}+\dfrac{\partial^2 M_y}{\partial y^2}\end{bmatrix}$$

$$\underset{-}{N}_f=\begin{bmatrix}\dfrac{\partial N_{fx}}{\partial x}+\dfrac{\partial N_{fxy}}{\partial y}\\ \dfrac{\partial N_{fy}}{\partial y}+\dfrac{\partial N_{fyx}}{\partial x}\\ \dfrac{\partial}{\partial x}[(N_x+N_{fx})\dfrac{\partial w_3}{\partial x}]+\dfrac{\partial}{\partial y}[(N_y+N_{fy})\dfrac{\partial w_3}{\partial y}]+\dfrac{\partial}{\partial x}[(N_{xy}+N_{fxy})\dfrac{\partial w_3}{\partial y}]\end{bmatrix}$$

$$N_x=\frac{Eh}{1-v^2}\left(\frac{\partial w_1}{\partial x}+\gamma\frac{\partial w_2}{\partial y}\right)+N_x^0$$

$$N_y=\frac{Eh}{1-v^2}\left(\frac{\partial w_2}{\partial y}+\gamma\frac{\partial w_1}{\partial x}\right)+N_y^0$$

$$N_{xy}=\frac{Eh}{2(1-v)}\left(\frac{\partial w_1}{\partial y}+\frac{\partial w_2}{\partial x}\right)+N_{xy}^0$$

$$M_x=-D\left(\frac{\partial^2 w_3}{\partial x^2}+\gamma\frac{\partial^2 w_3}{\partial y^2}\right)$$

$$M_y=-D\left(\frac{\partial^2 w_3}{\partial y^2}+\gamma\frac{\partial^2 w_3}{\partial x^2}\right)$$

$$M_{xy}=-D(1-v)\frac{\partial^2 w_3}{\partial x\partial y}$$

$$N_{fx}=\frac{Eh}{2(1-v^2)}\left[\left(\frac{\partial w_3}{\partial x}\right)^2+\gamma\left(\frac{\partial w_3}{\partial y}\right)^2\right]$$

$$N_{fy}=\frac{Eh}{2(1-v^2)}\left[\left(\frac{\partial w_3}{\partial y}\right)^2+\gamma\left(\frac{\partial w_3}{\partial x}\right)^2\right]$$

$$N_{fxy}=\frac{Eh(1-\gamma)}{2(1-v^2)}\frac{\partial w_3}{\partial x}\frac{\partial w_3}{\partial y}$$

式中 E 为弹性模量,v 为泊松比,$D=\dfrac{Eh^3}{12(1-v^2)}$为板的抗弯刚度,$h$ 为板的厚度,N_x^0,N_y^0,N_{xy}^0为作用在板上的面内荷载,w_1、w_2、w_3 分别为 x、y、z 的挠度. $\underset{-}{w}$ 为挠度向量.

采用 Timoshenko－Midlin 关于弹性薄板的有关假设[10],板上非中面上任意点的变形位移可用其对应中面上点的变形位移来表示

$$\underset{-}{u}(x,y,z,t)=\underset{-}{w}(x,y,t)+\underset{-}{\Delta} \quad (7)$$

其中 $\underset{-}{\Delta}$ 为中面变形位移之间的耦合变形,表示为

$$\underset{-}{\Delta}=\begin{bmatrix}-z\dfrac{\partial w_3(x,y,t)}{\partial x}+\dfrac{1}{2}\displaystyle\int_0^x\left(\frac{\partial w_3(\xi,y,t)}{\partial\xi}\right)^2\mathrm{d}\xi\\ -z\dfrac{\partial w_3(x,y,t)}{\partial y}+\dfrac{1}{2}\displaystyle\int_0^y\left(\frac{\partial w_3(x,\eta,t)}{\partial\eta}\right)^2\mathrm{d}\eta\\ 0\end{bmatrix}$$

将(7)式变分可得

$$\delta\underset{-}{u}(x,y,z,t)=\delta\underset{-}{w}(x,y,t)+\delta\underset{-}{\Delta} \quad (8)$$

其中

$$\delta\underset{-}{\Delta}=\begin{bmatrix}-z\delta\left(\dfrac{\partial w_3}{\partial x}\right)+\dfrac{\partial w_3}{\partial x}\delta w_3+\dfrac{1}{2}\displaystyle\int_0^x\left(\frac{\partial^2 w_3(\xi,y,t)}{\partial\xi^2}\right)\delta w_3\mathrm{d}\xi\\ -z\delta\left(\dfrac{\partial w_3}{\partial y}\right)+\dfrac{\partial w_3}{\partial y}\delta w_3+\dfrac{1}{2}\displaystyle\int_0^y\left(\frac{\partial^2 w_3(x,\eta,t)}{\partial\eta^2}\right)\delta w_3\mathrm{d}\eta\\ 0\end{bmatrix}$$

