第 4 卷第 3 期
2006 年 9 月

动 力 学 与 控 制 学 报
JOURNAL OF DYNAMICS AND CONTROL

Vol.4 No.3
Sep. 2006

非线性地基上正交异性矩形板的非线性热振动

杨立军 吴 晓

(湖南文理学院土木建筑工程系,常德 415000)

摘要 研究了非线性地基上正交异性矩形板的非线性固有热振动.采用常规的 L-P 法分析非线性地基上正交异性矩形板的非线性热振动难以得到高精度的近似解,为此,先对该强非线性振动系统进行参数变换,将该强非线性振动系统转化为弱非线性振动系统.然后采用改进的 L-P 法进行求解,得到了强非线性振动系统的高精度近似解.此外,讨论了温度、地基特征参数、长宽比等因素对非线性地基上正交异性矩形板非线性热振动固有频率的影响,得到了非线性地基上正交异性矩形板热振动频率随温度下降、地基特征参数变大、长宽比变大而增大的结论.

关键词 非线性,地基,正交异性,热振动

引言

在土木、建筑等工程中,许多正交异性矩形板处于热状态下的工作环境中,因此研究正交异性矩形板在非均匀热分布载荷作用下,温度升高对板的非线性热振动固有频率的影响是有实际工程意义的.一般经典的地基模型有双参数地基模型、Winkler 模型[1],但近年来许多研究结果表明[2],这两种模型不足以描写地基的力学性质.由于某些基础材料的应力——应变关系曲线在应变较大时会出现“软化”现象,一种非线性‘软化’弹性基础模型自然受到研究者的注意[3],本文以非线性基础上正交异性矩形板为例,研究了其非线性固有热振动.由于非线性地基上正交异性矩形板的非线性热振动是一个强非线性振动系统,因此采用常规的 L-P 法难以得到高精度的近似解.对强非线性振动系统高精度近似解的研究,一直是广大学者一个重要研究课题[4,5].本文根据文献[6]的方法对非线性地基上正交异性矩形板的非线性热振动方程进行参数变换,把强非线性振动方程转化为弱非线性振动方程,然后再进行求解即可得到高精度的近似解,并且讨论分析了温度、地基特征参数、长宽比等因素对非线性地基上正交异性矩形板非线性热振动固有频率的影响.

1 振动控制方程

非线性地基模型可以用下式描述为[7]

$$q(x,y,t) = k_1 w - G_P \nabla^2 w - k_2 w^3 \tag{1}$$

其中,k_1,G_P,k_2 为地基特征参数.

参阅文献[8 ~ 9]可以得到非线性地基上正交异性矩形板的非线性热振动方程为

$$L_1 w + \nabla^2 M^{\mathrm{T}} + k_1 w - G_P \nabla^2 w - k_2 w^3 + \rho h \frac{\partial^2 w}{\partial t^2} = L(w,\Phi) \tag{2}$$

$$L_2 \Phi + \nabla^2 N^{\mathrm{T}} = -\frac{h}{2} L(w,w) \tag{3}$$

其中

$$L() = \frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2} - 2\frac{\partial^2}{\partial x \partial y}\frac{\partial^2}{\partial x \partial y} + \frac{\partial^2}{\partial y^2}\frac{\partial^2}{\partial x^2};$$

$$L_1() = D_1 \frac{\partial^4}{\partial x^4} + 2D_3 \frac{\partial^4}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4}{\partial y^4};$$

$$L_2() = \frac{1}{E_2}\frac{\partial^4}{\partial x^4} + \left(\frac{1}{G} - \frac{2\mu_1}{E_1}\right)\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{1}{E_1}\frac{\partial^4}{\partial y^4};$$

$$\nabla^2 M^{\mathrm{T}} = \frac{\partial^2 M_x^{\mathrm{T}}}{\partial x^2} + \frac{\partial^2 M_y^{\mathrm{T}}}{\partial y^2};$$

$$\nabla^2 N^{\mathrm{T}} = \frac{1}{E_1}\left(\frac{\partial^2 N_x^{\mathrm{T}}}{\partial y^2} - \mu_1 \frac{\partial^2 N_x^{\mathrm{T}}}{\partial x^2}\right) + \frac{1}{E_2}\left(\frac{\partial^2 N_y^{\mathrm{T}}}{\partial x^2} - \mu_2 \frac{\partial^2 N_y^{\mathrm{T}}}{\partial y^2}\right);$$

