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载流梁在磁场中的横向固有振动分析

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摘要 在考虑结构变形对电磁场的影响基础上,假设载流梁的变形为小变形,把变形后载流梁中的电流方向改变看成是电流矢量的刚性旋转,建立了载流梁在磁场中的横向固有振动控制方程.方程表明载流梁在磁场中的横向固有振动是一个典型的非线性问题.采用摄动法求得了其近似解,得到了载流梁在磁场中的横向固有振动频率及位移解析表达式.并通过实例计算讨论分析了导线与载流梁间距、载流梁的电流与导线电流的方向及大小、载流梁梁长及其半径等因素对载流梁横向固有振动的影响,得到了一些有价值的结论.

关键词 载流梁,磁场,固有振动,电流

引言

在实际工程中经常会遇到载流导体在磁场中的受力和变形问题,如处于强电磁场中的铁磁弹性载流梁在现代核工程、磁悬浮装置以及强磁电器设备等高技术领域有着广泛的应用.近二十年来有关磁场中载流直导线及薄板的弹性变形问题的研究较为活跃,文献[1]研究了载流梁的磁弹性屈曲和过屈曲,文献[2]按弦的模型研究了载流导线的横向非线性振动,所以有关载流梁在磁场中的弹性变形问题已成为一个重要的前沿课题.本文在考虑结构变形对电磁场的影响基础上,按梁的模型建立了载流梁在磁场中的横向固有振动控制方程,采用摄动法求得其近似解.并通过实例计算,讨论分析了载流梁与导线间的距离及电流方向对载流梁横向固有振动的影响.

1 横向振动方程的建立

以图1所示载流梁在外加磁场中的变形问题为例研究载流梁在外加磁场中的横向固有振动问题.假设外加磁场由无限长刚性载流导线 AB 产生,其电流强度为 I_0;弹性载流梁 CD 为两端简支,长度为 l,直径为 $2R$,梁中电流为 I,载流梁与导线 AB 平行且间距为 d.

在很多机电问题上,电磁力除和外加磁场、导体中的电流大小有关外,还和结构的变形有关.计及机电耦合效应时,电磁力沿载流梁的长度方向将不再是常数,而是坐标的函数并与梁的变形有关.假定图1所示载流梁的变形为小变形,则变形后梁微段与轴的夹角为$\frac{\partial w}{\partial x}$,梁变形后导体梁中的电流方向也会发生变化,可以看成是电流矢量的刚性旋转[3,4].这样,梁微段内电流矢量为

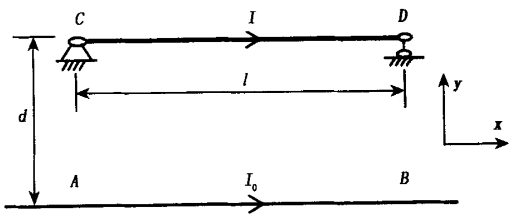

图1 载流梁模型

Fig.1 The model of beam carrying electric current

$$\vec{I} = I\cos\left(\frac{\partial w}{\partial x}\right)\vec{i} + I\sin\left(\frac{\partial w}{\partial x}\right)\vec{j} = I\left[1 - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\vec{i} + I\frac{\partial w}{\partial x}\vec{j} \tag{1}$$

弹性载流梁所在位置的外加磁场为

$$\vec{B} = \frac{\mu_0 I_0}{\pi(d \pm w)^2}\vec{k} \tag{2}$$

在上式中,当导线中电流与载流梁电流同向时取"+",反向时取"-",μ_0 为真空磁导率.

