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梁摆系统耦合振动分析*

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摘要 分析了梁摆系统的耦合振动,梁和摆均考虑为线性. 研究发现该系统含有非线性动力行为,在某些条件下会发生叉形分叉. 用结构动力学理论建立了梁摆系统的耦合振动方程,用摄动法求出了系统的近似解,分析了该系统的动力响应及分叉. 最后用 MATHMATIC 软件对分叉点前后动力响应进行分析.

关键词 耦合振动,分叉,摄动法

前言

随着科学技术的发展,工业技术和工程技术的要求,无论机械构造和建筑结构都日趋复杂化、精细化,这使得对系统的各种分析更加困难,对建立分析模型也要求更加逼近于真实情况. 模型建立逼真如否直接关系着动力分析的精度,特别是在动力分析中,过于简化的模型甚至会导致动力特性的丢失. 当然了,太过复杂的模型又会带来数学研究的不便.

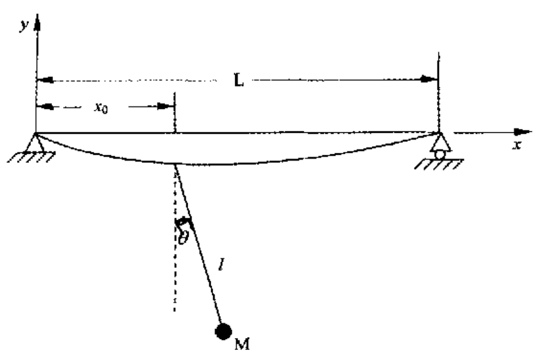

图 1 梁摆模型

Fig. 1 The pendulum and beam system

对于梁的研究及索的研究文献很多,在结构桥梁工程的研究方面,很多科学工作者研究了梁与质点系(包括静止和移动的质点系)之间的耦合振动问题[1-5]. 随着斜拉索桥的广泛应用,梁、索等刚柔耦合振动问题更加引起科研工作者的注意,特别是其中存在大量的非线性问题,都有待于进行研究.

本文研究梁摆系统的耦合振动,梁和摆均考虑为线性,梁摆模型如图 1 所示.

1 梁摆系统的耦合振动方程

对质量块进行受力分析,可得径向运动方程:

$$T - Mg\cos\theta = Ml\dot{\theta}^2 + M\ddot{Y}_1\cos\theta \tag{1}$$

即

$$T = Ml\dot{\theta}^2 + M\ddot{Y}_1\cos\theta + Mg\cos\theta \tag{2}$$

式中为索的拉力,M 为质量块的质量 $Y1$ 为摆悬挂点处梁的竖向位移,l 为索长. θ 为摆偏离平衡位置的角度.

质量块切向运动方程为

$$Ml\ddot{\theta} + M(g + \ddot{Y}_1)\sin\theta = 0 \tag{3}$$

对于梁运动方程

$$\rho A\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} = \delta(x - x_0)T\cos\theta \tag{4}$$

式中 ρ 为梁的线密度,A 为梁截面面积,w 为梁的竖向位移函数,δ 为阶跃函数,取

$$w = \varphi(t)\sin\frac{\pi x}{L} \tag{5}$$

则

$$Y_1 = \sin\frac{\pi x_0}{L}\varphi(t) \tag{6}$$

$$\ddot{Y}_1 = \sin\frac{\pi x_0}{L}\ddot{\varphi}(t) \tag{7}$$

代入式(4)

$$\rho A\ddot{\varphi}(t)\sin\frac{\pi x}{L} + EI\left(\frac{\pi}{L}\right)^4\varphi(t)\sin\frac{\pi x}{L} =$$

$$\delta(x - x_0)\cos\theta\left[Ml\dot{\theta}^2 + M\left(g + \ddot{\varphi}(t)\sin\frac{\pi x_0}{L}\right)\cos\theta\right] \tag{8}$$

