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横向磁场中矩形薄板在分布载荷作用下混沌分析(I)*

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摘要　在建立置于横向稳恒电磁场中,同时受横向均布载荷作用四边简支的金属矩形薄板的受力模型的基础上,推导了金属矩型薄板的磁弹性耦合动力学方程,求得了该模型振动系统的异宿轨道参数方程,并根据 Melnikov 函数方法,推导并求解了振动系统的异宿轨道的 Melnikov 函数,最后给出了判断该系统发生 Smale 马蹄变换意义下混沌运动的条件和混沌运动判据.由此可对矩形薄板在机械载荷和电磁载荷共同作用下的分岔和混沌进行分析.本文给出的方法可以推广到其他不同边界条件和不同外载荷条件下弹性薄板的磁弹性振动问题的研究.

关键词　矩形薄板,分布载荷,磁弹性,混沌运动,Melnikov 函数

引言

随着现代科技的发展,利用有磁场作用的薄板和薄壳作为结构元件已是屡见不鲜,如热核反应堆的防护壳.通常这些薄板元件都是在机械载荷和电磁场、以及温度场耦合作用的环境下工作的.在这些高科技装置中,其结构在电磁场作用下的力学行为直接影响着这些装置的安全性与品质因素,并由此促成磁弹性理论在近 30 年中形成了一门新型的交叉学科.

矩形薄板是船舶、航空、机械、水利和电力工程中不可缺少的构件.通常这些薄板元件都是在机械载荷和电磁场、以及温度场耦合作用的环境下工作的.因此,对薄板在耦合场作用下的振动问题的研究显得尤为重要[1].目前,国内外学者对机械载荷作用下板的混沌运动作了大量的研究[2-7],也取得了很多研究成果[8,9].当前,虽然有些学者对在热载荷及微扰外压作用下的分岔问题及壳体的非线性动力学特性进行了研究[10,11],但是对机械载荷与电磁场耦合作用下分岔问题的研究还不多见.

在建立横向磁场中,受横向均布载荷作用下金属矩形薄板的磁弹性耦合动力学方程基础上,本文应用 Melnikov 函数方法,给出了判断机械载荷作用与磁弹性耦合作用下系统发生混沌运动的条件.

1　磁弹性薄板振动系统同(异)宿轨道求解

当四边简支的金属矩形薄板,置于横向磁场 $\bar{B}(0,0,B_z)$ 中,同时受有横向均布载荷 $\bar{P}(0,0,P_3)$ 作用,其力学模型如图 1 所示.在笛卡儿直角坐标系 $OXYZ$ 中,OXY 为薄板的中面,Z 为法向坐标.不考虑热效应以及极化、磁化的影响,则薄板的磁弹性运动方程[12]

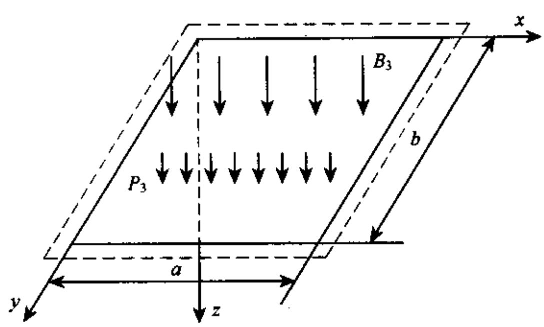

图 1　矩形薄板模型

Fig.1　Model of rectangular thin plate

$$\frac{\partial N_1}{\partial x} + \frac{\partial N_{12}}{\partial y} + (P_1 + \rho f_1) = \rho h \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial N_2}{\partial y} + \frac{\partial N_{12}}{\partial x} + (P_2 + \rho f_2) = \rho h \frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} + (P_3 + \rho f_3) = \rho h \frac{\partial^2 w}{\partial t^2}$$

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$$\frac{\partial M_1}{\partial x}+\frac{\partial M_{12}}{\partial y}+N_1\theta_1+N_{12}\theta_2-Q_1+m_1=\frac{\rho h^3}{12}\frac{\partial\theta_1^2}{\partial t^2}$$

$$\frac{\partial M_2}{\partial y}+\frac{\partial M_{12}}{\partial x}+N_2\theta_2+N_{12}\theta_1-Q_2+m_2=\frac{\rho h^3}{12}\frac{\partial\theta_2^2}{\partial t^2}\qquad(1)$$

