

Lorenz 系统的线性反馈控制*

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摘要 设计了状态变量的线性反馈控制器对 Lorenz 系统的平衡点和周期轨道进行控制.首先,利用 Routh-Hurwitz 准则对受控系统进行了稳定性分析,证明了达到控制目标反馈系数的选择原则.然后,通过数值计算证明了该方法能够有效地控制混沌系统达到稳定的平衡点同时也能使系统控制到 $1p$ 周期轨道,并且得到了相应的稳定的 $1p$ 周期轨道的控制参数和系统幅值的关系曲线.最后给出了控制到 $1p$ 周期轨道的控制参数的选取范围.

关键词 Lorenz 系统族,反馈控制,Routh-Hurwitz 准则,平衡点

引言

混沌和分岔理论以其理论的深奥性和工程上的应用性被称为非线性振动研究中最活跃,最前沿的课题之一.1963年美国科学家洛伦茨在研究气象预报时,对无穷维动力系统的瑞利-伯纳德热对流问题进行三维截断得到了著名的 Lorenz 系统,它是第一个极为简洁的且表现出奇异吸引子的动力系统. Lorenz 系统拥有极其复杂的非线性动力学行为^[1].在最近二十年内,国内外发现了一些类 Lorenz 系统,譬如, Rosser 系统, Chen 吸引子^[2], 吕氏吸引子^[3]等,这些统称为 Lorenz 系统族.

近年来,混沌的控制由于具有巨大的应用前景而成为非线性科学的一个研究热点.科研工作者提出了大量的控制与同步的方法,如 Ott, Grebogi, Yorke(OGY)控制法^[4], 反馈控制法^[5], 自适应反馈法^[6]等.陈对混沌系统的控制,同步及反控制等问题进行了综合介绍和评述^[7]. M. T. Yassen 和李等分别对一类混沌系统作了线性控制研究^[8-10].

本文所研究的 Lorenz 系统^[11]的数学模型分别为

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = -xz - y + cx \\ \dot{z} = xy - bz \end{cases} \quad (1)$$

其中 a, b, c 为正的常数.

$$\begin{cases} \dot{x} = -\sigma(x - y) \\ \dot{y} = \rho x - xz - y \\ \dot{z} = xy - \beta z \end{cases} \quad (2)$$

其中 σ, ρ, β 亦为正的常数.

1 Lorenz 系统的平衡点控制

1.1 系统的平衡点控制

对于系统当参数 $a = 10, b = 8/3, c = 28$ 时存在如图 1 所示的两个奇异吸引子,它们关于原点对称.系统(1)存在三个平衡点: $E_0(0, 0, 0), E_1 = (\sqrt{b(c-1)}, \sqrt{b(c-1)}, c-1), E_2 = (-\sqrt{b(c-1)}, -\sqrt{b(c-1)}, c-1)$.系统在领域内

的 Jacobi 矩阵为 $J_0 = \begin{bmatrix} -a & a & 0 \\ c & -1 & 0 \\ 0 & 0 & -b \end{bmatrix}$. J_0 对应

的特征值为 $\lambda_1 = -\frac{1+a}{2} +$

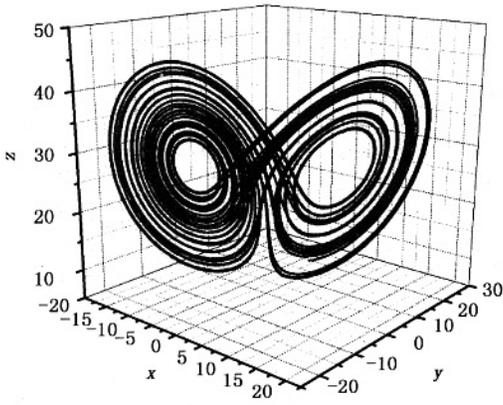
$\sqrt{(\frac{1+a}{2})^2 - a(1-c)}, \lambda_2 = -\frac{1+a}{2} -$