由 Hamilton 最小作用原理,有

$$\delta L = \int_1^{t2}(\delta t - \delta \Pi + \delta W) = 0 \tag{9}$$

式中的 δW 为外力所作的虚功. 将式(5)、(6)代入式(9),则可得弹性薄板动力学控制方程为

$$\delta L = \int_1^{t2}\{\int_m[\ddot{\underline{u}} + 2\underline{\tilde{\omega}}\dot{\underline{u}} + (\dot{\underline{\tilde{\omega}}} - \underline{\hat{\omega}})\underline{u} + (\dot{\underline{\tilde{\omega}}} - \underline{\hat{\omega}})\underline{\rho}_0 + \dot{\underline{v}}_0 + \underline{\tilde{\omega}}\underline{v}_0]\delta\underline{u}\mathrm{d}m - \int_A(\underline{N} + \underline{N}_f)\delta\underline{w}\mathrm{d}A + \int_m\underline{f}\delta\underline{w}\mathrm{d}m\}\mathrm{d}t = 0 \tag{10}$$

式中 $\underline{f}$ 为作用在薄板上的外力,若令 $\underline{u} = \underline{w}$,则上式为传统建模方法并考虑几何非线性的动力学方程.

考虑变形与大范围运动引起大的耦合并忽略高阶非线性项,动力学方程可化为

$$\ddot{\underline{w}} - \frac{h^2}{12}\underline{L}_1\ddot{\underline{w}} + 2\underline{\tilde{\omega}}\dot{\underline{w}} - \frac{h^2}{6}\omega^2\underline{L}_2\dot{\underline{w}} + (\dot{\underline{\tilde{\omega}}} + \underline{\hat{\omega}})\underline{w} + \frac{1}{\rho h}(\underline{L}_3 - \underline{N} - \underline{N}_f + (\dot{\underline{\tilde{\omega}}} - \underline{\hat{\omega}})\underline{\rho}_0 + \dot{\underline{v}}_0 + \underline{\tilde{\omega}}\underline{v}_0 - \underline{N}^0) + \underline{f} = 0 \tag{11}$$

其中

$$\underline{L}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \end{bmatrix}$$

$$\underline{L}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \end{bmatrix}$$

$$\underline{L}_3 = \begin{bmatrix} 0 \\ 0 \\ \frac{\partial}{\partial x}(\frac{\partial w}{\partial x}F^{0x}) + \frac{\partial}{\partial y}(\frac{\partial w}{\partial y}F^{0y}) \end{bmatrix}$$

$$F^{0x} = -\int_0^x \rho h(-x\omega^2 + \dot{v}_{01} + \omega v_{03})\mathrm{d}\xi - \frac{\rho h^3}{12}\omega^2$$

$$F^{0y} = -\int_0^y \rho h(y\omega^2 + \dot{v}_{02})\mathrm{d}\xi - \frac{\rho h^3}{12}\omega^2$$

这就是弹性薄板作一维高速转动时耦合非线性动力学方程. 其中第 1、3、5、7 及 9 以后的项为传统建模方法所得到的只考虑中面变形与一维高速转动之间的耦合项,第一项为中面轴向振动,第三项为陀螺力,第五项为中面变形与一维高速转动之间的耦合所引起的惯性力,这一项使系统产生了一个随一维高速转动速度增加而增加的负动力刚度项,在传统的建模方法中系统将成为一负刚度系统,导致失稳发散;第 2、4、6 项是中面耦合变形与一维高速转动之间的耦合项;第 8 项为几何非线性带入的非线性项.