弹性关系式和几何关系式[13]

$$N_1=D_N(\varepsilon_1+v\varepsilon_2);\quad N_2=D_N(\varepsilon_2+v\varepsilon_1);\quad N_{12}=D_N\frac{1-v}{2}\Omega;$$
$$M_1=D_M(K_1+VK_2);\quad M_2=D_M(K_2+VK_1);\quad M_{12}=D_M(1-V)\tau\qquad(2)$$

$$\varepsilon_1=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2;\quad \varepsilon_2=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2;$$
$$\kappa_1=-\frac{\partial^2 w}{\partial x^2};\kappa_2=-\frac{\partial^2 w}{\partial y^2};$$
$$\tau=-\frac{\partial^2 w}{\partial x\partial y};\Omega=\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\qquad(3)$$

式(1)中，ρf_1、ρf_2、ρf_3、m_1、m_2 为洛仑兹力和力矩[14]，且

$$\rho f_1=\sigma hB_z\left(e_y+\frac{\partial w}{\partial t}B_x-\frac{\partial u}{\partial t}B_z\right);$$
$$\rho f_2=\sigma hB_z\left(-e_x+\frac{\partial w}{\partial t}B_Y-\frac{\partial v}{\partial t}B_z\right);$$
$$\rho f_3=\sigma hB_y\left(e_x-\frac{\partial w}{\partial t}B_y+\frac{\partial v}{\partial t}B_z\right)-\sigma hB_x\left(e_y+\frac{\partial w}{\partial t}B_x-\frac{\partial u}{\partial t}B_z\right);$$
$$m_1=\frac{\sigma h^3}{12}B_z^2\frac{\partial^2 w}{\partial t\partial x};\quad m_2=\frac{\sigma h^3}{12}B_z^2\frac{\partial^2 w}{\partial t\partial y}$$

在式(1)～(3)中，$D_N=\frac{Eh}{1-v^3}$ 为拉伸刚度；$D_M=\frac{Eh^3}{12(1-v^2)}$ 为弯曲刚度；N_1、N_2、N_{12}、Q_1、Q_2、M_1、M_2、M_{12} 分别为薄板中面的内力和内力矩；w、u、v 分别为中面内点在 x、y、z 方向上的位移；P_1、P_2、P_3 为机械力；h 为板厚、ρ 为质量密度、E 为弹性模量、v 为泊松比、t 为时间、σ 为电导率、e_x、e_y 为磁感应电场强度分量. 下角标 1,2,3 分别对应 x、y、z 方向上的分量.

考虑几何非线性，忽略惯性项，可推导得到系统的振动方程为

$$D_M\nabla^4 w+\frac{D_N}{2}\left[3\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial y}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial y^2}+3\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial y}\right)^2+4\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y}\right]^2-\frac{\sigma h^3}{12}B_z^2\frac{\partial^2(\nabla^2 w)}{\partial t}+\rho h\frac{\partial^2 w}{\partial t^2}-P_3=0\qquad(4)$$

其中算子：$\nabla^4=\frac{\partial^4}{\partial x^4}+2\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2}+\frac{\partial^4}{\partial y^4}$；$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$. 边界条件为

$x=0,a$ 时，$w=\frac{\partial^2 w}{\partial x^2}=0$；$y=0,b$ 时，$w=\frac{\partial^2 w}{\partial y^2}=0$.

设满足方程(4)的一阶主振型解为

$$w(x,y,t)=X(t)w_0\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)\qquad(5)$$

其中 w_0 为振动幅值. 机械载荷为

$$P_3=p\cos(\omega t)\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)\qquad(6)$$

其中 p 为机械载荷幅值. 将式(5)和(6)带入式(4)并化简得

$$D_M\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]^2 w_0X(t)\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)+\frac{D_N}{2}\left[3\left(\frac{\pi}{a}\right)^4\cos^2\left(\frac{\pi}{a}x\right)\sin^2\left(\frac{\pi}{b}y\right)+\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2\sin^2\left(\frac{\pi}{a}x\right)\cos^2\left(\frac{\pi}{b}y\right)+\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2\cos^2\left(\frac{\pi}{a}x\right)\sin^2\left(\frac{\pi}{b}y\right)+3\left(\frac{\pi}{b}\right)^4\sin^2\left(\frac{\pi}{a}x\right)\cos^2\left(\frac{\pi}{b}y\right)+4\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2\cos^2\left(\frac{\pi}{a}x\right)\cos^2\left(\frac{\pi}{b}y\right)\right]\times w_0^3X^3(t)\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)-\frac{\sigma h^3}{12}B_z^2\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]\times w_0\dot{X}(t)\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)+\rho hw_0\ddot{X}(t)\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)-p\cos(\omega t)\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)=0\qquad(7)$$

