第 3 卷第 3 期
2005 年 9 月
动力学与控制学报
JOURNAL OF DYNAMICS AND CONTROL
Vol.3 No.3
Sep. 2005

有初始缺陷复合材料梁在湿热状态下混沌运动*

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摘要 在考虑剪切变形的影响基础上,研究了有初始缺陷复合材料梁在湿热状态下的混沌运动,并讨论分析了剪切变形、初始缺陷、温度、湿度等因素对混沌运动区域的影响.得到了以下结论:当温度升高时,非完善复合材料梁发生混沌运动区域增大;当初始缺陷程度增加,非完善复合材料梁的混沌运动区域越来越小;湿度升高时,非完善复合材料梁发生混沌运动区域增大;当考虑剪切变形影响时非完善复合材料梁的混沌运动区域变大;在湿热状态下理想复合材料梁的混沌运动区域要比非完善复合材料梁的混沌运动区域大.

关键词 初始缺陷,复合材料,混沌运动

引言

由于复合材料具有强度高、刚度大的特点,往往以薄壁结构应用于工程之中,在工作荷载下处于大挠度状态,加上日益发展的高强度复合材料,由于 E/G 的比值很大,也会产生较大的剪切挠度致使复合材料结构发生大挠度问题.文献[1-3]研究了混沌运动,得到了一些有益的结论,文献[4]研究了有初始缺陷梁在热状态下的混沌运动.由于温度与湿度结合可能会对聚合物基复合材料性能产生较大危害,因此工程设计人员极为关注温度和湿度效应对复合材料结构变形的影响.本文在考虑剪切变形的基础上,研究了有初始缺陷复合材料梁在湿热状态下的混沌运动,并讨论分析了剪切变形、初始缺陷、温度、湿度等因素对混沌运动区域的影响.

1 湿热振动控制方程

由复合材料力学理论可以得到考虑剪切变形时,有初始缺陷复合材料梁在湿热状态下的振动控制微分方程组为

$$\begin{cases} D\dfrac{\partial^2\theta}{\partial x^2}+C(\dfrac{\partial y}{\partial x}-\theta)=0 \\ C(\dfrac{\partial^2 y}{\partial x^2}-\dfrac{\partial\theta}{\partial x})-N\dfrac{\partial^2(y+y_0)}{\partial x^2}+q(x,t)+ \\ \qquad \rho A\dfrac{\partial^2 y}{\partial t^2}+\gamma_0\dfrac{\partial y}{\partial t} \end{cases} \tag{1}$$

式中,$N=N_T+N_m-\frac{B}{2l}\int_0^l[(\frac{\partial y}{\partial x})^2+2\frac{\partial y}{\partial x}\frac{\partial y_0}{\partial x}]\mathrm{d}x$,$B=b\sum_{i=1}^{n}E_i(z_i-z_{i-1})$,$C=k_s b\sum_{i=1}^{n}G_i(z_i-z_{i-1})$,$D=\frac{b}{3}\sum_{i=1}^{n}(z_i^3-z_{i-1}^3)$,$\rho A=b\sum_{i=1}^{n}\rho_i(z_i-z_{i-1})$.

E_i,G_i 分别为复合材料梁第 i 层材料的弹性模量、剪切弹性模量,b,l,h,A 为梁宽、梁长、梁高、梁的截面积、k_s 为梁截面剪切系数,ρ_i,z_i 为梁第 i 层密度、梁第 i 层高度,γ_0 为阻尼系数,N_T 为热力,N_m 为湿力,$q(x,t)$ 为外荷载,$\theta(x,t)$ 为横振位移,$y_0(x)$ 为初始缺陷.

热力、湿力的表达式为

$$N_T=b\sum_{i=1}^{n}E_i\alpha_{si}\int_{z_{i-1}}^{z_i}T(z)\mathrm{d}z \tag{2}$$

$$N_m=b\sum_{i=1}^{n}E_i\beta_{si}\int_{z_{i-1}}^{z_i}m(z)\mathrm{d}z \tag{3}$$

式(2)和式(3)中 α_{si},β_{si} 分别为热胀系数、湿胀系数.

由于温度、湿度沿厚度方向分布对结构影响最大,取热分布、湿分布为线性分布

$$T(z)=T_0+\frac{T_1 z}{h} \tag{4}$$

$$m(z)=m_0+\frac{m_1 z}{h} \tag{5}$$

考虑梁的基频振动,可设初始缺陷为

2005-03-21 收到第 1 稿.
* 湖南省自然科学基金资助项目(JJY-20114)

$$y_0(x) = rh\sin\frac{\pi x}{l} \tag{6}$$

上式中，r 为缺陷系数.

