

## 2 - 自由度强非线性振动系统的参数识别\*

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**摘要** 提出了非线性多自由度系统的一种新的参数识别方法,研究了二次非线性的2-自由度系统. 基于保守系统存在能量积分的特点,由系统的运动微分方程导出了哈密顿函数,并用它作为参数识别的数学模型. 利用系统自由振荡条件下相坐标测量值集合对系统的哈密顿函数进行拟合,并用最小二乘法进行参数识别. 不管系统非线性度的强弱如何,只要系统是保守的,这种方法就有效.

**关键词** 非线性多自由度系统,参数识别,哈密顿函数

### 引言

相对于线性模型而言,非线性模型能更好地描述动力系统的动力学行为,因而在近二十年中,非线性系统的建模、识别和控制一直是动力学研究的重要课题.

在参数识别方面,我们已经有充足的先验知识来写出系统的运动微分方程,只要我们根据实验数据确定了运动微分方程中的系数(参数),系统的数学模型就唯一地被确定了. 目前对于非线性参数识别有很多方法<sup>[1-14]</sup>.

本文中,我们将文献<sup>[8]</sup>的方法扩展到2-自由度非线性保守系统中,并基于非线性保守系统的能量积分建立一种新的参数识别方法. 这个参数识别方法是能识别系统所有参数的完整方法. 首先,将系统的运动微分方程进行一次积分,并从这些积分中得到哈密顿函数,将它作为参数识别的数学模型. 然后,我们让系统自由振荡并测定系统的相变量,再用相变量的测量值集合对哈密顿函数进行拟合. 因为哈密顿函数是待识别系统参数的线性函数,故可直接用最小二乘法辨别系统的参数.

### 1 参数识别原理

我们考虑以下的2-自由度保守非线性系统

$$\begin{cases} \ddot{u}_1 + \omega_1^2 u_1 = f_1(u_1, u_2, \boldsymbol{\alpha}) \\ \ddot{u}_2 + \omega_2^2 u_2 = f_2(u_1, u_2, \boldsymbol{\alpha}) \end{cases} \quad (1)$$

$f_1(u_1, u_2, \boldsymbol{\alpha})$ 和 $f_2(u_1, u_2, \boldsymbol{\alpha})$ 是关于 $u_1$ 和 $u_2$ 的非线性函数, $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, \dots, \alpha_n\}^T$ , $\omega_1, \omega_2$ 和 $\boldsymbol{\alpha}$ 是待识别的参数.

$N$ 自由度保守系统的一个重要特性是系统拥有 $N$ 个运动积分<sup>[5]</sup>. 因此,对于保守系统(1),存在两个运动积分

$$\begin{cases} J_1 = F_1(u_1, u_2; \dot{u}_1, \dot{u}_2; \omega_1, \omega_2, \boldsymbol{\alpha}) = c_1 \\ J_2 = F_2(u_1, u_2; \dot{u}_1, \dot{u}_2; \omega_1, \omega_2, \boldsymbol{\alpha}) = c_2 \end{cases} \quad (2)$$

其中, $c_1, c_2$ 是积分常量. 从这些运动积分中我们能获得系统(1)的哈密顿函数

$$H = H(J_1, J_2) = H(u_1, u_2; \dot{u}_1, \dot{u}_2; \omega_1, \omega_2, \boldsymbol{\alpha}) = h(c_1, c_2) \quad (3)$$

对于保守系统,在系统的运动过程中哈密顿量不变,因此我们可以用它作为系统参数识别的数学模型. 我们让系统(1)自由振荡,随着时间的变化测得相变量 $u_1, u_2, \dot{u}_1$ 和 $\dot{u}_2$ ,可以获得测量数据集 $\{u_i, u_2, \dot{u}_1, \dot{u}_2\} (i=1, 2, \dots, m)$ ,然后将这些数据代入哈密顿函数(3). 因为测量误差是不可避免的,当我们将测试数据 $\{u_i, u_2, \dot{u}_1, \dot{u}_2\} (i=1, 2, \dots, m)$ 代入方程(3)时引起的误差为

$$R_i = H(u_{1i}, u_{2i}; \dot{u}_{1i}, \dot{u}_{2i}; \omega_1, \omega_2, \boldsymbol{\alpha}) - h \quad i=1, 2, \dots, m \quad (4)$$