$\sqrt{(\frac{1+a}{2})^2 - a(1-c)}, \lambda_3 = -b$.当 $a = 10, b =$

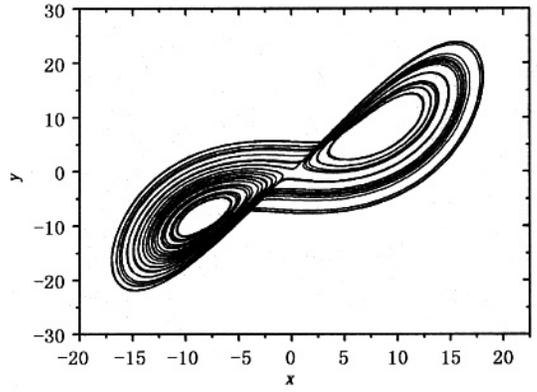
$8/3, c = 28$ 时,可得 $\lambda_1 = 11.83 > 0, \lambda_2 = -22.83 < 0, \lambda_3 = -2.67 < 0$.因此平衡点 E_0 是一个不稳定的鞍点.设计线性状态反馈控制器^[12]将混沌系统控制到平衡点 E_0 .我们只对系统的状态变量 $X,$

$$Y \text{ 施加控制,控制器为 } U = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \text{ 其}$$

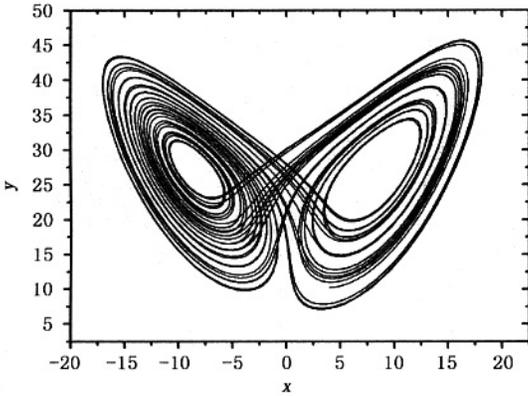
中 k_1, k_2 为控制参数,则受控系统变为



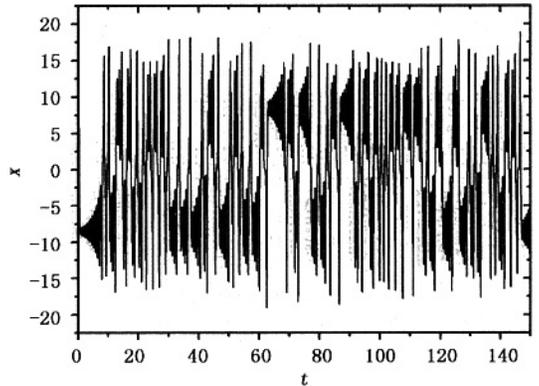
(a) x-y-z trajectories



(b) x-y trajectories



(c) x-y trajectories



(d) t-x curve

图1 系统(1)的混沌图和时间响应图

Fig.1 Trajectories of the system(1)

$$\begin{cases} \dot{x} = a(y-x) + k_1 x \\ \dot{y} = -xz - y + cx + k_2 y \\ \dot{z} = xy - bz \end{cases} \quad (3)$$

当 $k_1 = k_2 = 0$ 受控系统退化为原系统(1).显然此时受控系统只有一个平衡点 E_0 , 根据Routh-Hurwitz 准则知:当受控系统(3)的所有特征值均具有负实部时系统不再做混沌运动而是做收敛到平衡点 E_0 .受控系统(3)的Jacobi 矩阵为 $J_0 =$

$$\begin{bmatrix} -a+k_1 & a & 0 \\ c & -1+k_2 & 0 \\ 0 & 0 & -b \end{bmatrix}, \text{则线性部分的特征}$$

矩阵为

$$m = \begin{bmatrix} -a+k_1-\lambda & a & 0 \\ c & -1+k_2-\lambda & 0 \\ 0 & 0 & -b-\lambda \end{bmatrix},$$