2006-03-14 收到第 1 稿,2006-04-13 收到修改稿.

$$D_1 = \frac{E_1 h^3}{12(1-\mu_1\mu_2)},$$

$$D_2 = \frac{E_2 h^3}{12(1-\mu_1\mu_2)},$$

$$D_3 = \mu_2 D_1 + 2D_k,$$

$$D_k = \frac{Gh^3}{12}, \frac{\mu_1}{E_1} = \frac{\mu_2}{E_2}$$

D_1, D_2, D_k 分别为抗弯、抗扭刚度，E_1、E_2、μ_1 及 μ_2 分别是 x 轴和 y 轴方向的弹性模量和泊松比，G 为剪切弹性模量，ρ 为密度，h 为板厚，w 为横振位移，Φ 为应力函数，M^T、N^T 分别为热矩、热力.

假定正交异性矩形板四边为不可移简支，那么其边界条件为

$$x = 0, a \text{ 时}, w = \frac{\partial^2 w}{\partial x^2},$$

$$N_{xy} = 0, \Delta_x = 0 \tag{4}$$

$$y = 0, b \text{ 时}, w = \frac{\partial^2 w}{\partial y^2},$$

$$N_{xy} = 0, \Delta_y = 0 \tag{5}$$

板的端部伸长表达式为

$$\Delta_x = \frac{1}{b}\int_0^a\int_0^a \left[\frac{1}{h}\left(\frac{1}{E_1}\frac{\partial^2 \Phi}{\partial y^2} - \frac{\mu_2}{E_2}\frac{\partial^2 \Phi}{\partial x^2}\right) - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{h}\left(\frac{1}{E_1}N_x^T - \frac{\mu_2}{E_2}N_y^T\right)\right]\mathrm{d}x\mathrm{d}y \tag{6}$$

$$\Delta_y = \frac{1}{a}\int_0^a\int_0^a \left[\frac{1}{h}\left(\frac{1}{E_2}\frac{\partial^2 \Phi}{\partial x^2} - \frac{\mu_1}{E_1}\frac{\partial^2 \Phi}{\partial y^2}\right) - \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{1}{h}\left(\frac{1}{E_2}N_y^T - \frac{\mu_1}{E_1}N_x^T\right)\right]\mathrm{d}x\mathrm{d}y \tag{7}$$

其中，由文献[6] 可把沿和方向的热力及热矩表示为

$$\begin{cases} N_x^T = \dfrac{E_1(\alpha_1 + \mu_1\alpha_2)}{1-\mu_1\mu_2}\displaystyle\int_{-(h/2)}^{h/2} T(x,y,z)\mathrm{d}z \\ N_y^T = \dfrac{E_2(\alpha_2 + \mu_1\alpha_1)}{1-\mu_1\mu_2}\displaystyle\int_{-(h/2)}^{h/2} T(x,y,z)\mathrm{d}z \end{cases} \tag{8}$$

$$\begin{cases} M_x^T = \dfrac{E_1(\alpha_1 + \mu_1\alpha_2)}{1-\mu_1\mu_2}\displaystyle\int_{-(h/2)}^{h/2} ZT(x,y,z)\mathrm{d}z \\ M_y^T = \dfrac{E_2(\alpha_2 + \mu_1\alpha_1)}{1-\mu_1\mu_2}\displaystyle\int_{-(h/2)}^{h/2} ZT(x,y,z)\mathrm{d}z \end{cases} \tag{9}$$