2 算例

2.1 四边简支弹性薄板的离散动力学方程

现以四边简支弹性薄板为例,在浮动坐标系下,采用一阶模态动力学控制方程(11)进行 Garlerkin 模态截断,设变形位移形式为

$$\underline{w} = \underline{q}(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \tag{12}$$

代入(11)式可得系统的振动方程为

$$\ddot{\underline{q}} + \underline{C}\dot{\underline{q}} + \bar{\underline{K}}\underline{q} + \bar{\underline{K}}_f\underline{q}^3 + \underline{G} + \frac{2}{\rho h}\underline{f} = 0 \tag{13}$$

其中

$$\underline{C} = \begin{bmatrix} 0 & -2\omega & 0 \\ 2\omega & 0 & 0 \\ 0 & 0 & \frac{\alpha h^2\pi}{6}(\frac{1}{a^2} + \frac{1}{b^2}) \end{bmatrix}$$

$$\underline{K}_f^T = [0\ 0\ \frac{3\pi^4}{16}\sqrt{\frac{E}{\rho(1-v^2)}}(\frac{1}{a^4} + \frac{1}{b^4} + \frac{2v-1}{3a^2b^2})]$$

$$\underline{K} = \begin{bmatrix} \sqrt{\frac{E}{\rho(1-v^2)}}\frac{\pi^2}{ab}(\frac{b}{a} + \frac{a(1-v)}{2b}) - \omega^2 & 0 & 0 \\ 0 & \sqrt{\frac{E}{\rho(1-v^2)}}\frac{\pi^2}{ab}(\frac{b}{a} + \frac{a(1-v)}{2b}) - \omega^2 & 0 \\ 0 & 0 & k_{33} \end{bmatrix}$$

$$k_{33} = \alpha\{\frac{\pi^4h^2}{12}\sqrt{\frac{E}{\rho(1-v^2)}}(\frac{1}{a^2} + \frac{1}{b^2})^2 + \frac{\pi^2}{\rho h}(\frac{N_x^0}{a^2} + \frac{N_y^0}{b^2}) - \omega^2 + [\frac{\pi^2 + \pi - 2}{2} - \frac{h^2\pi^2}{12}(\frac{1}{a^2} + \frac{1}{b^2})]\omega^2\}$$

$$\frac{1}{\alpha} = 1 + \frac{h^2\pi^2}{12}(\frac{1}{a^2} + \frac{1}{b^2})$$

$$\underline{G} = \begin{bmatrix} \frac{8}{\pi^2}(2\dot{v}_{01} + b\dot{\omega} + a\omega^2 - 2v_{02}\omega) \\ \frac{8}{\pi^2}(2\dot{v}_{02} - a\dot{\omega} + b\omega^2 - 2v_{03}\omega) \\ \frac{16\alpha}{\pi^2}\dot{v}_{03} + (\frac{\pi}{2a} + \frac{\pi}{2b} - \frac{\pi^2}{a^2} - \frac{\pi^2}{b^2})(\dot{v}_{01} - \omega v_{02}) \end{bmatrix}$$

其中 $\underline{K}_f^T$ 为 $\underline{K}_f$的转置矩阵.

2.2 考虑几何非线性和中面耦合变形的动力学响应

当转动角速度 $\omega < Min[\sqrt{\frac{E}{\rho(1-v^2)}}\pi\sqrt{\frac{1}{a^2} + \frac{1-v}{2b^2}},$

$\sqrt{\frac{E}{\rho(1-v^2)}}\pi\sqrt{\frac{1}{b^2}+\frac{1-v}{2b^2}}$]时,三个方向的振动刚度均为正,且用 $\overline{\omega}_1^2$、$\overline{\omega}_2^2$、$\overline{\omega}_3^2$ 表示,对方程(13)无量纲化,引入

$$\begin{cases} x_1 = \dfrac{q_1}{a} \\ x_2 = \dfrac{q_2}{b} \\ x_3 = \dfrac{q_3}{h} \end{cases} \tag{14}$$

设无外激励和面内作用力,进行坐标变换,将上式代入方程(13)可得到无量纲化动力学方程为

$$\begin{cases} \ddot{x}_1 + \overline{\omega}_2^2 x_1 = 0 \\ \ddot{x}_2 + \overline{\omega}_2^2 x_2 = -\dfrac{2h\omega}{b}\dot{x}_3 \\ \ddot{x}_3 + \overline{\omega}_3^2 x_3 = \alpha[\dfrac{2a\omega}{h}\dot{x}_1 - \dfrac{h\pi^2}{6}(\dfrac{1}{a^2} + \\ \quad \dfrac{1}{b^2})\dot{x}_3 - \dfrac{a\dot{\omega}}{h}x_1 - K_{f3}h^2x_3^3] \end{cases} \tag{15}$$

其中

$$K_{f3} = \frac{3\pi^4}{16}\sqrt{\frac{E}{\rho(1-v^2)}}\left(\frac{1}{a^4}+\frac{1}{b^4}+\frac{2v-1}{3a^2b^2}\right)$$

从(15)式可知,板作高速转动时,对 x_1 方向的振动没有影响,x_2 方向振动影响较弱,可以看作 x_3 方向振动的扰动. 则在 x_3 方向的振动相平面上,相轨线为绕奇点(0,0)为中心的同心圆,因此系统是稳定系统.