为了求解方便，可设

$$\theta(x,t) = \frac{\partial F(x,t)}{\partial x} \tag{7}$$

$$y(x,t) = F(x,t) - \frac{D}{C}\frac{\partial^2 F(x,t)}{\partial x^2} \tag{8}$$

把式(7)及式(8)代入式(1)中解耦可得

$$D\frac{\partial^4 F}{\partial x^4} + \gamma_0\left(\frac{\partial F}{\partial t} - \frac{D}{C}\frac{\partial^3 F}{\partial x^2 \partial t}\right) + N\left(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 y_0}{\partial x^2} - \frac{D}{C}\frac{\partial^4 F}{\partial x^4}\right) + \rho A\left(\frac{\partial^2 F}{\partial t^2} - \frac{D}{C}\frac{\partial^4 F}{\partial x^2 \partial t^2}\right) = q(x,t) \tag{9}$$

对于两端为不可移的复合材料简支梁可设

$$F(x,t) = \phi(t)\sin\frac{\pi x}{l} \tag{10}$$

$$q(x,t) = P\cos(\omega t)\sin\frac{\pi x}{l} \tag{11}$$

把有关各式代入式(9)中利用伽辽金原理可得

$$\frac{d^2\phi}{dt^2} + \frac{\gamma_0}{\rho A}\frac{d\phi}{dt} - \alpha\phi + \beta\phi^2 + \gamma\phi^3 = P_0 + \frac{P}{\rho A g}\cos\omega t \tag{12}$$

式中，$\alpha = \frac{\pi^2}{\rho A l^2}\left(N_T + N_m - \frac{\pi^2 D}{l^2 g} - \frac{\pi^2 B h^2 r^2}{2l^2}\right)$，$g = 1 + \frac{\pi^2 D}{l^2 C}$，$\beta = \frac{3\pi^4 B g h r}{4\rho A l^4}$，$\gamma = \frac{\pi^4 B g^2}{4\rho A l^4}$，$P_0 = \frac{\pi^2(N_T + N_m)hr}{\rho A g l^2}$.

2 混沌运动的条件

对式(12)进行无量纲化变换可得

$$\frac{d^2\varphi}{d\tau^2} - \varphi + \lambda\varphi^2 + \varphi^3 = \epsilon\left(f_0 + f\cos\Omega\tau - \eta\frac{d\varphi}{d\tau}\right) \tag{13}$$

上式中，$t = \frac{\tau}{\sqrt{\alpha}}$，$\Omega = \frac{\omega}{\sqrt{\alpha}}$，$\eta = \frac{\gamma_0}{\epsilon\rho A\alpha^{1/2}}$，$\lambda = \frac{\beta}{\sqrt{\alpha\gamma}}$，$f_0 = \frac{P_0\gamma^{1/2}}{\epsilon\alpha^{3/2}}$，$f = \frac{P\gamma^{1/2}}{\epsilon\rho A g\alpha^{3/2}}$，$\phi = \varphi\sqrt{\frac{\alpha}{\gamma}}$.

令式(13)中的 $\epsilon = 0$，通过复杂的数学计算可以求得同宿轨道为

$$\varphi(\tau) = \frac{2C_0 e^{\pm\tau}}{(Ce^{\pm\tau} + \lambda/3)^2 + 1/2} \tag{14}$$

其中，C_0 为常数，由初始条件确定.

对应于式(14)的同宿轨道的 Melnikor 函数为[5]

$$M(\tau_0) = \int_{-\infty}^{+\infty}\dot{\varphi}[f_0 + f\cos\Omega(\tau + \tau_0) - \eta\dot{\varphi}]d\tau = \frac{4\pi\Omega f(ch\Omega\eta_1 - ch\Omega\eta_2)\sin\Omega(\xi + \tau)}{\sqrt{2}(ch2\pi\Omega - 1)} - \frac{4\eta}{3}\left[1 + \frac{\lambda^2}{3} + \frac{\lambda(2\lambda^2 + 9)}{9\sqrt{2}}\arcsin\frac{\sqrt{2}\lambda}{\sqrt{2\lambda^2 + 9}}\right] \tag{15}$$

上式中：$\xi = \ln\frac{\sqrt{2\lambda^2 + 9}}{3\sqrt{2}C}$，$\eta_1 = \pi + \arctan\frac{3}{\sqrt{2}\lambda}$，$\eta_2 = \pi - \arctan\frac{3}{\sqrt{2}\lambda}$.