一般情况下,哈密顿函数是一个关于被识别参数 $\omega_1^2, \omega_2^2, \alpha_1, \dots, \alpha_n$ 的线性函数,因此误差 $R_i$ 是这些参数的线性函数. 我们可以使用最小二乘法来识别这些参数. 所有误差 $R_i$ 的平方和为

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$$R = \sum_{i=1}^m R_i^2 = \sum_{i=1}^m [H(u_{1i}, u_{2i}; \dot{u}_{1i}, \dot{u}_{2i}; \omega_1, \omega_2, \alpha) - h]^2 \quad (5)$$

使(5)为最小值的充分必要条件  $R$  是对参数  $\omega_1^2, \omega_2^2, \alpha_1, \dots, \alpha_n$  的偏微分为零,即

$$\frac{\partial R}{\partial \omega_1^2} = 0, \frac{\partial R}{\partial \omega_2^2} = 0, \frac{\partial R}{\partial \alpha_1} = 0, \frac{\partial R}{\partial \alpha_2} = 0, \dots, \frac{\partial R}{\partial \alpha_n} = 0 \quad (6)$$

这些方程被称为正则方程. 方程组(6)是参数  $\omega_1^2, \omega_2^2, \alpha_1, \dots, \alpha_n$  的线性方程. 根据数值分析理论, 方程组(6)的解是唯一的, 它们会使平方误差的和最小<sup>[16]</sup>. 通过解方程(6), 系统(1)的参数可以被识别出来.

## 2 二次非线性系统

二次非线性系统与很多物理系统相关, 如感应加速器、弹性钟摆、轮船、液体分界面的运动, 栅栏纵向与横向的连接振动等等<sup>[16]</sup>. 我们考虑下面一般的具有二次非线性的保守系统

$$\begin{cases} \ddot{u}_1 = -\omega_1^2 u_1 + \alpha_1 u_1^2 + \alpha_2 u_1 u_2 + \alpha_3 u_2^3 \\ \ddot{u}_2 = -\omega_2^2 u_2 + \alpha_4 u_1^2 + \alpha_5 u_1 u_2 + \alpha_6 u_2^2 \end{cases} \quad (7)$$

这里,  $\omega_1^2, \omega_2^2, \alpha_1, \alpha_2, \dots, \alpha_6$  是待识别的参数. 对方程组(7)进行能量积分, 得

$$\begin{cases} E = \frac{1}{2} \dot{u}_1^2 + f_1(\dot{u}_2) + \frac{1}{2} \omega_1^2 u_1^2 - \frac{1}{3} \alpha_1 u_1^3 - \\ \quad \frac{1}{2} \alpha_2 u_1^2 u_2 - \alpha_3 u_1 u_2^2 + f_2(u_2) \\ E = \frac{1}{2} \dot{u}_2^2 + f_3(\dot{u}_1) + \frac{1}{2} \omega_2^2 u_2^2 - \alpha_4 u_1^2 u_2 - \\ \quad \frac{1}{2} \alpha_5 u_1 u_2^2 - \frac{1}{3} \alpha_6 u_2^3 + f_4(u_1) \end{cases} \quad (8)$$

这里  $E$  是系统的能量,  $f_1(\dot{u}_2), f_3(\dot{u}_1), f_2(u_2)$  和  $f_4(u_1)$  是待定的积分函数. 比较以上两个能量积分, 得到积分函数

$$\begin{cases} f_1(\dot{u}_2) = \frac{1}{2} \dot{u}_2^2, f_2(u_2) = \frac{1}{2} \omega_2^2 u_2^2 - \frac{1}{3} \alpha_6 u_2^3 \\ f_3(\dot{u}_1) = \frac{1}{2} \dot{u}_1^2, f_4(u_1) = \frac{1}{2} \omega_1^2 u_1^2 - \frac{1}{3} \alpha_1 u_1^3 \end{cases} \quad (9)$$