取非均匀热分布为抛物线型

$$T(x,y,z) = T_0 + T_1 z + T_2 z^2 \tag{10}$$

取板的横振位移为

$$w(x,y,t) = T_{mn}(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{11}$$

把有关各式代入式(3) 中可得

$$\Phi(x,y,t) = \frac{hT_{mm}^2}{32}\left(\frac{E_2 n^2 a^2}{m^2 b^2}\cos\frac{2m\pi x}{a} + \frac{E_1 m^2 b^2}{n^2 a^2}\cos\frac{2n\pi y}{b}\right) + \frac{1}{2}P_x y^2 + \frac{1}{2}P_y x^2 \tag{12}$$

其中

$$P_x = \frac{\pi^2 E_1 h(m^2 b^2 + \mu_2 n^2 a^2)T_{mn}^2}{8(1-\mu_1\mu_2)a^2 b^2} - \frac{E_1 h(\alpha_1 + \mu_2\alpha_2)(T_0 + T_2 h^2/12)}{1-\mu_1\mu_2}$$

$$P_y = \frac{\pi^2 E_2 h(n^2 a^2 + \mu_1 m^2 b^2)T_{mn}^2}{8(1-\mu_1\mu_2)a^2 b^2} - \frac{E_2 h(\alpha_2 + \mu_1\alpha_1)(T_0 + T_2 h^2/12)}{1-\mu_1\mu_2}$$

再把有关各式代入式(2) 中并利用伽辽金原理得

$$\frac{\mathrm{d}^2 T_{mn}}{\mathrm{d}t^2} + \omega_0^2 T_{mn} + \beta T_{mn}^3 = 0 \tag{13}$$

式中

$$\omega_0 = \frac{\pi^4(D_1 m^4 + 2D_3 m^2 n^2\lambda^2 + D_2 n^4\lambda^4)}{\rho h a^4} + \frac{k_1}{\rho h} + \frac{\pi^2 G_P(m^2 + n^2\lambda^2)}{\rho h a^2} - \frac{\pi^2(T_0 + T_2 h^2/12)}{(1-\mu_1\mu_2)\rho a^2}[m^2(\alpha_1 + \mu_2\alpha_2)E_1 + n^2\lambda^2(\alpha_2 + \mu_1\alpha_1)E_2]$$

$$\beta = \frac{E_1 hm^2\pi^4(m^2 + \mu_2 n^2\lambda^2) + E_2 hn^2\pi^4\lambda^2(n^2\lambda^2 + \mu_1 m^2)}{8(1-\mu_1\mu_2)\rho h a^4} - \frac{9k_2}{16\rho h} + \frac{E_1 m^4\pi^4 + E_2 n^4\pi^4\lambda^4}{6\rho a4}$$

$$\lambda = \frac{a}{b}$$

2　分析方法及近似解

在式(13) 中由于 β 是一个较大的数，所以式(13) 是一个强非线性方程，因此可令 $\tau = \omega t$，把式(13) 化为

$$\omega^2\frac{\mathrm{d}^2 T_{mn}}{\mathrm{d}\tau^2} + \omega_0^2 T_{mn} + \beta T_{mn}^3 = 0 \tag{14}$$

设

$$\omega = \omega_0^2 + \beta\omega_1 + \beta^2\omega_2 + \cdots \quad (15)$$

引入变换参数

$$\varepsilon = \frac{\beta\omega_1}{\omega_0^2 + \beta\omega_1} \quad (16)$$

利用式(16)可将式(15)化为

$$\omega^2 = \frac{\omega^2}{1-\varepsilon}(1 + \varepsilon^2\alpha_2 + \varepsilon^2\alpha_3 + \cdots) \quad (17)$$

则

$$\omega = \omega_0[1 + \frac{1}{2}\varepsilon + \varepsilon^2(\frac{3}{8} + \frac{\alpha_2}{2}) + \cdots] \quad (18)$$