用伽辽金法对式(7)积分得

$$D_M\lambda^2 w_0 X(t) - \frac{1}{32}D_N\left[9\lambda^2 - 20\left(\frac{\pi}{a}\right)^2 \times \left(\frac{\pi}{b}\right)^2\right] w_0^3 X^3(t) + \frac{\sigma h^3}{12}B_z^2\lambda w_0 \dot{X}(t) + \rho h w_0 \ddot{X}(t) - p\cos(\omega t) = 0$$

为 Duffing 方程. 对其简化可得

$$\ddot{X} + \delta\dot{X} + \alpha X - \beta X^3 = \gamma\cos(\omega t) \tag{8}$$

其中 $\alpha = \dfrac{D_M\lambda^2}{\rho h}$,

$$\beta = \frac{1}{32}\,\frac{D_N\left[9\lambda^2 - 20\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2\right]w_0^2}{\rho h},$$

$$\delta = \frac{1}{12}\,\frac{\sigma h^2\lambda}{\rho}B_z^2,\ \gamma = \frac{p}{\rho h w}.$$

令 $\dot{X} = Y$, $\delta = \varepsilon\delta_1$, $\gamma = \varepsilon\gamma_1$,则系统(8)的等价系统为

$$\begin{cases}\dot{X} = Y \\ Y = -\alpha X + \beta X^3 + \varepsilon(-\delta_1 Y + \gamma_1\cos(\omega t))\end{cases} \tag{9}$$

当 $\varepsilon = 0$ 时,即 $\delta = 0$, $\gamma = 0$ 时,系统是无扰的 Hamilton 系统,即

$$\begin{cases}\dot{X} = Y \\ Y = -\alpha X + \beta X^3\end{cases} \tag{10}$$

由

$$\begin{cases}\dot{X} = Y = 0 \\ Y = -\alpha X + \beta X^3 = 0\end{cases}$$

解得三个不动点 $(\pm\sqrt{\frac{\alpha}{\beta}},0)$ 和 $(0,0)$. 系统(10)的特征方程为

$$\begin{vmatrix} 0-\lambda & 1 \\ -\alpha + 3\beta X^2 & 0-\lambda \end{vmatrix} = 0$$

即

$$\lambda^2 + \alpha - 3\beta X^2 = 0 \tag{11}$$

解(11)得 $\lambda = \pm\sqrt{3\beta X^2 - \alpha}$.

当不动点为 $(0,0)$ 时, $\lambda = \pm\sqrt{\alpha}i$,所以 $(0,0)$ 为中心 当不动点为 $(\pm\sqrt{\frac{\alpha}{\beta}},0)$ 时, $\lambda = \pm\sqrt{2\alpha}$,所以 $(\pm\sqrt{\frac{\alpha}{\beta}},0)$ 为鞍点. 系统(10)的 Hamilton 量为

$$H(X,Y) = \frac{1}{2}Y^2 + \frac{1}{2}\alpha X^2 - \frac{1}{4}\beta X^4$$

当 $H(\pm\sqrt{\frac{\alpha}{\beta}},0) = \frac{\alpha^2}{4\beta}$ 时,存在两条连接鞍点 $(\pm\sqrt{\frac{\alpha}{\beta}},0)$ 的异宿轨道 $q_i^0(t)$. 当 $\alpha \approx 0.0906$, $\beta \approx 1.3588 \times 10^7 w_0^2$, $w_0 = 1 \times 10^{-3}$ m 时,如图 2 所示. 异宿轨道可通过求解下式得到

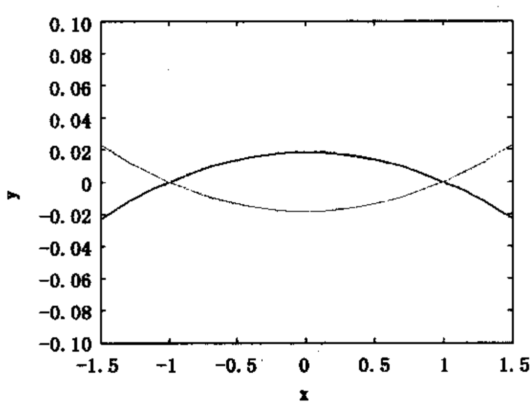

图 2 异宿轨道

Fig. 2 Heteroclinic orbit

$$\begin{cases}\dot{X} = Y \\ H(\pm\sqrt{\frac{\alpha}{\beta}},0) = \frac{1}{2}Y^2 + \frac{1}{2}\alpha X^2 - \frac{1}{4}\beta X^4 = \frac{\alpha^2}{4\beta}\end{cases} \tag{12}$$