对应的特征值为

$$\lambda_1 = \frac{k_1 + k_2 - a - 1}{2} +$$

$$\sqrt{\left(\frac{k_1 + k_2 - a - 1}{2}\right)^2 - 4[(k_1 - a)(k_2 - 1) - ac]} \quad (4)$$

$$\lambda_2 = \frac{k_1 + k_2 - a - 1}{2} -$$

$$\sqrt{\left(\frac{k_1 + k_2 - a - 1}{2}\right)^2 - 4[(k_1 - a)(k_2 - 1) - ac]} \quad (5)$$

$$\lambda_3 = -b \quad (6)$$

为了使系统控制到平衡点 E_0 , 所以应有 $\lambda_1 < 0, \lambda_2 < 0, \lambda_3 < 0$ 由式(4)~(6)可知有 $\lambda_3 = -b < 0$ 恒成立.对于 λ_1, λ_2 只需满足

$$(k_1 - a)(k_2 - 1) - ac > 0 \quad (7)$$

就能使得 $\lambda_1 < 0, \lambda_2 < 0$.

1.2 系统的平衡点控制

对于系统当参数 $\sigma = 4, \beta = 1, \rho = 16$ 时存在如图 2 所示的两个奇异吸引子, 它们关于原点对称. 系统存在三个平衡点: $E_0(0, 0, 0), E_1 = (\sqrt{\beta(\rho - 1)}, \sqrt{\beta(\rho - 1)}, \rho - 1), E_2 = (-\sqrt{\beta(\rho - 1)}, -\sqrt{\beta(\rho - 1)}, \rho - 1)$. 系统(2)在 E_0

领域内的 Jacobi 矩阵为 $J_0 = \begin{bmatrix} -\sigma & \sigma & 0 \\ \rho & -1 & 0 \\ 0 & 0 & -\beta \end{bmatrix}$. J_0

对应的特征值为 λ_1

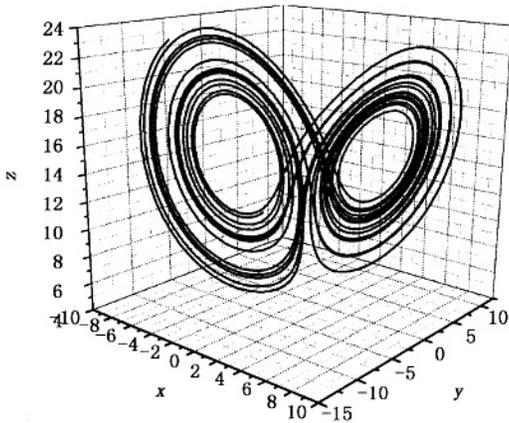
$$= -\frac{1 + \sigma}{2} + \sqrt{\left(\frac{1 + \sigma}{2}\right)^2 - \alpha(1 - \rho)}, \lambda_2 = -\frac{1 + \sigma}{2}$$

$-\sqrt{\left(\frac{1 + \sigma}{2}\right)^2 - \alpha(1 - \rho)}, \lambda_3 = -\beta$. 当 $\sigma = 4, \beta = 1, \rho = 16$ 时, 可得 $\lambda_1 = 6.51 > 0, \lambda_2 = -11.51 < 0, \lambda_3 = -1 < 0$. 因此平衡点 E_0 也是一个不稳定的鞍点. 设计线性状态反馈控制器将混沌系统控制到平衡点 E_0 . 同样只对系统的状态变量 X, Y 施加控

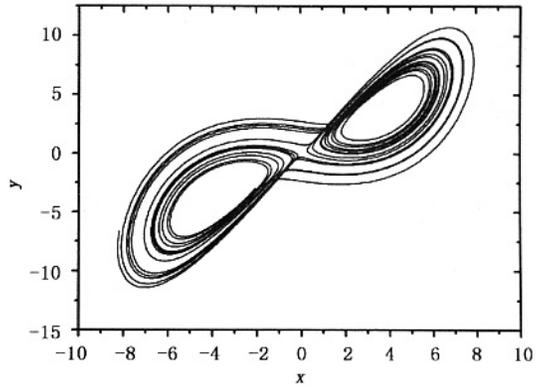
制, 控制器为 $U = \begin{bmatrix} k_3 & 0 & 0 \\ 0 & k_4 & 0 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, 其中 k_3, k_4

