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恒定磁场中简支圆柱壳的磁弹性振动分析

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摘要 依据电磁场方程及相应的电磁本构关系,给出了作用于圆柱壳体上的电磁力及力矩表达式.在此基础上,分别推得了纵向和横向磁场中圆柱壳体的磁弹性轴对称振动方程.针对两端简支约束条件,通过位移函数的设定,得到了相应的有阻尼振动微分方程.通过算例,给出了系统衰减振动的响应曲线图和相图,分析了磁感应强度和壳体厚度对系统振幅衰减速度的影响.结果表明,通过改变磁感应强度可以达到控制系统振动的目的.

关键词 磁弹性,圆柱薄壳,振动,电磁力

引言

变形场与电磁场相互耦合作用的现象在航空航天、核工业及电力电子等现代工业领域中经常遇到.一些导电弹性体尤其是薄壁结构在电磁场环境下将受到力、电、磁等多种效应间的相互耦合作用,其力学行为较为复杂[1-11],并将影响着系统的运行性能.国内外学者在这方面做了不少理论上的研究工作,取得了许多重要成果,如:文[1~3]研究了传导板的磁弹性振动及稳定性问题;文[4]建立了磁场中软铁磁壳体的磁弹性理论模型,其数值计算结果与实验结果具有较好的吻合;文[5]对磁场环境中圆柱壳体的非线性振动问题进行了研究,并用多尺度法进行了求解,分析了磁场对振动频率的影响.本文在[5]的基础上,研究了两端简支圆柱薄壳分别处于纵向和横向磁场中的自由振动问题,导出了圆柱壳体的振动控制微分方程,通过数值计算,得到了相应的振动响应图和相图,分析了电磁和机械参数对系统振动的影响.

1 壳体磁弹性运动方程与电磁力表达式

对于圆柱薄壳体,建立正交曲线坐标系 α、β、γ,其中坐标 α、β 分别沿中面的轴向和环向,γ 为法向坐标.在略去面内及转动惯性力影响情况下,可得电磁场环境中圆柱壳体的轴对称运动方程[5]

$$\frac{\partial N_\alpha}{\partial \alpha} + F_\alpha = 0 \tag{1}$$

$$\frac{\partial^2 M_\alpha}{\partial \alpha^2} - \frac{N_\beta}{R} + F_\gamma + \frac{\partial m_\alpha}{\partial \alpha} = \rho h \frac{\partial^2 w}{\partial t^2} \tag{2}$$

式中:N_α、N_β、M_α 为相应方向的内力和弯矩,F_α、F_γ 分别为轴向和法向电磁力,m_α 为电磁力矩,ρ 为材料密度,h 为壳体厚度,R 为壳体半径,t 为时间变量.

这里不记热效应及位移电流的影响,且假定材料为非极化、非磁化的良导体,依文[6],麦克斯韦方程及电磁本构关系式为

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{3}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \tag{4}$$

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{5}$$

式中 E 为电场强度矢量,B 为磁感应强度矢量,J 为电流密度矢量,H 为磁场强度矢量,v 表示体内各点的速度矢量,σ 为材料的电导率.

单位体积电磁力的表达式为

$$\boldsymbol{f} = \boldsymbol{J} \times \boldsymbol{B} \tag{6}$$

当壳体在外界磁场 $\boldsymbol{B}_0(B_{0\alpha}, B_{0\beta}, B_{0\gamma})$ 中运动时,可将体内各电磁场量表示为

$$\boldsymbol{H}(H_\alpha, H_\beta, H_\gamma) = \boldsymbol{H}_0(H_{0\alpha}, H_{0\beta}, H_{0\gamma}) + \boldsymbol{h}(h_\alpha, h_\beta, h_\gamma) \tag{7}$$

$$\boldsymbol{B}(B_\alpha, B_\beta, B_\gamma) = \boldsymbol{B}_0(B_{0\alpha}, B_{0\beta}, B_{0\gamma}) + \boldsymbol{b}(b_\alpha, b_\beta, b_\gamma) \tag{8}$$

$$\boldsymbol{E}(E_\alpha, E_\beta, E_\gamma) = \boldsymbol{e}(e_\alpha, e_\beta, e_\gamma) \tag{9}$$

$$\boldsymbol{D}(D_\alpha, D_\beta, D_\gamma) = \boldsymbol{d}(d_\alpha, d_\beta, d_\gamma) \tag{10}$$

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式中 h、b、e、d 分别为扰动后激发产生的附加电磁矢量.