若不计弹性载流梁产生的自磁场对作用在梁

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上的电磁力影响时,则弹性载流梁的电磁力分布载荷为

$$\vec{q}(x) = \vec{I}\times\vec{B} = I\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 & \frac{\partial w}{\partial x} & 0 \\ 0 & 0 & \frac{\mu_0 I_0}{\pi(d\pm w)^2}\end{vmatrix} = \frac{\mu_0 I_0 I}{\pi(d\pm w)^2}\left[\frac{\partial w}{\partial x}\vec{i} - \left(1-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)\vec{j}\right] \tag{3}$$

在式(3)中,第一项使梁沿轴线方向发生变形,第二项使梁发生弯曲变形.若以 N 表示梁的轴力,M 和 Q 分别表示载流梁的弯矩和剪力,则有

$$\frac{\partial N}{\partial x} = \frac{\mu_0 I_0 I}{\pi d^2}\left(1\pm\frac{w}{d}\right)^{-2}\frac{\partial w}{\partial x} \tag{4}$$

$$\frac{\partial Q}{\partial x} = \frac{\mu_0 I_0 I}{\pi d^2}\left(1\pm\frac{w}{d}\right)^{-2}\left[1-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right] \tag{5}$$

对式(4)积分可以得到

$$N = \mp\frac{\mu_0 I_0 I}{\pi d}\left(1\pm\frac{w}{d}\right)^{-1} \tag{6}$$

而载流梁的平衡方程为

$$\frac{\partial^2 M}{\partial x^2} - \frac{\partial}{\partial x}\left(N\frac{\partial w}{\partial x}\right) - \frac{\partial Q}{\partial x} + \rho A\frac{\partial^2 w}{\partial t^2} = 0 \tag{7}$$

由式(4)～式(7)可以得到载流梁横向固有振动控制方程为

$$D\frac{\partial^4 M}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = \frac{\mu_0 I_0 I}{\pi d^2}\left(1\pm\frac{w}{d}\right)^{-2}\times\left[1-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right] + \frac{\mu_0 I_0 I}{\pi d}\left(1\pm\frac{w}{d}\right)^{-2}\times\left(\frac{\partial w}{\partial x}\right)^2 \mp \frac{\mu_0 I_0 I}{\pi d}\left(1\pm\frac{w}{d}\right)^{-1}\frac{\partial^2 w}{\partial x^2} \tag{8}$$

式中,$D = \frac{E\pi R^4}{4}$ 为梁的抗弯刚度.

从式(8)可以知道,载流梁在磁场中的横向固有振动是一个典型的非线性问题.

2 横向振动方程的近似解

首先研究图1所示载流梁与导线中的电流同向时载流梁的横向固有振动的近似解.由式(8)可以知道电流同向时的振动控制方程为

$$D\frac{\partial^4 M}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = \frac{\mu_0 I_0 I}{\pi d}\left(1+\frac{w}{d}\right)^{-2}\times\left[1-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right] + \frac{\mu_0 I_0 I}{\pi d}\left(1+\frac{w}{d}\right)^{-2}\times\left(\frac{\partial w}{\partial x}\right)^2 - \frac{\mu_0 I_0 I}{\pi d}\left(1+\frac{w}{d}\right)^{-1}\frac{\partial^2 w}{\partial x^2} \tag{9}$$

将式(9)的右端用 Taylor 级数展开,略去立方项以上高阶项可以得到

$$D\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial 2w}{\partial t^2} = \frac{\mu_0 I_0 I}{\pi d}\left[1-\frac{2w}{d}+\frac{3w^2}{d^2}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{3w}{d}\left(\frac{\partial w}{\partial x}\right)^2\right] - \frac{\mu_0 I_0 I}{\pi d}\left(\frac{\partial^2 w}{\partial x^2} - \frac{w}{d}\frac{\partial^2 w}{\partial x^2} + \frac{w^2}{d^2}\frac{\partial^2 w}{\partial x^2}\right) \tag{10}$$