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对(8)式两边同时乘以 $\sin\dfrac{\pi x}{L}$ 并对 x 在区间 $[0,l]$ 积分得

$$\rho A\ddot{\varphi}(t)+EI\left(\frac{\pi}{L}\right)^4\varphi(t)=$$
$$\cos\theta\left[Ml\dot{\theta}^2+M\left(g+\ddot{\varphi}(t)\sin\frac{\pi x_0}{L}\right)\cos\theta\right]\sin\frac{\pi x_0}{L} \tag{9}$$

研究 θ 为小角度情况,可使用简化

$\sin\theta\approx\theta,\cos\theta\approx 1$

则(9)式可写为

$$\rho A\ddot{\varphi}(t)+EI\left(\frac{\pi}{L}\right)^4\varphi(t)=Ml\sin\frac{\pi x_0}{L}\dot{\theta}^2+$$
$$Mg\sin\frac{\pi x_0}{L}+M\left(\sin\frac{\pi x_0}{L}\right)^2\ddot{\varphi}(t)$$

即

$$\left[\rho A-M\left(\sin\frac{\pi x_0}{L}\right)^2\right]\ddot{\varphi}(t)+EI\left(\frac{\pi}{L}\right)^4\varphi(t)=$$
$$Ml\sin\frac{\pi x_0}{L}\dot{\theta}^2+Mg\sin\frac{\pi x_0}{L} \tag{10}$$

也可写成

$$\ddot{\varphi}(t)=\frac{1}{\rho A-M\left(\sin\frac{\pi x_0}{L}\right)^2}\times$$
$$\left[M\sin\frac{\pi x_0}{L}(g+l\dot{\theta}^2)-EI\left(\frac{\pi}{L}\right)^4\varphi(t)\right] \tag{11}$$

将(7)代入(3)式可得

$$\ddot{\theta}+\frac{1}{l}\left(g+\sin\frac{\pi x_0}{L}\ddot{\varphi}(t)\right)\theta=0 \tag{12}$$

将(11)代入到(12)式即得

$$\ddot{\theta}+\frac{1}{l}\left(g+\sin\frac{\pi x_0}{L}\left\{\frac{1}{\rho A-M\left(\sin\frac{\pi x_0}{L}\right)^2}\left[M\sin\frac{\pi x_0}{L}(g+l\dot{\theta}^2)-EI\left(\frac{\pi}{L}\right)^4\varphi(t)\right]\right\}\right)\theta=0$$

整理得

$$\ddot{\theta}+\left(\frac{g}{l}+\frac{M\left(\sin\frac{\pi x_0}{L}\right)^2}{\rho A-M\left(\sin\frac{\pi x_0}{L}\right)^2}\right)\theta+\frac{\sin\frac{\pi x_0}{L}}{\rho A-M\left(\sin\frac{\pi x_0}{L}\right)^2}\left[M\sin\frac{\pi x_0}{L}l\dot{\theta}^2\theta-EI\left(\frac{\pi}{L}\right)^4\varphi\theta\right]=0$$

令

$$\omega_1^2=\frac{EI}{\rho A-M\left(\sin\frac{\pi x_0}{L}\right)^2}\left(\frac{\pi}{L}\right)^4,$$

$$\alpha_1=\frac{Ml\sin\frac{\pi x_0}{L}}{\rho A-M\left(\sin\frac{\pi x_0}{L}\right)^2},\quad \alpha_2=\frac{Mg\sin\frac{\pi x_0}{L}}{\rho A-M\left(\sin\frac{\pi x_0}{L}\right)^2}$$

$$\omega_2^2=\left(\frac{g}{l}+\frac{M\left(\sin\frac{\pi x_0}{L}\right)^2}{\rho A-M\left(\sin\frac{\pi x_0}{L}\right)^2}\right),$$

$$\beta_1=\alpha_1\sin\frac{\pi x_0}{L},\quad \beta_2=\omega_1^2\sin\frac{\pi x_0}{L}$$

则系统方程可写成

$$\ddot{\varphi}+\omega_1^2\varphi=\alpha_1\dot{\theta}^2+\alpha_2 \tag{13}$$
$$\ddot{\varphi}+\omega_2^2\theta=\beta_1\dot{\theta}^2\theta+\beta_2\varphi\theta \tag{14}$$