这样,通过式(6),沿壳体厚度方向进行积分,略去扰动产生的二阶小项,并考虑轴对称特性,可得到作用于壳体单位面积上的电磁力及力矩表达式为

$$F_\alpha = \sigma h B_{0\gamma}\left(e_\beta + \frac{\partial w}{\partial t}B_{0\alpha} - \frac{\partial u}{\partial t}B_{0\gamma}\right) \qquad (11)$$

$$F_\beta = 0 \qquad (12)$$

$$F_\gamma = -\sigma h B_{0\alpha}\left(e_\beta + \frac{\partial w}{\partial t}B_{0\alpha} - \frac{\partial u}{\partial t}B_{0\gamma}\right) \qquad (13)$$

$$m_\alpha = \frac{\sigma h^3 B_{0\gamma}^2}{12}\frac{\partial^2 w}{\partial\alpha\partial t} \qquad (14)$$

$$m_\beta = 0 \qquad (15)$$

式中 u、w 分别为中面内沿 α 和 γ 方向的位移.

2 恒定磁场中圆柱壳体的振动问题

下面分别研究纵向和横向磁场中圆柱壳体的振动问题.

当圆柱壳体处于恒定纵向磁场 $\boldsymbol{B}_0(B_{0\alpha},0,0)$ 环境中时,电磁力的表达式将得到进一步的简化,将其代入运动方程,并考虑到几何方程和物理方程,可推得纵向磁场中圆柱壳体的振动方程为

$$-D_M\frac{\partial^4 w}{\partial\alpha^4} - \frac{Eh}{R^2}w - \sigma h B_{0\alpha}^2\frac{\partial w}{\partial t} = \rho h\frac{\partial^2 w}{\partial t^2} \qquad (16)$$

式中 $D_M = \frac{Eh^3}{12(1-\mu^2)}$ 为弯曲刚度,E 为弹性模量,μ 为泊松系数.

当圆柱壳体处于恒定横向磁场 $\boldsymbol{B}_0(0,0,B_{0\gamma})$ 环境中时,同样可推得横向磁场中圆柱壳体的振动方程为

$$-D_M\frac{\partial^4 w}{\partial\alpha^4} - \frac{Eh}{R^2}w + \frac{\sigma h^3 B_{0\gamma}^2}{12}\frac{\partial^3 w}{\partial\alpha^2\partial t} = \rho h\frac{\partial^2 w}{\partial t^2} \qquad (17)$$

对于两端简支的圆柱壳体,可将满足边界条件的解设为

$$w = T(t)\sin\frac{m\pi}{l}\alpha \quad (m = 1,2,3\cdots) \qquad (18)$$

这样,将上式分别代入式(16)和(17)中.整理后,可得到如下壳体磁弹性有阻尼自由振动微分方程的统一表达式

$$\ddot{T}(t) + 2\zeta\omega_n\dot{T}(t) + \omega_n^2 T(t) = 0 \qquad (19)$$

这里 $\dot{T}$ 和 $\ddot{T}$ 分别表示 $T(t)$ 对时间 t 的一阶和二阶导数,$\zeta = \frac{n}{\omega_n}$ 为相对阻尼系数,$\omega_n = \sqrt{\frac{E}{\rho R^2} + \frac{D_M}{\rho h}\left(\frac{m\pi}{l}\right)^4}$ 为无阻尼固有频率;纵向磁场中 $2n = \frac{\sigma B_{0\alpha}^2}{\rho}$,横向磁场中 $2n = \frac{\sigma h^2 B_{0\gamma}^2}{12\rho}\left(\frac{m\pi}{l}\right)^2$;而有阻尼固有频率 $\omega_d = \omega_n\sqrt{1-\zeta^2}$.

3 算例分析

对于铝制材料壳体,其主要物理参数为:密度 $\rho = 2670\ \mathrm{kg/m^3}$,泊松比 $\mu = .34$,弹性模量 $E = 71$ GPa,电导率 $\sigma = 3.63\times10^7(\Omega\cdot\mathrm{m})^{-1}$.通过编程计算,得到了不同条件下圆柱壳体的振动特性曲线图.

图 1 ~ 图 6 为纵向磁场环境下圆柱壳体的振动特性曲线图(取 $m = 1$).图 1 为不同加载磁场下壳体在欠阻尼状态时的振动响应图,由图可见,此时系统为振幅按指数规律衰减的周期振动,而磁场的存在相当于阻尼的作用,且磁感应强度越强,振幅衰减得越快,但三种情况下的衰减频率基本一致.图 2 ~ 4 为三种情形对应的相图曲线,它们充分体现了振动衰减及磁场的影响效应,即 $B = 1.2$ T 时幅值很快降为零,而 $B = 0.4$ T 时需经很多运动周期后幅值才降为零.