在工程实际中,一般对结构的的基频最感兴趣,可设式(8)的解为

$$w(x,t) = T(t)\sin\frac{\pi x}{l} \tag{11}$$

把式(11)代入式(10)中,利用 Galerkin 原理可得

$$\frac{\mathrm{d}^2 T}{\mathrm{d}t^2} + \omega_0^2 T + \beta T^2 = k \tag{12}$$

式中,$\omega_0^2 = \frac{\pi^4 D}{\rho A l^4} + \frac{\mu_0 I I_0}{\pi\rho A d^2}\left(\frac{2}{d} - \frac{\pi^2 d}{l^2}\right)$,$\beta = \frac{\mu_0 I_0 I}{\pi\rho A d^2}\left(\frac{2\pi}{l} - \frac{8}{\pi d^2}\right)$,$k = \frac{4\mu_0 I_0 I}{\pi^2\rho A d^2}$.

令 $\tau = \omega t$,$T = \varepsilon\varphi$ 可把式(12)化为如下

$$\omega^2\frac{\mathrm{d}^2\omega}{\mathrm{d}\tau^2} + \omega_0^2\varphi + \varepsilon\beta\varphi^2 = \frac{k}{\varepsilon} \tag{13}$$

设

$$\begin{cases}\varphi = \varphi_0 + \varepsilon\varphi_1 + \varepsilon\varphi_2 + \cdots \\ \omega = \omega_0 + \varepsilon\omega_1 + \varepsilon\omega_2 + \cdots\end{cases} \tag{14}$$

把式(14)代入式(13)中可得

$$\begin{cases}\omega_0^2\frac{\mathrm{d}^2\varphi_0}{\mathrm{d}\tau^2} + \omega_0^2\varphi_0 = \frac{k}{\varepsilon} \\ \omega_0^2\frac{\mathrm{d}^2\varphi_1}{\mathrm{d}\tau^2} + \omega_0^2\varphi_1 = -\beta\varphi_0^2 - 2\omega_0\omega_1\frac{\mathrm{d}^2\varphi_0}{\mathrm{d}\tau^2}\end{cases} \tag{15}$$

设初始条件为

$$\tau = 0, \varphi(0) = a, \frac{\mathrm{d}\varphi(0)}{\mathrm{d}\tau} = 0 \tag{16}$$

由式(15)及式(16)可以求得

$$\varphi_0(\tau) = \left(a - \frac{k}{\varepsilon\omega_0^2}\right)\cos\tau + \frac{k}{\varepsilon\omega_0^2} \tag{17}$$

设

$$\varphi_1(\tau) = B_0 + \sum_{i=2}^{\infty} B_i\cos i\tau \tag{18}$$

把式(18)代入式(15)第2分式可得

$$\omega_0^2\left[B_0 + \sum_{i=2}^{\infty} B_i(1-i^2)\cos i\tau\right] = -\frac{\beta k^2}{\varepsilon^2\omega_0^4} - \frac{\beta}{2}\left(a - \frac{k}{\varepsilon\omega_0^2}\right)^2 + \left(a - \frac{k}{\varepsilon\omega_0^2}\right)\left(2\omega_0\omega_1 - \right.$$

$$\left.\frac{2\beta k}{\varepsilon\omega_0^2}\right)\cos\tau - \frac{\beta}{2}\left(a - \frac{k}{\varepsilon\omega_0^2}\right)^2\cos 2\tau \qquad (19)$$

由式(19)可以求得

$$B_0 = -\frac{\beta k^2}{\varepsilon^2\omega_0^6} - \frac{\beta}{2\omega_0^2}\left(a - \frac{k}{\varepsilon\omega_0^2}\right)^2,$$

$$B_2 = \frac{\beta}{6\omega_0^2}\left(a - \frac{k}{\varepsilon\omega_0^2}\right)^2, \omega_1 = \frac{\beta k}{\varepsilon\omega_0^3} \qquad (20)$$