假设式(14)的初始条件为

$$\tau = 0, T_{mn}(0) = A, \frac{dT_{mn}(0)}{d\tau} = 0 \quad (19)$$

设

$$T_{mn}(\tau) = A\cos\tau + \varepsilon T_{mn}^{(1)}(\tau) + \varepsilon^2 T_{mn}^{(2)}(\tau) + \cdots \quad (20)$$

把式(17)和式(19)代入式(14),比较同次幂系数得

$$\frac{d^2T_{mn}^{(1)}}{d\tau^2} + T_{mn}^{(1)} = T_{mn}^{(0)} - \frac{1}{\omega_1}(T_{mn}^{(0)})^3 \quad (21)$$

$$\frac{d^2T_{mn}^{(2)}}{d\tau^2} + T_{mn}^{(2)} = -\alpha_2\frac{d^2T_{mn}^{(0)}}{d\tau^2} + T_{mn}^{(1)} - \frac{3}{\omega_1}(T_{mn}^{(0)})^2T_{mn}^{(1)} \quad (22)$$

设

$$T_{mn}^{(1)}(\tau) = B_0 + \sum_{i=2}^{\infty}B_i\cos i\tau + \sum_{i=1}^{\infty}B_i^*\sin i\tau \quad (23)$$

把式(23)代入式(21)中可得

$$B_0 + \sum_{i=2}^{\infty}(1-i^2)B_i\cos i\tau + \sum_{i=2}^{\infty}(1-i^2)B_i^*\sin i\tau = (a - \frac{3a^3}{4\omega_1})\cos\tau - \frac{a^3}{4\omega_1}\cos 3\tau \quad (24)$$

由式(24)得

$$\omega_1 = \frac{3A^2}{4}, B_3 = \frac{A}{24} \quad (25)$$

由初始条件可知

$$T_{mn}^{(1)}(\tau) = \frac{A}{24}(\cos 3\tau - \cos\tau) \quad (26)$$

设

$$T_{mn}^{(2)}(\tau) = C_0 + \sum_{i=2}^{\infty}C_i\cos i\tau + \sum_{i=2}^{\infty}C_i^*\sin i\tau \quad (27)$$

把式(27)代入式(22)中可以得到

$$C_0 + \sum_{i=2}^{\infty}(1-i^2)C_i\cos i\tau + \sum_{i=2}^{\infty}(1-i^2)C_i^*\sin i\tau = \alpha_2A\cos\tau + \frac{A}{24}(\cos 3\tau - \cos\tau) - \frac{A}{6}\cos^2\tau(\cos 3\tau - \cos\tau) \quad (28)$$

由式(28)可以求得

$$\alpha_2 = -\frac{1}{24}, C_5 = \frac{A}{576} \quad (29)$$

由初始条件可知

$$T_{mn}^{2}(\tau) = \frac{A}{576}(\cos 5\tau - \cos\tau) \quad (30)$$

所以式(13)的近似解为

$$\begin{cases} \omega = \omega_0(1 + \varepsilon/2 + 17\varepsilon^2/48) \\ \varepsilon = 3\beta A^2/(4\omega_0^2 + 3\beta A^2) \\ w(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}[A\cos\omega t + A(\cos 3\omega t - \cos\omega t)/24 + A(\cos 5\omega t - \cos\omega t)/576] \times \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \end{cases} \quad (31)$$

3 实例计算及讨论

为了分析温度、地基特征参数、长宽比等因素对正交异性矩形板固有热振动频率的影响,本文以混凝土矩形板为例进行计算. 具体参数为:$T_0 = 3T_2$, $a = 20$ m, $T_0 = 3T_2$, $\rho = 2.5\times10^3$ kg/m^3, $E_1 = 2.1\times10^8$ N/m^2, $E_2 = 4.2\times10^8$ N/m^2, $G = 0.82\times10^8$ N/m^2, $\mu_1 = 0.28$, $\mu_2 = 0.56$, $\alpha_1 = 12.7\times10^{-6}$ ℃, $\alpha_2 = 6.8\times10^{-6}$ ℃,具体计算结果见表1、表2、表3、表4(表中括号内数字为采用椭圆函数计算的非线性地基上正交异性矩形板非线性热振动固有频率).