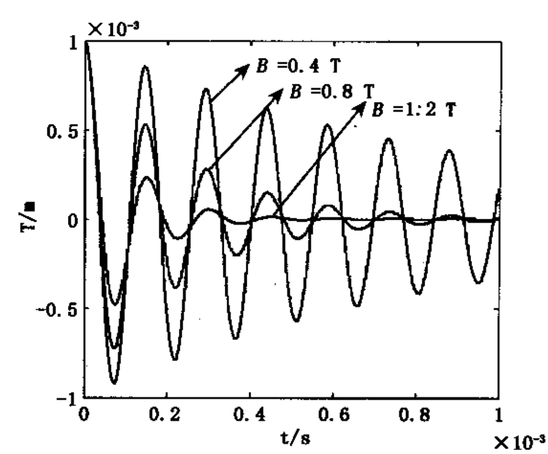

图 1 纵向磁场中欠阻尼振动响应图($h = 2.5$ mm)

Fig.1 Light damping vibrating response of the system under the lengthways magnetic field

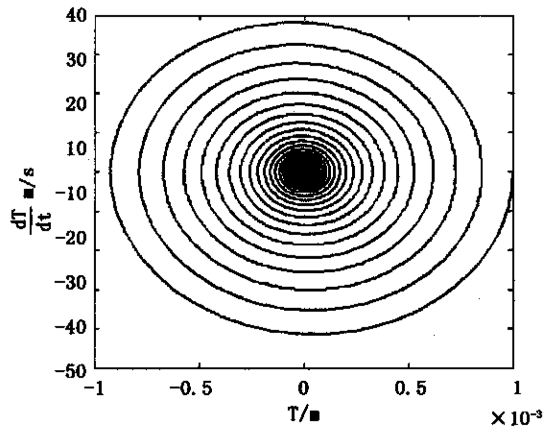

图 2　纵向磁场中欠阻尼振动相图(B = 0.4 T)

Fig.2　Light damping vibrating phase figure of the system under lengthways magnetic field

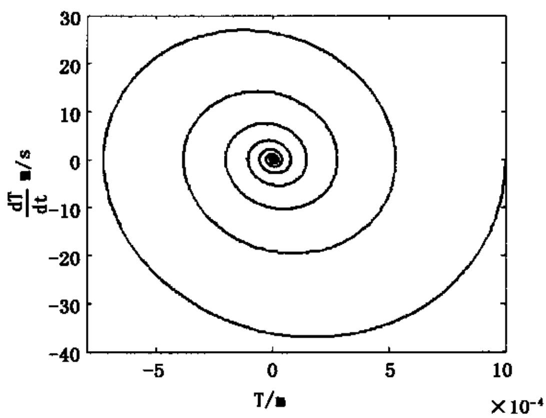

图 3　纵向磁场中欠阻尼振动相图(B = 0.8 T)

Fig.3　Light damping vibrating phase figure of the system under the lengthways magnetic field

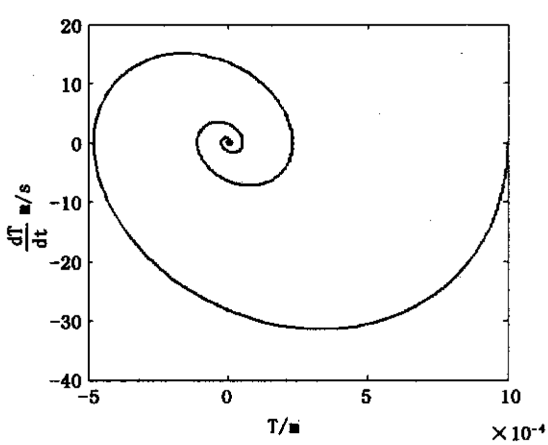

图 4　纵向磁场中欠阻尼振动相图(B = 1.2 T)

Fig.4　Light damping vibrating phase figure of the system under the lengthways magnetic field

随着加载磁感应强度的增强,当相对阻尼系数大于 1 时,系统将处于按指数规律衰减的非周期过阻尼运动状态.图 5 给出的是磁感应强度变化对过阻尼状态下系统振动的影响,由图可见,随着磁感应强度的增强,壳体振幅的衰减逐渐趋缓.图 6 则给出了板厚变化对壳体振动的影响,由图可见,三种情形下的曲线基本重合,即给定的板厚变化影响很小.