为控制参数, 则受控系统变为

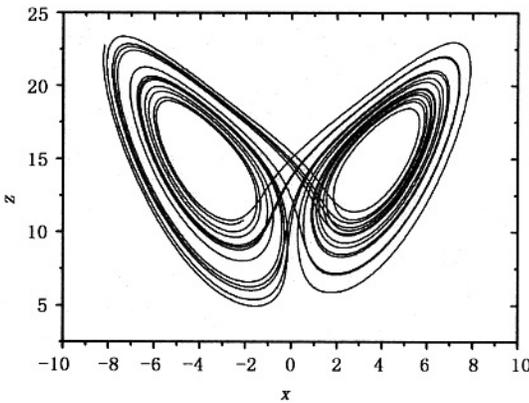
$$\begin{cases} \dot{x} = -\alpha(y - x) + k_3 x \\ \dot{y} = \rho x - xz - y + k_4 y \\ \dot{z} = xy - \beta z \end{cases} \quad (8)$$



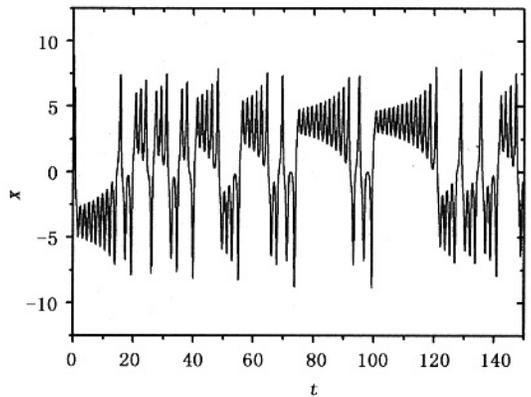
(a) x-y-z trajectories



(b) x-y trajectories



(c) x-z trajectories



(d) t-x curve

图 2 系统(2)的混沌图和时间响应图

Fig.2 Trajectories of the system(2)

受控系统(6)的 Jacobi 矩阵为 $J_0 =$

$$\begin{bmatrix} -\sigma + k_3 & \sigma & 0 \\ \rho & -1 + k_4 & 0 \\ 0 & 0 & -\beta \end{bmatrix}$$

则特征矩阵为 $m' =$

$$\begin{bmatrix} -\sigma + k_3 - \lambda & \sigma & 0 \\ \rho & -1 + k_4 - \lambda & 0 \\ 0 & 0 & -\beta - \lambda \end{bmatrix}$$

对应的

特征值为

$$\lambda_1' = \frac{k_3 + k_4 - \sigma - 1}{2} +$$

$$\sqrt{\left(\frac{k_3 + k_4 - \sigma - 1}{2}\right)^2 - 4[(k_3 - a)(k_4 - 1) - \sigma\rho]} \quad (9)$$

$$\lambda_2' = \frac{k_3 + k_4 - \sigma - 1}{2} -$$

$$\sqrt{\left(\frac{k_3 + k_4 - \sigma - 1}{2}\right)^2 - 4[(k_3 - a)(k_4 - 1) - \sigma\rho]} \quad (10)$$

$$\lambda_3' = -\beta \quad (11)$$

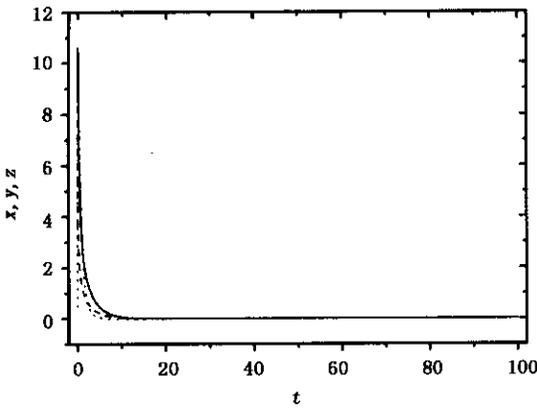
同样 $\lambda_3' = -\beta < 0$ 成立. 如 1.2 可知为了控制到平衡点 E_0' 只需满足

$$(k_3 - a)(k_4 - 1) - \sigma\rho > 0 \quad (12)$$

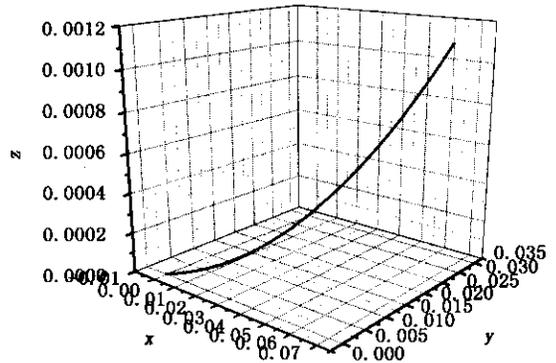
即可使得 $\lambda_1 < 0, \lambda_2 < 0$.