对表1、表2进行分析,可以得出如下结论:

(l)非线性地基上混凝土矩形板固有热振动频率随着温度升高而下降,但变化幅度不大,这说明混凝土矩形板固有热振动频率对温度变化不是十分敏感.

(2)随着地基特征参数k_1, G_P, k_2的变大,正交异性矩形板固有热振动频率变大.

(3)长宽比λ越大,非线性地基上正交异性矩形板热振动频率也越大;且非线性地基上正交异性矩形板热振动呈‘硬弹簧”特性,即振幅增大,频率

也增大.

(4) 采用改进的 L-P 法计算的非线性地基上正交异性矩形板非线性热振动固有频率与采用椭圆函数计算的非线性地基上正交异性矩形板非线性热振动固有频率的误差非常小,这说明改进的 L-P 法计算精度非常高.同时,从本文的推导计算过程可以看出改进的 L-P 法计算简便,更易于工程技术人员掌握和应用.

表 1　$\lambda = 1$ 时正交异性矩形板固有热振动频率(单位:赫兹)

Table 1　The natural thermal vibration frequency of rectangular orthotropic plates when $\lambda = 1$(unit:Hz)

Characteristic parameter of foundation	A (m)	T_0 (℃)			
		0	4	8	12
$k_1 = 0$	0.025	4.6602(4.6602)	4.6206(4.6206)	4.5807(4.5807)	4.5404(4.5404)
$G_P = 0$	0.05	4.6688(4.6688)	4.6292(4.6292)	4.5894(4.5894)	4.5491(4.5491)
$k_2 = 0$	0.075	4.6830(4.6830)	4.6436(4.6436)	4.6038(4.6039)	4.5637(4.5638)
$k_1 = 3 \times 10^7$	0.025	173.2678(173.2678)	173.2667(173.2667)	173.2656(173.2656)	173.2646(173.2646)
$G_P = 0$	0.05	173.2687(173.2684)	173.2669(173.2669)	173.2659(173.2659)	173.2648(173.2648)
$k_2 = 0$	0.075	173.2684(173.2684)	173.2673(173.2673)	173.2663(173.2663)	173.2652(173.2652)
$k_1 = 3 \times 10^7$	0.025	178.8731(178.8731)	178.8721(178.8721)	178.8711(178.8711)	178.8732(178.8756)
$G_P = 4 \times 10^7$	0.05	178.8734(178.8734)	178.8723(178.8723)	178.8713(178.8713)	178.8703(178.8703)
$k_2 = 0$	0.075	178.8737(178.8737)	178.8727(178.8727)	178.8717(178.8717)	178.8706(178.8706)
$k_1 = 3 \times 10^7$	0.025	178.9321(178.9321)	178.9311(178.9311)	178.9359(178.9359)	178.9295(178.9294)
$G_P = 4 \times 10^7$	0.05	179.1093(179.1091)	179.1085(179.1081)	179.1072(179.1073)	179.1059(179.1062)
$k_2 = -8 \times 10^7$	0.075	179.4035(179.4037)	179.4024(179.4027)	179.4014(179.4017)	179.4004(179.4007)
$k_1 = 3 \times 10^7$	0.025	179.4616(179.462)	179.4606(179.4609)	179.4596(179.4599)	179.4585(179.4589)
$G_P = 4 \times 10^7$	0.05	181.2131(181.2189)	181.2121(181.2179)	181.2111(181.2169)	181.2187(181.2159)
$k_2 = -8 \times 10^8$	0.075	184.0819(184.1148)	184.0809(184.1138)	184.0799(184.1128)	184.0789(184.1118)

表 2　$\lambda = 1$ 时正交异性矩形板大挠度固有热振动频率(单位:赫兹)

Table 2　The natural thermal vibration frequency of rectangular orthotropic plates when $\lambda = 1$(unit:Hz)