和以下的关系

$$\alpha_4 = \frac{1}{2} \alpha_2, \quad \alpha_3 = \frac{1}{2} \alpha_5 \quad (10)$$

这样我们求得了系统(7)的能量, 即系统的哈密尔顿函数

$$H = \frac{1}{2} \dot{u}_1^2 + \frac{1}{2} \dot{u}_2^2 + \frac{1}{2} \omega_1^2 u_1^2 + \frac{1}{2} \omega_2^2 u_2^2 - \frac{1}{3} \alpha_1 u_1^3 - \frac{1}{2} \alpha_2 u_1^2 u_2 - \alpha_3 u_1 u_2^2 - \frac{1}{3} \alpha_6 u_2^3 \quad (11)$$

因为系统(7)是保守的, 能量守恒, 即

$$H = \frac{1}{2} \dot{u}_1^2 + \frac{1}{2} \dot{u}_2^2 + \frac{1}{2} \omega_1^2 u_1^2 + \frac{1}{2} \omega_2^2 u_2^2 - \frac{1}{3} \alpha_1 u_1^3 - \frac{1}{2} \alpha_2 u_1^2 u_2 - \alpha_3 u_1 u_2^2 - \frac{1}{3} \alpha_6 u_2^3 = \frac{1}{2} \dot{u}_{10}^2 + \frac{1}{2} \dot{u}_{20}^2 + \frac{1}{2} \omega_1^2 u_{10}^2 + \frac{1}{2} \omega_2^2 u_{20}^2 - \frac{1}{3} \alpha_1 u_{10}^3 - \frac{1}{2} \alpha_2 u_{10}^2 u_{20} - \alpha_3 u_{10} u_{20}^2 - \frac{1}{3} \alpha_6 u_{20}^3 \quad (12)$$

这里  $\dot{u}_{10} = \dot{u}_1(0), \dot{u}_{20} = \dot{u}_2(0), u_{10} = u_1(0), u_{20} = u_2(0)$  是系统速度和坐标的初值. 方程(12)能写成如下形式

$$H - h = \frac{1}{2} (\dot{u}_1^2 - \dot{u}_{10}^2) + \frac{1}{2} (\dot{u}_2^2 - \dot{u}_{20}^2) + \frac{1}{2} \omega_1^2 (u_1^2 - u_{10}^2) + \frac{1}{2} \omega_2^2 (u_2^2 - u_{20}^2) - \frac{1}{3} \alpha_1 (u_1^3 - u_{10}^3) - \frac{1}{2} \alpha_2 (u_1^2 u_2 - u_{10}^2 u_{20}) - \alpha_3 (u_1 u_2^2 - u_{10} u_{20}^2) - \frac{1}{3} \alpha_6 (u_2^3 - u_{20}^3) = 0 \quad (13)$$

选此函数作为非线性系统(7)参数识别的数学模型很明显,  $H - h$  是待识别参数  $\omega_1^2, \omega_2^2, \alpha_1, \alpha_2, \alpha_3, \alpha_6$  的线性函数, 因而我们可以用最小二乘法的方法决定这些参数. 为此, 我们有必要测量系统的相坐标. 在前面提到的初始条件下, 我们让系统(7)自由振荡并测得它的坐标  $(u_1, u_2)$  和速度  $(\dot{u}_1, \dot{u}_2)$ , 将数据  $\{u_{1i}, u_{2i}, \dot{u}_{1i}, \dot{u}_{2i}\} (i=1, 2, \dots, m)$  待入到方程(13), 由于测量误差的存在, 将测量值代入方程(13)后将产生如下的偏差

$$R_i = \frac{1}{2} (\dot{u}_{1i}^2 - \dot{u}_{10}^2) + \frac{1}{2} (\dot{u}_{2i}^2 - \dot{u}_{20}^2) + \frac{1}{2} \omega_1^2 (u_{1i}^2 - u_{10}^2) + \frac{1}{2} \omega_2^2 (u_{2i}^2 - u_{20}^2) - \frac{1}{3} \alpha_1 (u_{1i}^3 - u_{10}^3) - \frac{1}{2} \alpha_2 (u_{1i}^2 u_{2i} - u_{10}^2 u_{20}) - \alpha_3 (u_{1i} u_{2i}^2 - u_{10} u_{20}^2) - \frac{1}{3} \alpha_6 (u_{2i}^3 - u_{20}^3) \quad i=1, 2, \dots, m \quad (14)$$