由(12)的后一式导出

$$Y = \pm\frac{\alpha}{\sqrt{2\beta}}\left(1 - \frac{\beta}{\alpha}X^2\right) \tag{13}$$

把(13)代入(12)的前一式中并分离变量和积分可得

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \dot{X} = \pm\frac{\alpha}{\sqrt{2\beta}}\left(1 - \frac{\beta}{\alpha}X^2\right)$$

$$\frac{\mathrm{d}X}{1 - \frac{\beta}{\alpha}X^2} = \pm\frac{\alpha}{\sqrt{2\beta}}\mathrm{d}t,$$

$$X = \pm\sqrt{\frac{\alpha}{\beta}}\,\mathrm{th}\left(\sqrt{\frac{\alpha}{2}}t\right) + c$$

其中 c 为积分常数. 当 $t = 0$, $X_0 = 0$ 时,有 $c = 0$. 所以有: $X(t) = \pm\sqrt{\frac{\alpha}{\beta}}\,\mathrm{th}\left(\sqrt{\frac{\alpha}{2}}t\right)$. 对 $X(t)$ 求导得到: $Y(t) = \pm\frac{\alpha}{\sqrt{2\beta}}\mathrm{sech}^2\left(\sqrt{\frac{\alpha}{2}}t\right)$. 可知, $t = 0$, $Y_0 = 0$. 从而得到两条异宿轨道的参数方程

$$\begin{cases}X_i(t) = \pm\sqrt{\frac{\alpha}{\beta}}\,\mathrm{th}\left(\sqrt{\frac{\alpha}{2}}t\right) \\ Y_i(t) = \pm\frac{\alpha}{\sqrt{2\beta}}\mathrm{sech}^2\left(\sqrt{\frac{\alpha}{2}}t\right)\end{cases}\quad (i = 1,2) \tag{14}$$

2 求解(Melnikov 函数)混沌判据

根据 Melnikov 函数 $M_i(t_0)$ 的定义[15],把(9)

和(14) 代入异宿轨的 Melnikov 函数 $M_i(t_0)$

$$M_i(t_0) = \int_{-\infty}^{+\infty} f(q_i^0(t)) \wedge g(q_i^0(t), t + t_0)\mathrm{d}t, (i = 1,2,\cdots,n)$$

其中 g 为外加周期策动力函数,f 为非线性项函数,q^0 为同(异) 宿轨线表达式,$\wedge$ 是法向投影算子,具体运算为,若 $a = (a_1, a_2), b = (b_1, b_2)$ 则

$$a \wedge b = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 = (-a_2, a_1)(b_1, b_2)$$

并注意到 $Y_i(t)$ 是偶函数,则可以得到扰动作用下异宿轨道的 Melnikov 函数

$$M_i(t_0) = \int_{-\infty}^{+\infty} [\delta_1 Y_i(t) = \gamma_1 \cos(t + t_0)] Y_i(t)\mathrm{d}t = \delta_1 \int_{-\infty}^{+\infty} [Y_i(t)]^2 \mathrm{d}t + \gamma_1 \int_{-\infty}^{+\infty} Y_i(t)\cos(t + t_0)\mathrm{d}t = \delta_1 \int_{-\infty}^{+\infty} \frac{\alpha^2}{2\beta}\mathrm{sech}^4\left(\sqrt{\frac{\alpha}{2}}t\right)\mathrm{d}t \pm \gamma_1 \int_{-\infty}^{+\infty} \frac{\alpha}{\sqrt{2\beta}}\mathrm{sech}^2\left(\sqrt{\frac{\alpha}{2}}t\right) \times \cos(\omega t)\mathrm{d}t\cos(\omega t_0) = \frac{4\alpha}{3\beta}\sqrt{\frac{\alpha}{2}}\delta_1 \pm \gamma_1 I\cos(\omega t_0), i = 1,2 \tag{15}$$

其中 $I = \int_{-\infty}^{+\infty} \frac{\alpha}{\sqrt{2\beta}}\mathrm{sech}^2\left(\sqrt{\frac{\alpha}{2}}t\right)\cos(\omega t)\mathrm{d}t = \mathscr{R}es\left[\int_{-\infty}^{+\infty} \frac{\alpha}{\sqrt{2\beta}}\mathrm{sech}^2\left(\sqrt{\frac{\alpha}{2}}z\right)e^{\mathrm{i}\omega z}\mathrm{d}t\right]$.