2.3　实例仿真计算

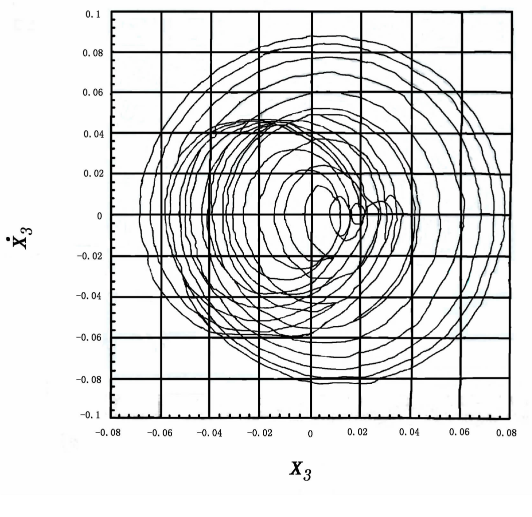

图 2　弹性薄板横向振动的相平面图

Fig. 2　Phase plane of transverse vibration of rotating elastic thin plate

以 $1m \times 0.5m$,厚度为 $2cm$ 的钢板为例,其转动角速度规律给定为

$$\omega = \begin{cases} \dfrac{2\pi n^*}{60t^*}(t - \dfrac{t^*}{2\pi}\sin(\dfrac{2\pi t}{t^*})) & t < t^* \\ \dfrac{2\pi n^*}{60} & t \geq t^* \end{cases}$$

其中 $t^* = 25s$,$n^* = 60rad/s$. 系统在 x_3 方向上振动的相平面和时间响应历程如下图所示

从耦合几何非线性动力学模型中横向振动的相平面图 2 中看出,奇点(0,0)是中心,相轨线为绕中心的同心圆,只是由于纵横振动的相互耦合,使相轨线稍有漂移,同时从横向振动的时间响应历程图 3 中可知,系统的横向振动的确是稳定的. 可见理论分析和仿真结果是吻合的.

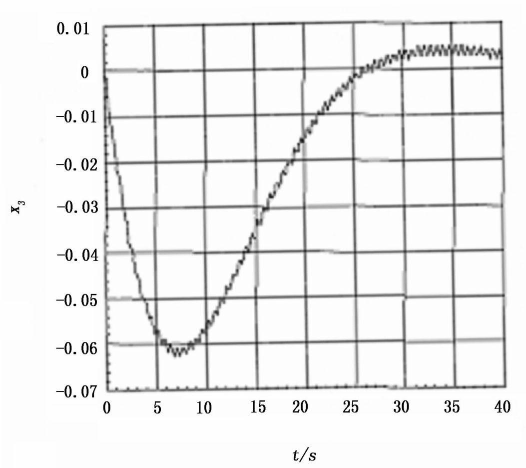

图 3　弹性薄板横向振动的时间响应历程图

Fig. 3　The history of transverse vibration of rotating elastic thin plate

3　结束语

本文从连续介质力学关于弹性薄板的变形理论和动力学入手,建立了一维转动弹性薄板考虑几何非线性和中面耦合非线性的动力学方程,也建立了该系统的离散动力学方程,通过理论分析和仿真计算,结果是吻合的.

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NONLINEAR ANALYSIS ON COUPLING DYNAMICS OF AXIAL ROTATION OF ELASTIC THIN PLATE *

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Abstract With the deformation theory on elastic thin plate in continuum mechanics, this paper investigated the dynamic properties of elastic thin plate rotating around an axis with large overall motions. In the absence of large overall motion, the effects of the deformation of elastic thin plate on the dynamic properties of the system are small and can be neglected. But if the deformation is coupled with large overall motion, its effects on the dynamic properties are significant. This paper established a geometrically nonlinear dynamic equation for elastic thin plate in the case of large overall motion with the strain-deformation geometrically nonlinear relation, and established a dynamic discrete equation of the system with Garlerkin's mode shapes method. Numerical simulation was given to verify the correctness of the theoretical analysis, which showed that the transverse vibration of the system was stable.

Key words axial rotation, thin plate, coupling dynamics, geometrically nonlinear

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