2 数值计算及 1P 周期轨道控制

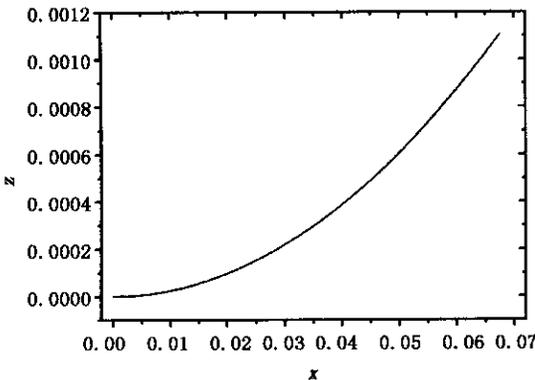
对于受控系统 采用四阶龙格 - 库塔进行数值计算, 系统参数取为 $a = 10, b = 8/3, c = 28$, 初值 $x(0) = 10, y(0) = 10, z(0) = 0$, 步长 $h = 0.005$, 在满足控制条件(7)的前提下取 $k_1 = 5, k_2 = -60$, 施加控制后状态向量 $x(t), y(t), z(t)$ 逐渐向平衡点靠近, 最终完全控制到不稳定的平衡点 $E_0(0, 0, 0)$.



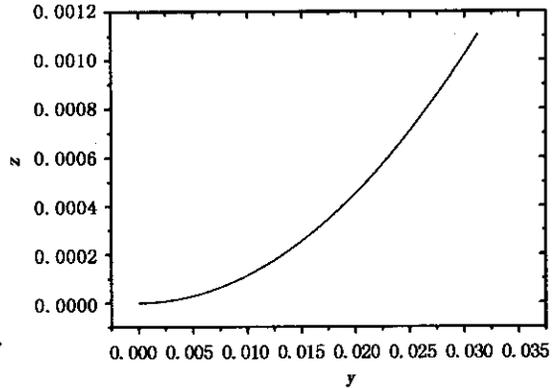
(a) t-x curve



(b) x-y-z trajectories



(c) x-z trajectories



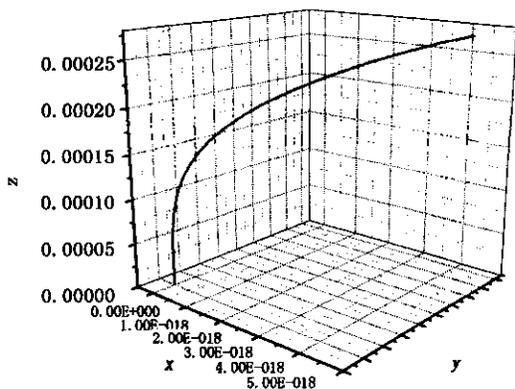
(d) y-z trajectories

图3 受控系统(3)的稳定轨迹和时间响应图

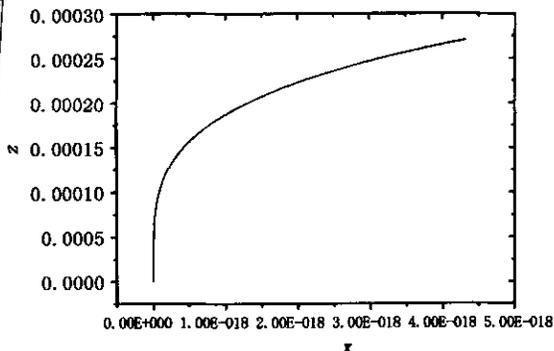
Fig.3 Trajectories of the controlling system(3)

对于受控系统,采用四阶龙格-库塔法进行数值计算,系统参数取为 $\sigma = 4, \beta = 1, \rho = 16$,初值 $x(0) = 10.0, y(0) = -5.0, z(0) = 5.7$,步长 $h = 0.005$,在满足控制条件的前提下取 $k_3 = -7, k_4$