求以上误差的平方和, 得到

$$R = \sum_{i=1}^m R_i^2 = \sum_{i=1}^m \left[ \frac{1}{2} (\dot{u}_{1i}^2 - \dot{u}_{10}^2) + \frac{1}{2} (\dot{u}_{2i}^2 - \dot{u}_{20}^2) + \frac{1}{2} \omega_1^2 (u_{1i}^2 - u_{10}^2) + \frac{1}{2} \omega_2^2 (u_{2i}^2 - u_{20}^2) - \frac{1}{3} \alpha_1 (u_{1i}^3 - u_{10}^3) - \frac{1}{2} \alpha_2 (u_{1i}^2 u_{2i} - u_{10}^2 u_{20}) - \alpha_3 (u_{1i} u_{2i}^2 - u_{10} u_{20}^2) - \frac{1}{3} \alpha_6 (u_{2i}^3 - u_{20}^3) \right]^2 \quad (15)$$

根据最小二乘法,使最小的正则方程为

$$\begin{aligned}
& \frac{1}{2} \left( \sum_{i=1}^m (u_{1i}^2 - u_{10}^2)^2 \right) \omega_1^2 + \frac{1}{2} \left( \sum_{i=1}^m (u_{2i}^2 - u_{20}^2) (u_{1i}^2 - u_{10}^2) \right) \omega_2^2 - \frac{1}{3} \left( \sum_{i=1}^m (u_{1i}^3 - u_{10}^3) (u_{1i}^2 - u_{10}^2) \right) \alpha_1 - \frac{1}{2} \left( \sum_{i=1}^m (u_{1i}^2 u_{2i} - \right. \\
& \quad \left. u_{10}^2 u_{20}) (u_{1i}^2 - u_{10}^2) \right) \alpha_2 - \left( \sum_{i=1}^m (u_{2i}^2 u_{1i} - u_{10}^2 u_{20}) (u_{1i}^2 - u_{10}^2) \right) \alpha_3 - \frac{1}{3} \left( \sum_{i=1}^m (u_{2i}^3 - u_{20}^3) (u_{1i}^2 - u_{10}^2) \right) \alpha_6 + \\
& \quad \frac{1}{2} \sum_{i=1}^m [(\dot{u}_{1i}^2 - \dot{u}_{10}^2) + (\dot{u}_{2i}^2 - \dot{u}_{20}^2)] (u_{1i}^2 - u_{10}^2) = 0 \\
& \frac{1}{2} \left( \sum_{i=1}^m (u_{1i}^2 - u_{10}^2) (u_{2i}^2 - u_{20}^2) \right) \omega_1^2 + \frac{1}{2} \left( \sum_{i=1}^m (u_{2i}^2 - u_{20}^2)^2 \right) \omega_2^2 - \frac{1}{3} \left( \sum_{i=1}^m (u_{1i}^3 - u_{10}^3) (u_{2i}^2 - u_{20}^2) \right) \alpha_1 - \frac{1}{2} \left( \sum_{i=1}^m (u_{1i}^2 u_{2i} - \right. \\
& \quad \left. u_{10}^2 u_{20}) (u_{2i}^2 - u_{20}^2) \right) \alpha_2 - \left( \sum_{i=1}^m (u_{2i}^2 u_{1i} - u_{20}^2 u_{10}) (u_{2i}^2 - u_{20}^2) \right) \alpha_3 - \frac{1}{3} \left( \sum_{i=1}^m (u_{2i}^3 - u_{20}^3) (u_{2i}^2 - u_{20}^2) \right) \alpha_6 + \\
& \quad \frac{1}{2} \sum_{i=1}^m [(\dot{u}_{1i}^2 - \dot{u}_{10}^2) + (\dot{u}_{2i}^2 - \dot{u}_{20}^2)] (u_{2i}^2 - u_{20}^2) = 0 \\
& \frac{1}{3} \left( \sum_{i=1}^m (u_{1i}^2 - u_{10}^2) (u_{1i}^3 - u_{10}^3) \right) \omega_1^2 + \frac{1}{3} \left( \sum_{i=1}^m (u_{2i}^2 - u_{20}^2) (u_{1i}^3 - u_{10}^3) \right) \omega_2^2 - \frac{2}{9} \left( \sum_{i=1}^m (u_{1i}^3 - u_{10}^3)^2 \right) \alpha_1 - \frac{1}{3} \left( \sum_{i=1}^m (u_{1i}^2 u_{2i} - \right. \\
& \quad \left. u_{10}^2 u_{20}) (u_{1i}^3 - u_{10}^3) \right) \alpha_2 - \frac{2}{3} \left( \sum_{i=1}^m (u_{2i}^2 u_{1i} - u_{10}^2 u_{20}) (u_{1i}^3 - u_{10}^3) \right) \alpha_3 - \frac{2}{9} \left( \sum_{i=1}^m (u_{2i}^3 - u_{20}^3) (u_{1i}^3 - u_{10}^3) \right) \alpha_6 + \\