Characteristic parameter of foundation	A (m)	T_0 (℃)			
		16	20	24	28
$k_1 = 0$	0.025	4.4997(4.49969)	4.4587(4.4587)	4.4172(4.4172)	4.3754(4.3754)
$G_P = 0$	0.05	4.5086(4.5086)	4.4676(4.4676)	4.4263(4.4263)	4.3846(4.3846)
$k_2 = 0$	0.075	4.5233(4.5233)	4.4825(4.4825)	4.4413(4.4413)	4.3997(4.3997)
$k_1 = 3 \times 10^7$	0.025	173.2635(173.2635)	173.2625(173.2625)	173.2614(173.2614)	173.2603(173.2603)
$G_P = 0$	0.05	173.2638(173.2638)	173.2627(173.2627)	173.2616(173.2616)	173.2606(173.2606)
$k_2 = 0$	0.075	173.2641(173.2641)	173.2631(173.2631)	173.2623(173.2623)	173.2616(173.2617)
$k_1 = 3 \times 10^7$	0.025	178.8692(178.8695)	178.8687(178.8685)	178.8679(178.8673)	178.8659(178.8659)
$G_P = 4 \times 10^7$	0.05	178.8692(178.8692)	178.8682(178.8682)	178.8672(178.8672)	178.8662(178.8662)
$k_2 = 0$	0.075	178.8696(178.8696)	178.8686(178.8686)	178.8676(178.8676)	178.8665(178.8665)
$k_1 = 3 \times 10^7$	0.025	178.9283(178.9285)	178.9269(178.9272)	178.9259(178.9259)	178.9249(178.9249)
$G_P = 4 \times 10^7$	0.05	179.1049(179.105)	179.1039(179.1039)	179.1029(179.1029)	179.1018(179.1019)
$k_2 = -8 \times 10^7$	0.075	179.3994(179.3996)	179.3983(179.3986)	179.3973(179.3976)	179.3963(179.3966)
$k_1 = 3 \times 10^7$	0.025	179.4575(179.4579)	179.4565(179.4568)	179.4555(179.4558)	179.4544(179.4548)
$G_P = 4 \times 10^7$	0.05	181.209(181.2149)	181.208(181.2138)	181.207(181.2128)	181.206(181.2118)
$k_2 = -8 \times 10^7$	0.075	184.0779(184.1108)	184.0769(184.1098)	184.0759(184.1088)	184.0749(184.1078)

表 3 $\lambda = 2$ 时正交异性矩形板大挠度固有热振动频率(单位 赫兹)

Table 3 The natural thermal vibration frequency of rectangular orthotropic plates when $\lambda = 2$(unit :Hz)

Characteristic parameter of foundation	$A(m)$	T_0(℃)			
		0	4	8	12
$k_1 = 0$	0.025	17.8378(17.8378)	17.8103(17.8103)	17.7828(17.7828)	17.7551(17.7551)
$G_P = 0$	0.05	17.8601(17.8601)	17.8326(17.8326)	17.8051(17.8051)	17.7775(17.7775)
$k_2 = 0$	0.075	17.8972(17.8972)	17.8697(17.8698)	17.8422(17.8423)	17.8147(17.8148)
$k_1 = 3 \times 10^7$	0.025	174.1212(174.1212)	174.1184(174.1184)	174.1156(174.1156)	174.1127(174.1127)
$G_P = 0$	0.05	174.1235(174.1235)	174.1207(174.1207)	174.1178(174.1178)	174.1151(174.1152)
$k_2 = 0$	0.075	174.1273(174.1273)	174.1245(174.1245)	174.1216(174.1216)	174.1188(174.1188)
$k_1 = 3 \times 10^7$	0.025	187.7575(187.7575)	187.7549(187.7549)	187.7523(187.7523)	187.7497(187.7497)
$G_P = 4 \times 10^7$	0.05	187.7596(187.7596)	187.7576(187.7575)	187.7544(187.7544)	187.7518(187.7518)
$k_2 = 0$	0.075	187.7632(187.7632)	187.7606(187.7606)	187.7579(187.7579)	187.7553(187.7553)
$k_1 = 3 \times 10^7$	0.025	187.8137(187.8137)	187.8111(187.8111)	187.8085(187.8085)	187.8059(187.8059)
$G_P = 4 \times 10^7$	0.05	187.9842(187.9842)	187.9816(187.9816)	187.9796(187.9794)	187.9763(187.9764)
$k_2 = -8 \times 10^7$	0.075	188.2679(188.2681)	188.2653(188.2655)	188.2627(188.2629)	188.2601(188.2603)
$k_1 = 3 \times 10^7$	0.025	188.3183(188.3186)	188.3157(188.3159)	188.3131(188.3133)	188.3104(188.3107)
$G_P = 4 \times 10^7$	0.05	189.9903(189.9953)	189.9878(189.9928)	189.9852(189.9902)	189.9826(189.9876)
$k_2 = -8 \times 10^8$	0.075	192.7338(192.7619)	192.7313(192.7593)	192.7287(192.7568)	192.7262(192.7542)