所以,图 1 所示载流梁与导线中的电流同向时,载流梁的横向固有振动的近似解为

$$\omega = \omega_0 + \frac{\beta k}{\omega_0^3} \qquad (21)$$

$$\omega(x,t) = \left\{\frac{k}{w_0^2} + \left(A' - \frac{k}{w_0^2}\right)\cos\omega t - \frac{\beta k^2}{\omega_0^6} - \frac{\beta}{2\omega_0^2}\left(A' - \frac{k}{\omega_0^2}\right)^2 + \left[\frac{\beta k^2}{\omega_0^6} + \frac{\beta}{3\omega_0^2}\left(A' - \frac{k}{\omega_0^2}\right)^2\right]\cos\omega t + \frac{\beta}{6\omega_0^2}\left(A' - \frac{k}{\omega_0^2}\right)^2\cos 2\omega t\right\}\sin\frac{\pi x}{l} \qquad (22)$$

式中,$A' = \varepsilon a$ 为振幅.

同理,当图 1 所示载流梁与导线中的电流反向时,载流梁的横向固有振动的近似解为

$$\omega = \omega_0' + \frac{\beta k}{\omega_0'^3} \qquad (23)$$

$$\omega(x,t) = \left\{\frac{k}{w_0'^2} + \left(A' - \frac{k}{w_0'^2}\right)\cos\omega t - \frac{\beta k^2}{\omega_0'^6} - \frac{\beta}{2\omega_0'^2}\left(A' - \frac{k}{\omega_0'^2}\right)^2 + \left[\frac{\beta k^2}{\omega_0'^6} + \frac{\beta}{3\omega_0'^2}\left(A' - \frac{k}{\omega_0'^2}\right)^2\right]\cos\omega t + \frac{\beta}{6\omega_0'^2}\left(A' - \frac{k}{\omega_0'^2}\right)^2\cos 2\omega t\right\}\sin\frac{\pi x}{l} \qquad (24)$$

式中,$\omega_0'^2 = \frac{\pi^4 D}{\rho A l^4} + \frac{\mu_0 I I_0}{\pi\rho A d^2}\left(\frac{\pi^2 d}{l^2} - \frac{2}{d}\right)$.

3　实例计算及讨论

为了讨论载流梁与导线间的距离 d、电流 I、I_0 的方向及其大小、载流梁长度 l 和载流梁半径 R 等因素对载流梁在磁场中的横向固有振动的影响,取计算参数如下:$E = 2.06\times10^{11}\ \mathrm{N/m^2}$,$\rho = 7.8\times10^3\ \mathrm{kg/m^3}$,$\mu = 4\pi\times10^{-7}\ \mathrm{H/m}$,$A' = 0.01$ m,由式(21)~式(24)可以得到图 2~图 8 所示的曲线.

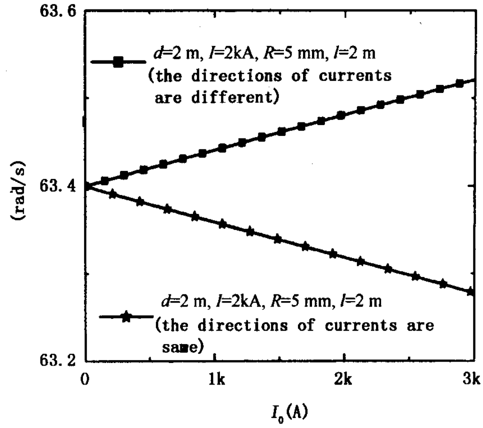


图 2　载流梁自振频率随导线电流变化曲线

Fig. 2　Natural vibration frequency of beam carrying electric current versus current of wire