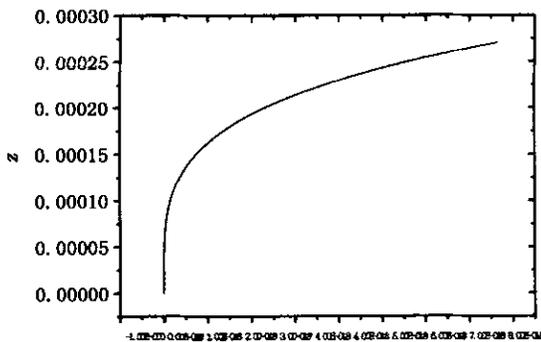
$= -12$,施加控制后状态向量 $x(t), y(t), z(t)$ 逐渐向平衡点靠近,最终完全控制到不稳定的平衡点 $E'_0(0, 0, 0)$.



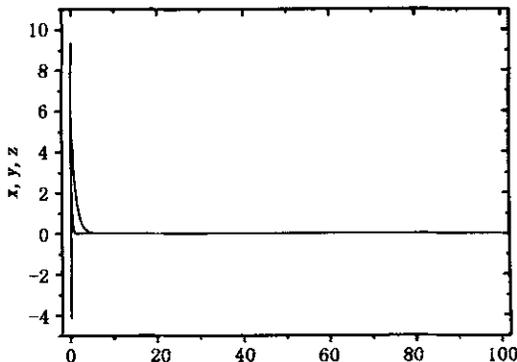
(a) x-y-z trajectories



(b) x-z trajectories



(c) y-z trajectories



(d) t-x-y-z curve

图 4 受控系统(3)的稳定轨迹和时间响应图

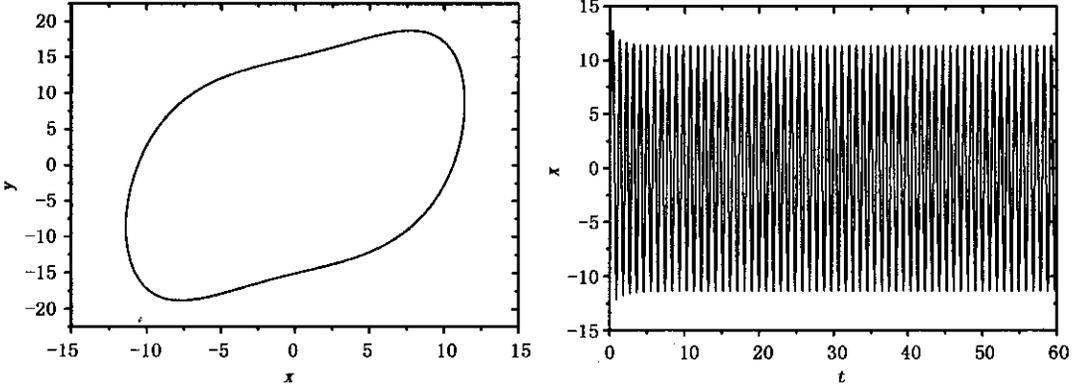
Fig.4 Trajectories of the controlling system(3)

对于系统(2)当设计控制器为 $U =$

$$\begin{bmatrix} k_5 & 0 & 0 \\ 0 & k_6 & 0 \\ 0 & 0 & k_7 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

时,通过调整控制参数可以将混沌系统控制到稳定的 $1p$ 周期轨道,当系统参数仍然取为 $\sigma = 4, \beta = 1, \rho = 16$,初值 $x(0) = 10.0, y(0) = -5.0, z(0) = 5.7$,步长 $h = 0.005$,控制参