表 4 $\lambda = 2$ 时正交异性矩形板大挠度固有热振动频率(单位 赫兹)

Table 4 The natural thermal vibration frequency of rectangular orthotropic plates when $\lambda = 2$(unit :Hz)

Characteristic parameter of foundation	$A(m)$	T_0(℃)			
		16	20	24	28
$k_1 = 0$	0.025	17.7275(17.7275)	17.6998(17.6998)	17.6721(17.6721)	17.6443(17.6443)
$G_P = 0$	0.05	17.7499(17.7499)	17.7222(17.7222)	17.6945(17.6945)	17.6669(17.6668)
$k_2 = 0$	0.075	17.7872(17.7872)	17.7596(17.7596)	17.7319(17.7320)	17.7042(17.7043)
$k_1 = 3 \times 10^7$	0.025	174.1099(174.1099)	174.1071(174.1071)	174.1043(174.1043)	174.1015(174.1015)
$G_P = 0$	0.05	174.1122(174.1122)	174.1094(174.1094)	174.1066(174.1066)	174.1037(174.1037)
$k_2 = 0$	0.075	174.1163(174.1169)	174.1132(174.1132)	174.1104(174.1104)	174.1076(174.1076)
$k_1 = 3 \times 10^7$	0.025	187.7471(187.7471)	187.7445(187.7445)	187.7418(187.7418)	187.7392(187.7392)
$G_P = 4 \times 10^7$	0.05	187.7492(187.7492)	187.7466(187.7466)	187.7440(187.7440)	187.7414(187.7414)
$k_2 = 0$	0.075	187.7527(187.7527)	187.7501(187.7501)	187.7475(187.7475)	187.7449(187.7449)
$k_1 = 3 \times 10^7$	0.025	187.8032(187.8032)	187.8006(187.8006)	187.7981(187.7982)	187.7954(187.7954)
$G_P = 4 \times 10^7$	0.05	187.9737(187.9738)	187.9711(187.9712)	187.9685(187.9686)	187.9659(187.9660)
$k_2 = -8 \times 10^7$	0.075	188.2575(188.2577)	188.2549(187.9712)	188.2523(188.2525)	188.2497(188.2499)
$k_1 = 3 \times 10^7$	0.025	188.3078(188.3081)	188.3052(188.3055)	188.3026(188.3029)	188.3004(188.3003)
$G_P = 4 \times 10^7$	0.05	189.9844(189.9850)	189.9774(189.9824)	189.9748(189.9798)	189.9723(189.9773)
$k_2 = -8 \times 10^8$	0.075	192.7236(192.7517)	192.7211(192.7492)	192.7185(192.7466)	192.716(192.7441)