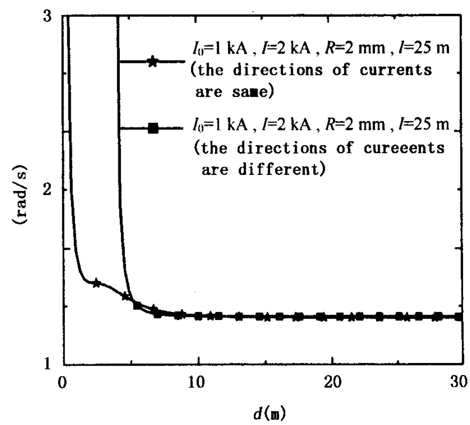


图 3　载流梁自振频率随载流梁与导线间的距离变化曲线

Fig. 3　Natural vibration frequency of beam carrying electric current versus distance between wire and beam

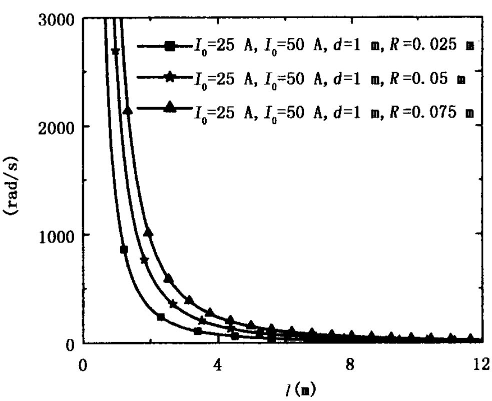


图 4　载流梁自振频率随梁长变化曲线(电流同向)

Fig. 4　Natural vibration frequency of beam carrying electric current versus length of beam (with the same direction as the current)

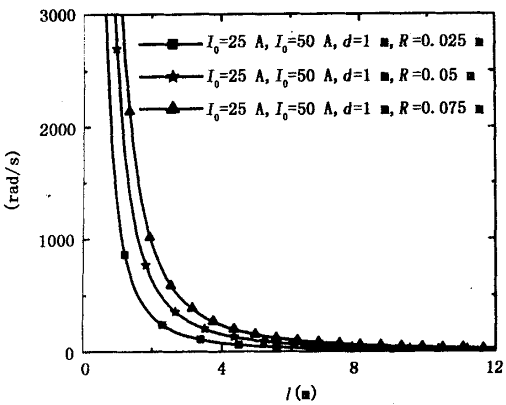
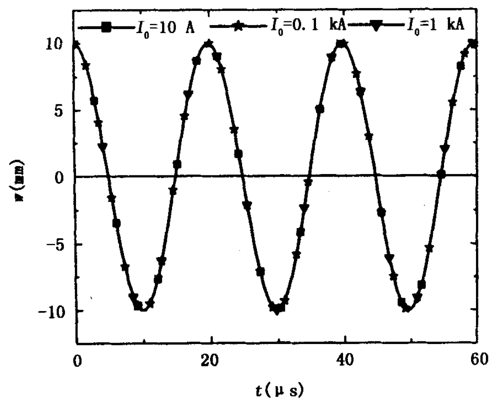


图5 载流梁自振频率随梁长变化曲线(电流异向)

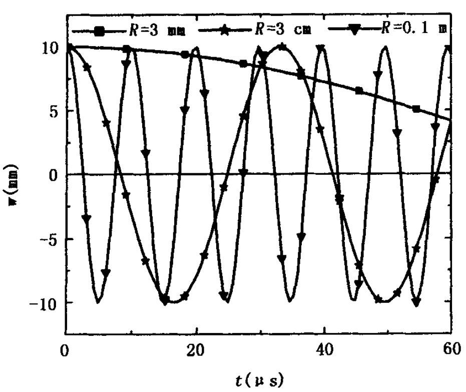
Fig.5 Natural vibration frequency of beam carrying electric current versus length of beam (with different direction from the current direction)



(d = 2 m, I = 100 A, R = 0.025 m, l = 2 m)

图6 载流梁的横振位移变化曲线(电流同向)