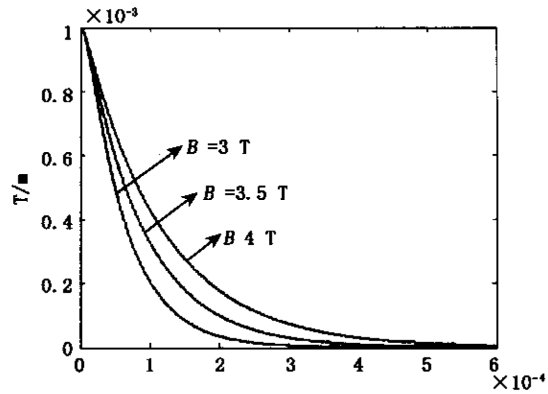

图 5　纵向磁场中过阻尼振动响应图(h = 2.5 mm)

Fig.5　Heavy damping vibrating response of the system under the lengthways magnetic field

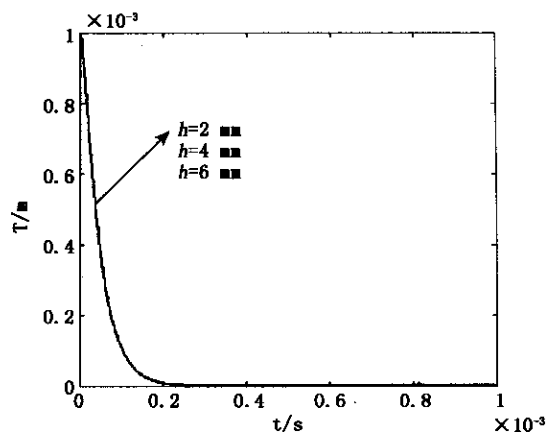

图 6　纵向磁场中过阻尼振动响应图(B = 3 T)

Fig.6　Heavy damping vibrating response of the system under the lengthways magnetic field

图 7 和图 8 为横向磁场环境下壳体的振动响应图(取 m = 1),此时系统仅表现出欠阻尼状态下的周期衰减运动状态,且相对于纵向磁场环境,衰减很慢,而若想达到欠阻尼情形,则需施加相当大的磁场强度.由图亦见,在改变磁感应强度和壳体厚度情况下,振动形式基本相同,即在本文给定参数

下,磁感应强度和壳体厚度对振动的影响极小.

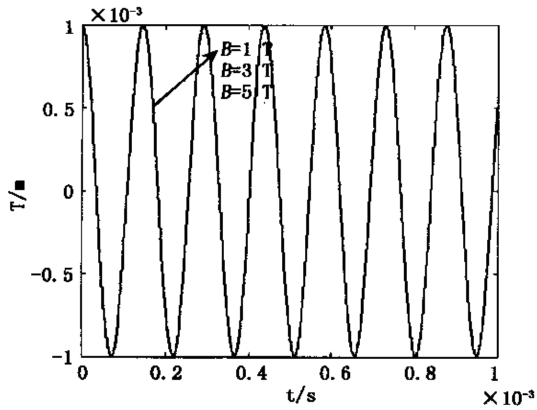

图7 横向磁场中振动响应图(h = 4 mm)

Fig.7 Vibrating response of the shell under the transverse magnetic field

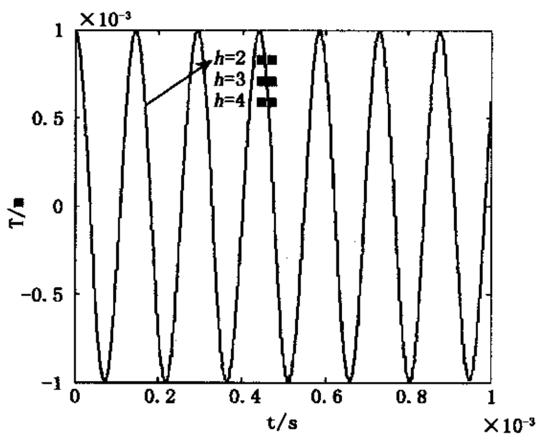

图8 横向磁场中振动响应图(B = 5 T)

Fig.8 Vibrating response of the system under the transverse magnetic field

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MAGNETOELASTIC VIBRATION ANALYSIS OF A SIMPLY SUPPORTED CYLINDRICAL SHELL IN CONSTANT MAGNETIC FIELD

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Abstract Based on the electrodynamics equations, the electromagnetic forces and moments applied on a cylindrical shell were obtained. Magnetoelastic axisymmetric vibration equations of the cylindrical shell under a longitudinal and a transverse magnetic field were derived respectively. Applying the assuming displacement function, the damped vibration differential equations of the cylindrical shell with two opposite simply supported sides were obtained. For examples, response curves and phase figures of damped vibration were demonstrated. The effects of magnetic induction and the thickness of the shell on amplitude damping were discussed. The results show that the vibration can be controlled by adjusting the magnetic induction.

Key words magnetoelastic, thin cylindrical shell, vibration, electromagnetic force

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