令

$$\phi=\varphi-\frac{\alpha_2}{\omega_1^2} \tag{15}$$

则方程(21)、(22)可写为

$$\ddot{\phi}+\omega_1^2\phi=\alpha_1\dot{\theta}^2 \tag{16}$$
$$\ddot{\theta}+\left(\omega_2^2-\frac{\alpha_2\beta_2}{\omega_1^2}\right)\theta=\beta_1\dot{\theta}^2\theta+\beta_2\phi\theta \tag{17}$$

2 耦合振动方程的求解

采用多尺度方法[6-9]求解梁摆系统的耦合振动方程,设

$$\varphi=\varphi_0(T_0,T_1)+\varepsilon\varphi_1(T_0,T_1)+\cdots \tag{18}$$
$$\theta=\theta_0(T_0,T_1)+\varepsilon\theta_1(T_0,T_1)+\cdots \tag{19}$$

其中 $T_n=\varepsilon^n t$,则

$$\frac{d}{dt}=\frac{\partial}{\partial T_0}+\varepsilon\frac{\partial}{\partial T_1}=D_0+\varepsilon D_1,$$
$$\frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0D_1+\varepsilon^2D_1 \tag{20}$$

将(18)-(20)式代入(16)、(17)式并令 ε 的各幂次系数为零可得

$$\varepsilon^0:\quad D_0^2\phi_0+\omega_1^2\phi_0=0$$
$$D_0^2\theta_1+\left(\omega_2^2-\frac{\alpha_2\beta_2}{\omega_1^2}\right)\theta_1=0 \tag{21}$$

$$\varepsilon^1:\quad D_0^2\phi_1+\omega_1^2\phi_1=\alpha_1(D_0\theta_0)^2-2D_0D_1\phi_0$$

$$D_0^2\theta_1+\left(\omega_2^2-\frac{\alpha_2\beta_2}{\omega_1^2}\right)\theta_1=$$
$$\beta_1(D_0\theta_0)^2\theta_0+\beta_2\phi_0\theta_0-2D_0D_1\phi_0 \tag{22}$$

方程(21)的解形式可设为:

$$\phi_0=A_1(T_1)e^{i\omega_1T_0}+\bar{A}_1(T_1)e^{-i\omega_1T_0}$$
$$\theta_0=A_2(T_1)e^{i\omega_2T_0}+\bar{A}_2(T_1)e^{-i\omega_2T_0} \tag{23}$$

将(23)式代入到(22)式可得

$$D_0^2\phi_1+\omega_1^2\phi_1=\alpha_1(-A_2^2\omega_2^2e^{2i\omega_2T_0}+A_2\bar{A}_2)-2i\omega_1D_1A_1e^{i\omega_1T_0}+cc \tag{24}$$

$$D_0^2\theta_1+\omega_2^2\theta_1=\beta_1(-\omega_2^2A_2^3e^{3i\omega_2T_0}+\omega_2^2A_2^2\bar{A}_2e^{i\omega_2T_0})+\beta_2(A_1A_2e^{i(\omega_1+\omega_2)T_0}+A_1\bar{A}_2e^{i(\omega_1-\omega_2)T_0})-2i\omega_2D_1A_2e^{i\omega_2T_0}+cc \tag{25}$$

2.1　非共振情况

消除(29)、(30)式中的永年项可得

$$\begin{cases}D_1A_1=0\\ \beta_1\omega_2^2A_2^2\bar{A}_2-2i\omega_2D_1A_2\end{cases} \tag{26}$$

由第一式可得

$$A_1=const$$

令

$$A_2=a_2e^{i\psi_2}$$

其中 a_2、ψ_2 为关于 T_1 的实函数,代入第二式并分离实虚部可得

$$\begin{cases}\dfrac{da_2}{dT_1}=0\\ 2\omega_2a_2\dfrac{d\psi_2}{dT_1}+2\beta_1a_2^3\omega_2^2=0\end{cases} \tag{27}$$

解上式可得

$$\begin{cases}a_2=C_{20}\\ \psi_2=\beta_1\omega_2a_2^2T_1+C_{21}\end{cases} \tag{28}$$

其中实常数 C_{20}、C_{21} 由初始条件得到.