根据留数定理解 I 的解析表达式

$$R(z) = \frac{\varphi(z)}{\psi(z)} = \frac{\alpha}{\sqrt{2\beta}}\mathrm{sech}^2\left(\sqrt{\frac{\alpha}{2}}z\right)e^{\mathrm{i}\omega z} = \frac{4\alpha}{\sqrt{2\beta}}\frac{e^{(\sqrt{2\alpha}+\mathrm{i}\omega)z}}{(e^{\sqrt{2\alpha}z}+1)^2}.$$

其中

$$\begin{cases} \varphi(z) = \dfrac{4\alpha}{\sqrt{2\beta}}e^{(\sqrt{2\alpha}+\mathrm{i}\omega)z} \\ \psi(z) = (e^{\sqrt{2\alpha}z}+1)^2 \end{cases}$$

函数 $R(z)$ 的奇点为方程 $(e^{\sqrt{2\alpha}z}+1)^2 = 0$ 的根,即 $z_k = \frac{\pi \pm 2k\pi}{\sqrt{2\alpha}}i, (k = 0,1,2,\cdots)$. 选取围道如图 3 所示,其中 P 点处纵坐标值为 $\frac{\pi}{\sqrt{2\alpha}}$. 由留数定理得

$$\oint R(z)\mathrm{d}(z) = \int_{l_1} R(z)\mathrm{d}z + \int_{l_2} R(z)\mathrm{d}z + \int_{l_3} R(z)\mathrm{d}z + \int_{l_4} R(z)\mathrm{d}z = 2\pi i\sum_{k=1}^{n}\mathscr{R}es[R(z), z_k] \tag{16}$$

在所选围道内,只有一个二级极点 $z_0 = \frac{\pi}{\sqrt{2\alpha}}i$. 由留数定理公式

$$\mathscr{R}es\left[\frac{\varphi(z)}{\psi(z)}, z_0\right] = \frac{2\dot{\varphi}(z_0)\ddot{\psi}(z_0) - \frac{2}{3}\varphi(z_0)\dddot{\psi}(z_0)}{[\ddot{\psi}(z_0)]^2} \tag{17}$$

求出 $\varphi(z), \psi(z)$ 和它们的各阶导数在 z_0 处的值

$$\varphi(z_0) = -\frac{4\alpha}{\sqrt{2\beta}}e^{-\frac{\pi\omega}{\sqrt{2\alpha}}}$$

$$\dot{\varphi}(z_0) = -(\sqrt{2\alpha} + \mathrm{i}\omega)\frac{4\alpha}{\sqrt{2\beta}}e^{-\frac{\pi\omega}{\sqrt{2\alpha}}}$$

$$\ddot{\psi}(z_0) = 4\alpha, \dddot{\psi}(z_0) = 12\sqrt{2}\alpha^{3/2} \tag{18}$$

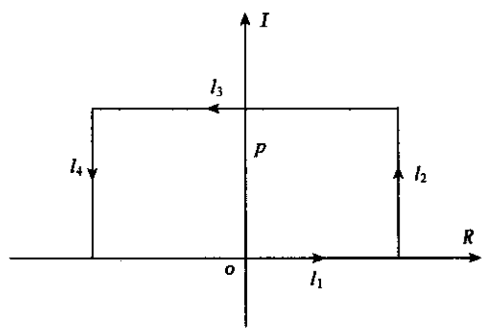

图 3　围道选择

Fig. 3　Choice of the roads

将(18) 各式代入(17) 得

$$\mathscr{R}es\left[\frac{\varphi(z)}{\psi(z)}, z_0\right] = \left\{2\left[-(\sqrt{2\alpha} + \mathrm{i}\omega)\frac{4\alpha}{\sqrt{2\beta}}e^{-\frac{\pi\omega}{\sqrt{2\alpha}}}\right](4\alpha) - \frac{2}{3}\left[-\frac{4\alpha}{\sqrt{2\beta}}e^{-\frac{\pi\omega}{\sqrt{2\alpha}}}\right] \times (12\sqrt{2}\alpha^{3/2})\right\}\Big/(4\alpha)^2 = -\sqrt{\frac{2}{\beta}}\mathrm{i}\omega e^{-\frac{\pi\omega}{\sqrt{2\alpha}}} \tag{19}$$