& \quad \frac{1}{3} \sum_{i=1}^m [(\dot{u}_{1i}^2 - \dot{u}_{10}^2) + (\dot{u}_{2i}^2 - \dot{u}_{20}^2)] (u_{1i}^3 - u_{10}^3) = 0 \\
& \frac{1}{2} \left( \sum_{i=1}^m (u_{1i}^2 - u_{10}^2) (u_{1i}^2 u_{2i} - u_{10}^2 u_{20}) \right) \omega_1^2 + \frac{1}{2} \left( \sum_{i=1}^m (u_{2i}^2 - u_{20}^2) (u_{1i}^2 u_{2i} - u_{10}^2 u_{20}) \right) \omega_2^2 - \frac{1}{3} \left( \sum_{i=1}^m (u_{1i}^3 - u_{10}^3) (u_{1i}^2 u_{2i} - \right. \\
& \quad \left. u_{10}^2 u_{20}) \right) \alpha_1 - \frac{1}{2} \left( \sum_{i=1}^m (u_{1i}^2 u_{2i} - u_{10}^2 u_{20})^2 \right) \alpha_2 - \left( \sum_{i=1}^m (u_{2i}^2 u_{1i} - u_{10}^2 u_{20}) (u_{1i}^2 u_{2i} - u_{10}^2 u_{20}) \right) \alpha_3 - \frac{1}{3} \left( \sum_{i=1}^m (u_{2i}^3 - \right. \\
& \quad \left. u_{20}^3) (u_{1i}^2 u_{2i} - u_{10}^2 u_{20}) \right) \alpha_6 + \frac{1}{2} \sum_{i=1}^m [(\dot{u}_{1i}^2 - \dot{u}_{10}^2) + (\dot{u}_{2i}^2 - \dot{u}_{20}^2)] (u_{1i}^2 u_{2i} - u_{10}^2 u_{20}) = 0 \\
& \left( \sum_{i=1}^m (u_{1i}^2 - u_{10}^2) (u_{2i}^2 u_{1i} - u_{20}^2 u_{10}) \right) \omega_1^2 + \left( \sum_{i=1}^m (u_{2i}^2 - u_{20}^2) (u_{2i}^2 u_{1i} - u_{20}^2 u_{10}) \right) \omega_2^2 - \frac{2}{3} \left( \sum_{i=1}^m (u_{1i}^3 - u_{10}^3) (u_{2i}^2 u_{1i} - \right. \\
& \quad \left. u_{20}^2 u_{10}) \right) \alpha_1 - \left( \sum_{i=1}^m (u_{1i}^2 u_{2i} - u_{10}^2 u_{20}) (u_{2i}^2 u_{1i} - u_{20}^2 u_{10}) \right) \alpha_2 - 2 \left( \sum_{i=1}^m (u_{2i}^2 u_{1i} - u_{20}^2 u_{10})^2 \right) \alpha_3 - \frac{2}{3} \left( \sum_{i=1}^m (u_{2i}^3 - u_{20}^3) \times \right. \\
& \quad \left. (u_{2i}^2 u_{1i} - u_{20}^2 u_{10}) \right) \alpha_6 + \sum_{i=1}^m [(\dot{u}_{1i}^2 - \dot{u}_{10}^2) + (\dot{u}_{2i}^2 - \dot{u}_{20}^2)] (u_{2i}^2 u_{1i} - u_{20}^2 u_{10}) = 0 \\
& \frac{1}{3} \left( \sum_{i=1}^m (u_{1i}^2 - u_{10}^2) (u_{2i}^3 - u_{20}^3) \right) \omega_1^2 + \frac{1}{3} \left( \sum_{i=1}^m (u_{2i}^2 - u_{20}^2) (u_{2i}^3 - u_{20}^3) \right) \omega_2^2 - \frac{2}{9} \left( \sum_{i=1}^m (u_{1i}^3 - u_{10}^3) (u_{2i}^3 - u_{20}^3) \right) \alpha_1 - \\
& \quad \frac{1}{3} \left( \sum_{i=1}^m (u_{1i}^2 u_{2i} - u_{10}^2 u_{20}) (u_{2i}^3 - u_{20}^3) \right) \alpha_2 - \frac{2}{3} \left( \sum_{i=1}^m (u_{2i}^2 u_{1i} - u_{20}^2 u_{10}) (u_{2i}^3 - u_{20}^3) \right) \alpha_3 - \frac{2}{9} \left( \sum_{i=1}^m (u_{2i}^3 - u_{20}^3)^2 \right) \alpha_6 + \\
& \quad \frac{1}{3} \sum_{i=1}^m [(\dot{u}_{1i}^2 - \dot{u}_{10}^2) + (\dot{u}_{2i}^2 - \dot{u}_{20}^2)] (u_{2i}^3 - u_{20}^3) = 0 \tag{16}
\end{aligned}$$