2.2　共振情况(内共振)

考虑 $2\omega_2\approx\omega_1$ 时,令

$$\omega_1=2\omega_2+\varepsilon\sigma,\quad \omega=\omega_2 \tag{29}$$

则

$$\begin{cases}D_0^2\phi_1+4\omega^2\phi_1=\alpha_1(D_0\theta_0)^2-2D_0D_1\phi_0-4\varepsilon\sigma\phi_0\\ D_0^2\theta_1+\omega^2\theta_1=\beta_1(D_0\theta_0)^2\theta_0+\beta_2\phi_0\theta_0-2D_0D_1\theta_0\end{cases} \tag{30}$$

其中

$$\begin{cases}\phi_0=A_1(T_1)e^{2i\omega T_0}+\bar{A}_1(T_1)e^{-2i\omega T_0}\\ \theta_0=A_2(T_1)e^{i\omega T_0}+\bar{A}_1(T_1)e^{-i\omega T_0}\end{cases} \tag{31}$$

式(36)代入式(35),消除永年项得

$$\begin{cases}\alpha_1A_2^2\omega^2+4i\omega D_1A_1+4\omega\sigma A_1=0\\ 2\beta_1\omega^2A_2^2\bar{A}_2-2i\omega D_1A_2+\beta_2A_1\bar{A}_2=0\end{cases} \tag{32}$$

令 $A_1=a_1e^{i\psi_1}$,$A_2=a_2e^{i\psi_2}$,代入上式即得

$$\begin{cases}4\omega\dfrac{da_1}{dT_1}+\alpha_1\omega^2a_2^2\sin(2\psi_2-\psi_1)=0\\ 4\omega a_1\dfrac{d\psi_1}{dT_1}-4\omega\sigma a_1-\alpha_1\omega^2a_2^2\cos(2\psi_2-\psi_1)=0\\ 2\omega\dfrac{da_2}{dT_1}+\beta a_1a_2\sin(2\psi_2-\psi_1)=0\\ 2\omega a_2\dfrac{d\psi_2}{dT_1}+2\beta_1\omega^2a_2^3+\beta_2a_1a_2\cos(2\psi_2-\psi_1)=0\end{cases} \tag{33}$$

对于稳态解有

$$\frac{da_1}{dT_1}=\frac{d\psi_1}{dT_1}=\frac{da_2}{dT_1}=\frac{d\psi_2}{dT_1}=0$$

即得

$$\sin(2\psi_2-\psi_1)=0,\cos(2\psi_2-\psi_1)=0$$

$$\begin{cases}4\omega\sigma a_1+\alpha_1\omega^2a_2^2=0\\ 2\beta_1\omega^2a_2^3+\beta_2a_1a_2\end{cases} \tag{34}$$

由于 a_1 是受系统外界扰动决定,分析上式可知当满足条件

$$\beta_1\beta_2<0 \tag{35}$$

a_2 存在的可能解有

$$a_2=0,\quad a_2=\sqrt{-\frac{\beta_2a_1}{2\beta_1\omega^2}} \tag{36}$$

即会发生叉形分叉. 这对摆的运动稳定性是不利的,因此需要避免.

为了对系统稳定性进行研究,借助 MATHMATIC 软件进行数值计算,图 2 为取系统参数 $\omega_1=2$,$\omega_2=1$,$\alpha_1=0.1$,$\beta_1=0.1$,$\beta_2=-3$,取初始条件 $\phi(0)=\dot{\phi}(0)=0$,$\theta(0)=1$,$\dot{\theta}(0)=0$,图 3 取参数 $\omega_1=2$,$\omega_2=1$,$\alpha_1=0.1$,$\beta_1=0.1$,$\beta_2=0.1$,取初始条件 $\phi(0)=\dot{\phi}(0)=0$,$\theta(0)=0.1$,$\dot{\theta}(0)=0$.

由图可以看出,当时 $\beta_1\beta_2<0$,梁、摆内共振表现出拍的共振特性,而当 $\beta_1\beta_2>0$,即使摆的初扰动很小,系统解仍然发散,即系统不稳定.