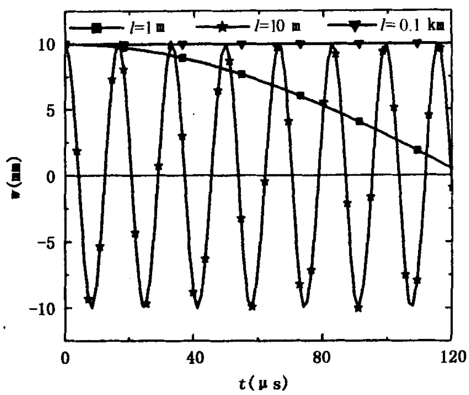
Fig.6 the lateral vibration displacement of beam carrying electric current versus time(with the same direction as the current)



(d = 2 m, I = 100 A, I_0 = 50 A, l = 2 m)

图7 载流梁的横振位移变化曲线(电流同向)

Fig.7 The lateral vibration displacement of beam carrying electric current versus time(with the same direction as the current)



(d = 2 m, I = 100 A, I_0 = 50 A, R = 0.025 m)

图8 载流梁的横振位移变化曲线(电流同向)

Fig.6 The lateral vibration displacement of beam carrying electric current versus time(with the same direction as the current)

对图2～图8所示的曲线进行分析,可以得到如下结论:

(1)载流梁的电流 I 与导线电流 I_0 异向时载流梁在磁场中的横向固有振动自振频率大于二者同向时的自振频率,但相差不大;二者同向时,载流梁在磁场中的横向固有振动自振频率 ω 是导线电流 I_0 的减函数,二者异向时,载流梁的自振频率 ω 则是导线电流 I_0 的增函数(因导线电流 I_0 与载流梁电流 I 是对称的,载流梁的自振频率 ω 与载流梁电流 I 亦有此关系).

(2)无论 I_0、I 是否同向,载流梁在磁场中的横向固有振动自振频率 ω 在载流梁与导线间的距离 d 约小于5 m时随 d 增大急剧减小,此后变化趋于平缓,I_0、I 同向和异向的相同计算参数2曲线基本重合.载流梁的横向固有振动自振频率 ω 与载流梁长度 l 的变化规律与此类似.

(3)当载流梁半径 R 变大时,载流梁在磁场中的横向固有振动自振频率 ω 和载流梁长度 l 之间的曲线上移,说明载流梁自振频率 ω 变大,即载流梁自振频率 ω 是载流梁半径 R 的增函数.

(4)载流梁的横振位移 w 一般约以 $w = 0$ 为中心呈余弦规律变化,但载流梁长度 l 为0.1 km时,载流梁的横振位移 w 约以 w = 10 mm为中心呈余弦规律变化;载流梁长度 l 增大时,横振位移 w 变化周期变小,振幅变小,当 l = 0.1 km时载流梁的横振位移 w 变化曲线几乎成为直线(变化周期相当小,振幅相当小);载流梁横振位移 w 变化周期是半径 R 的减函数;横振位移 w 与导线电流 I_0

关系不大,同样可以得到,载流梁与导线间的距离 d 及 I、I_0 是否同向等因素对横振位移 w 影响也不大.

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ANALYSIS OF THE NATURAL LATERAL VIBRATION OF BEAM CARRYING ELECTRIC CURRENT IN MAGNETIC FIELD

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Abstract Taking into account the influence of structural deformations on electromagnetic field, assuming that the beam's deformations are small, and regarding the change of current direction after beam's deformations as current vector's stiff rotation, the natural lateral vibration control equation of beam carrying electric current in magnetic field was established. The equation indicated that the beam's natural lateral vibration was a typical issue. The perturbation method was employed to get the control equation's approximate solution. The analytical expressions of beam's natural lateral vibration frequency and displacement were derived. The main factors, such as the distance of wire and beam, the quantity and direction of current, and the length and radius of beam, which affect the natural lateral vibration of beam carrying electric current, were discussed with an example, and some valuable conclusions were obtained from the example.

Key words beam carrying electric current, magnetic field, natural vibrations, current

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