这些方程是参数  $\omega_1^2, \omega_2^2, \alpha_1, \alpha_2, \alpha_3, \alpha_6$  的线性方程. 根据数值分析理论, 正则方程的系数矩阵是可逆的, 因此它的解是唯一的, 此解能使误差达到最小<sup>[16]</sup>. 通过解方程(16), 参数值  $\omega_1^2, \omega_2^2, \alpha_1, \alpha_2, \alpha_3, \alpha_6$  即被确定. 因此非线性系统(7)的参数得到识别.

### 3 总结

本文提出了一个基于哈密尔顿函数的保守非线性系统的参数识别方法. 通过这种方法, 对具有二次非线性的 2-自由度保守系统的参数进行了识别. 以

上的讨论表明不管系统非线性的强弱, 对于保守系统这种方法总是有效的. 而且, 很明显这种方法能被用于一般的非线性 2-自由度和多自由度保守系统.

这种方法是能量积分和最小二乘方法的合成. 因为能量积分是一种解析运算, 不会产生任何误差, 本方法的准确性完全依靠于最小二乘法的精度.

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## PARAMETER IDENTIFICATION OF STRONGLY NONLINEAR VIBRATION SYSTEMS OF 2 – DOF \*

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**Abstract** A new parameter identification method for nonlinear multi – degree – of – freedom systems was presented. The two – degree – of – freedom systems with quadratic or cubic nonlinearities were studied. Based on the property that there exists energy integral in conservative systems, the Hamiltonian was derived and selected as the mathematic model of parameter identification. Hamiltonian function was fitted with the test data, which were the value sets of phase coordinates measured in free oscillation of the systems, and the parameters were identified with the least square method. No matter the nonlinearity of the system is strong or weak, the presented technique is valid as long as the system is conservative.

**Key words** nonlinear multi – degree – of – freedom systems, parameter identification, Hamiltonian function

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