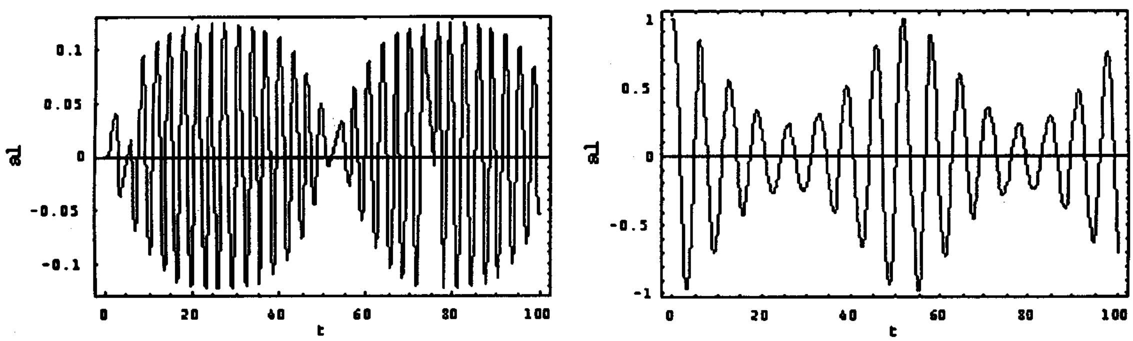

图 2 梁、摆稳定运动时间历程图

Fig. 2 Stable time - displacement curve

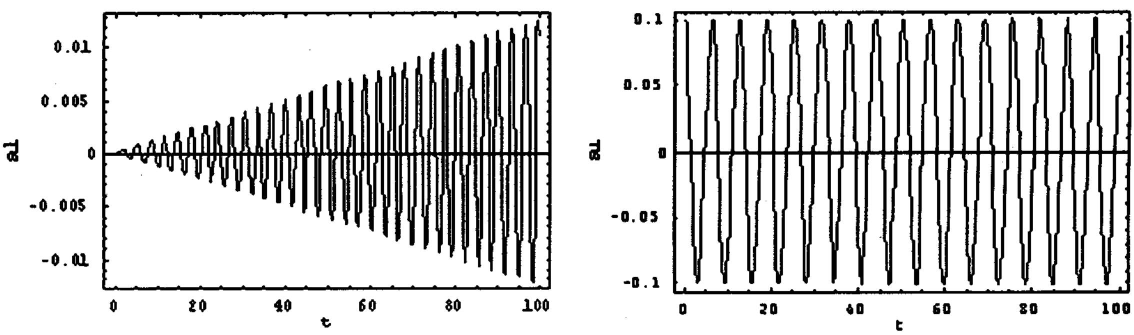

图 3 梁、摆不稳定运动时间历程图

Fig. 3 Stable time - displacement curve

3 结论

本文对连接点静止的梁摆动力系统的耦合振动进行了理论分析,得到该动力系统运动方程. 系统方程表明,尽管不考虑梁、摆的几何非线性和物理非线性,但耦合振动方程仍然是非线性问题,因此,系统存在非线性特性. 利用多尺度法对非线性耦合方程进行分析,分析表明,在满足内共振条件时,在一定条件下会发生叉形分岔,这对系统运动是不利的,在工程中应当避免. 同时,本文的理论分析对结构工程中研究该类系统破坏机理提供了一定的帮助,也为进一步分析该类系统的分叉控制奠定了基础.

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DYNAMICS ANALYSIS FOR COUPLED PENDULUM AND BEAM SYSTEM *

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Abstract The dynamical characteristics for the coupled pendulum and beam system were studied. The nonlinear characteristics, including geometrically nonlinear and physically nonlinear, were not considered in the beam and pendulum. But the nonlinear items were found in the coupled vibration equations derived from structure dynamics. Using the perturbation method, the dynamical response and bifurcation were studied. Using the software MATHMATIC, the responses of point near the bifurcation point were obtained.

Key words coupled vibration, bifurcation, perturbation method

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