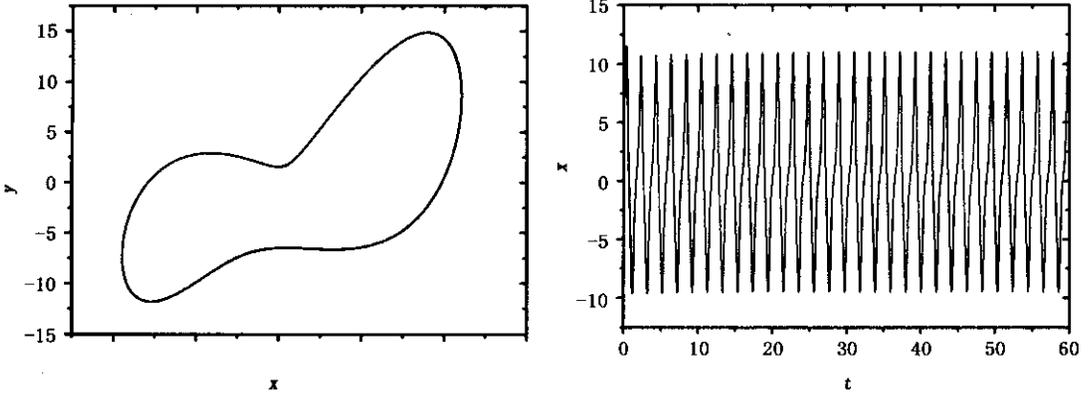
数取为 $k_5 = 0.8, k_6 = 3.0, k_7 = -0.9$ 时系统被稳定到 $1p$ 周期轨道,如图 4 所示.控制参数取为 $k_5 = 0.8, k_6 = 1.3, k_7 = -0.9$ 时系统同样被稳定到 $1p$ 周期轨道,如图 5 所示.由数值计算表明只变化控制参数 k_6 ,当 $1.3 \leq k_6 \leq 3.0$ 时系统能被稳定到 $1p$ 周期轨道.



(a) x - y trajectories (b) x - t curve

图5 $k_5 = 0.8, k_6 = 3.0, k_7 = -0.9$ 时控制到稳定的 $1p$ 周期轨道

Fig.5 Trajectories of the system with $k_5 = 0.8, k_6 = 3.0, k_7 = -0.9$



(a) x - y trajectories (b) x - t curve

图6 $k_5 = 0.8, k_6 = 1.3, k_7 = -0.9$ 时控制到稳定的 $1p$ 周期轨道

Fig.6 Trajectories of the system with $k_5 = 0.8, k_6 = 1.3, k_7 = -0.9$

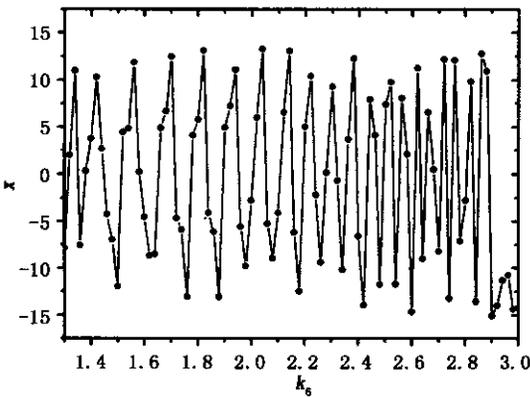


图7 控制参数 k_6 x 幅值的关系曲线

Fig.7 k_6 - x curve

3 结论

用线性反馈控制器对 Lorenz 系统的平衡点和 $1p$ 周期轨道进行了控制,用 Routh-Hurwitz 准则从理论上得出了将系统控制到稳定的平衡点的控制参数的选取范围,通过数值计算的方法证明所设计的控制器完全达到了预期的目的.数值计算说明了对 Lorenz 系统施加控制能将系统控制到稳定的 $1p$ 周期轨道.

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LINEAR STATE FEEDBACK CONTROL FOR LORENZ SYSTEM*

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Abstract The linear state variables feedback controllers were designed to control the equilibrium points and period trajectories of the Lorenz system. First, we applied the Routh-Hurwitz criterion to analyze the stability of the controlled system, and the choice principle of the feedback coefficients to attain control objective was proved strictly. Then, the numerical simulation were performed, whose results indicated that the method can effectively guide the system trajectories to equilibrium, especially, the method can also direct the controlled system to 1 periodic trajectories. And the curve of the controlling parameter with the amplitude of the controlled system was obtained. Finally, the range of the controlling parameter directing the controlled system to 1 periodic trajectory was given.

Key words Lorenz system, feedback control, Routh-Hurwitz criterion, equilibrium points