又知

$$\lim_{R\to\infty}\int_{l_1} R(z)\mathrm{d}z = I,$$

$$\lim_{R\to\infty}\int_{l_2} R(z)\mathrm{d}z = \lim_{R\to\infty}\int_{l_4} R(z)\mathrm{d}z = 0$$

$$\lim_{R\to\infty}\int_{l_3} R(z)\mathrm{d}z = -\lim\int_{-R}^{+R} \times \frac{\frac{4\alpha}{\sqrt{2\beta}}e^{(\sqrt{2\alpha}+\mathrm{i}\omega)(x+\sqrt{\frac{2}{\varepsilon}}\pi\mathrm{i})}}{[e^{\sqrt{2\alpha}(x+\sqrt{\frac{2}{\varepsilon}}\pi\mathrm{i})} + 1]^2}\mathrm{d}x = -e^{\sqrt{\frac{2}{\varepsilon}}\pi\omega} I \tag{20}$$

将(19)和(20)代入(16)得

$$2\pi i = \left(-\sqrt{\frac{2}{\beta}}\mathrm{i}\omega e^{-\frac{\pi\omega}{\sqrt{2\alpha}}}\right) = I - e^{-\sqrt{\frac{2}{\alpha}}\pi\omega} I$$

$$I = \sqrt{\frac{2}{\beta}}\pi\omega \operatorname{csch}\left(\frac{\pi\omega}{\sqrt{2\alpha}}\right) \tag{21}$$

将(21)代入(15)得

$$M_i(t_0) = \frac{4\alpha}{3\beta}\sqrt{\frac{\alpha}{2}}\delta_1 \pm \gamma_1\sqrt{\frac{2}{\beta}}\pi\omega \operatorname{csch} \times \left(\frac{\pi\omega}{\sqrt{2\alpha}}\right)\cos(\omega t_0), i = 1,2 \tag{22}$$

由 Melnikov 理论，令 $M_i(t_0) = 0$，则由(22)得到

$$\frac{\gamma_1}{\delta_1} = \frac{\frac{4\alpha}{3\beta}\sqrt{\frac{\alpha}{2}}}{\sqrt{\frac{2}{\beta}}\pi\omega \operatorname{csch}\left(\frac{\pi\omega}{\sqrt{2\alpha}}\right)\cos(\omega t_0)} \tag{23}$$

因为 $\dot{M}_i(t_0) \neq 0$，所以 $\cos(\omega t_0) \neq \pm 1$，即 $|\cos(\omega t_0)| < 1$. 所以当系统满足 $\frac{\gamma_1}{\delta_1} > \frac{\frac{4\alpha}{3\beta}\sqrt{\frac{\alpha}{2}}}{\sqrt{\frac{2}{\beta}}\pi\omega \operatorname{csch}\left(\frac{\pi\omega}{\sqrt{2\alpha}}\right)}$ 时，就可能产生 Smale 马蹄变换意义下的混沌运动.

3 结论

本文给出了在外磁场中，受有均布载荷作用的金属薄板的磁弹性耦合振动方程，采用 Melnikov 理论求得四边简支矩形金属薄板系统混沌运动的异宿轨道，并求得 Melnikov 函数 $M_i(t_0)$，得到了混沌运动判据. 该方法可以推广到其他不同边界条件和不同外载荷条件下弹性薄板的磁弹性振动问题的研究.

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THE CHAOS ANALYSIS OF RECTANGULAR PLATE UNDER DISTRIBUTED LOAD IN TRANSVERSE MAGNETIC FIELD(I) *

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Abstract In this paper, the model for thin rectangular plate coupled transverse invariable electro-magnetic with uniform transverse forces was built. The boundary of this model was that four edges were simply supported. Based on this, the coupled vibration equations of thin rectangular plate were derived, and the heteroclinic orbit parameter equations of this thin plate vibration system were solved. Using Melnikov function method, the heteroclinic orbit's Melnikov function of vibration system were derived and solved. Finally, the chaos condition and judging criterion of this system about Smale commutation were given. Thus, we can study the bifurcation and chaos of thin rectangular plate coupled mechanical loads with electro-magnetic. The method offered in this paper may be used to study the elastic-magnetic vibration of thin plate under different boundary conditions and different external loads.

Key words thin rectangular plate, distributed load, magneto-elastic, chaos, Melnikov function

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