参 考 文 献

1 杨端生,黄炎,潘军. 双参数弹性地基上板的自由振动. 动力学与控制学报,2004,2(1):92 ~ 96(Yang Duansheng, Huang Yan, Pan Jun. Free vibration of plates on bi-parameter elastic foundation. *Journal of Dynamics and Control*, 2004,2(1):92 ~ 96(in Chinese))

2 Dumir PC. Nonlinear dynamic response of isotropic thin rectangular plates on elastic foundation. New Delhi: Acta Mechanica,1988,71(4):233 ~ 244

3 沈惠申,张建武. 非线性弹性基础上矩形板热后屈曲分析. 应用力学学报,1997,14(1):29 ~ 35(Shen Hui-shen,

Zhang Jian-wu. Analysis of post bucking of rectangular plates on nonlinear elastic foundation. *Chinese Journal of Applied Mechanics*, 1997,14(1):29 ~ 35 (in Chinese))

4 孙中奎,徐伟,杨晓丽.求解强非线性动力系统响应的一种新方法.动力学与控制学报,2005,3(2):29 ~ 35(Sun Zhongkui, Xu Wei, Yang Xiaoli. A new analytic approximate technoque for strongly nonlinear dynamics systems. *Journal of Dynamics and Control*,2005,3(2):29 ~ 35 (in Chinese))

5 彭献,陈自力.一类强非线性系统共振周期解的渐近分析.动力学与控制学报,2004,2(1):46 ~ 50(Peng Xian, Chen Zili. Asymptotic analysis for resonance cycle solution of a type of strongly nonlinear systems. *Journal of Dynamics and Control*, 2004,2(1):46 ~ 50(in Chinese))

6 吴晓,禹金云.求强非线性振动系统的新摄动法.振动与冲击,2000,19(4):60 ~ 62(WU Xiao, Yu jin-yun. A new perturbation method of strongly nonlinear vibration system. *Journal of Vibration and Shock*, 2000, 19(4):60 ~ 62 (in Chinese))

7 王嘉新,罗雄辉.三参数地基上四边简支矩形薄板的弯曲问题.中南工业大学学报,1996(3):363 ~ 366 (Wang Jia-xin, Luo Xiong-hui. Solution to bending of rectangular thin plates simply supported on four edges on three-parametric modal of foundation. *Journal of Central South University of Technology*, 1996(3):363 ~ 366 (in Chinese))

8 贾春元.板的非线性分析.北京:科学出版社,1989,329 ~ 334 (Jia Chunyuan. Nonlinear analysis of plates .Beijing:Science Press, 1989, 329 ~ 334 (in Chinese))

9 息志臣,陈浩然.复合材料加筋板热变形和热应力分析.复合材料学报,1992(4):17 ~ 21.(Xi Zhichen, Chen Haoran. Analysis of thermal deformations and stresses in stiffened composite laminates. *Acta Materiae Compositae Sinica*, 1992 (4):17 ~ 21 (in Chinese))

NONLINEAR THERMAL VIBRATION OF RECTANGULAR ORTHOTROPIC PLATE ON NONLINEAR FOUNDATION

Yang Lijun　Wu Xiao

(*Department of Civil and Architectural Engineering, Hunan University of Arts and Science, Changde* 415000, *China*)

Abstract The nonlinear natural thermal vibration of a rectangular orthotropic plate on a nonlinear foundation was studied. To get a high-accuracy approximate solution which can't be obtained with the normal L-P method, the strongly nonlinear vibration system of the nonlinear natural thermal vibration of the rectangular orthotropic plate on the nonlinear foundation was changed to a weakly nonlinear vibration system firstly by the parameter transformation, and then the high-accuracy approximate solution can be obtained with the improved L-P method. The effects of temperature, characteristic parameter of foundation, ratio of length and width etc. on the natural thermal vibration frequency were discussed. The results showed that the natural thermal vibration frequency decreased with the increase of temperature and increased with the increase of the characteristic parameter of foundation and ratio of length and width.

Key words nonlinearity, foundation, orthotropy, thermal vibration

Received 14 March 2006, revised 13 April 2006.