所以，由式(8)可以得到有初始缺陷梁在湿热状态下发生混沌运动的临界条件为

$$\frac{f}{\eta} \geqslant \sqrt{2}(ch2\pi\Omega - 1)\left[1 + \frac{\lambda^2}{3} + \frac{\lambda(2\lambda^2 + 9)}{9\sqrt{2}} \times \arcsin\frac{\sqrt{2}\lambda}{\sqrt{2\lambda^2 + 9}}\right]/3\pi\Omega(ch\Omega\eta_1 - ch\Omega\eta_2) \tag{16}$$

当 $y_0 = 0$ 时式(16)可化为

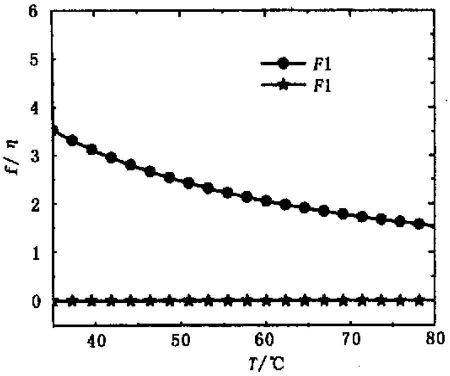
$$\frac{f}{\eta} \geqslant 4ch\frac{\pi\Omega}{2}/3\sqrt{2}\pi\Omega \tag{17}$$

式(17)即为完善复合材料梁在湿热状态下发生混沌运动的临界条件，这说明本文所得到的有初始缺陷复合材料梁在湿热状态下发生混沌运动的临界条件是正确的.

3 讨论分析

为了讨论分析初始缺陷、剪切变形、温度、湿度等因素对复合材料梁在湿热状态下发生混沌运动的影响，本文以材料 $T300/976$ 碳／环氧单向复合材料梁为例，取梁的计算参数为：$E = 134.45$ GPa，$G = 6.96$ GPa，$\rho = 1.6 \times 10^3$ kg/m^3，$\alpha_s = 0.9 \times 10^{-6}$ ℃，$\beta_s = 0.2$，$l = 1$ m，$h = 5 \times 10^{-3}$ m，$k_s = 10(1 + \mu)(12 + 11\mu)$，$\mu = 0.3$，$b = 1 \times 10^{-2}$ m，$\omega = 400$ rad/s，$T_0 = T_1 = T$，$m_0 = m_1$.

当非完善复合材料梁取不同的初始缺陷、温度、湿度时，把有关参数代入式(16)中，即可得到 $f\eta^{-1} - T$ 曲线. 令 $C = \infty$，即不考虑剪切变形，把由此得到的相关各式及有关参数代入式(16)中，可据此计算出不考虑剪切变形时 $f\eta^{-1} - T$ 曲线. 具体计算结果绘制在图1～图3中.



$m_0 = 0.01$　$r = 0.5$

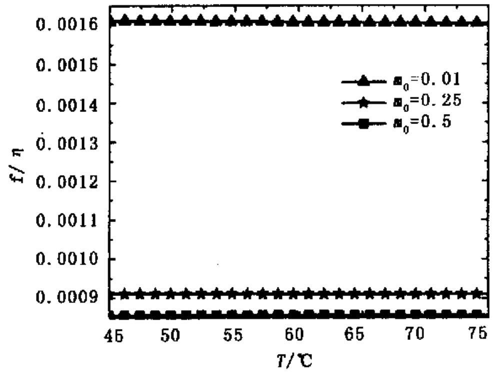
图 1　剪切变形对混沌区域的影响

F_1：不考虑剪切变形　F_2：考虑剪切变形

Fig. 1　The influence of shear deformation on chaotic motive region

F_1: not considering the influence of shear deformation

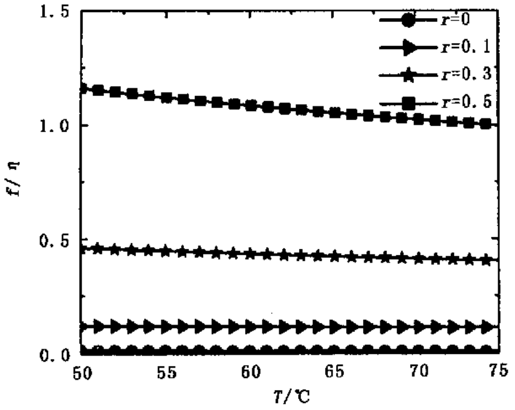
F_2: considering the influence of shear deformation



$r = 0.1$

图 2　湿度对混沌区域的影响

Fig. 2　The influence of humidity on chaotic motive region



$m_0 = 0.01$

图 3　初始缺陷对混沌运动区域的影响

Fig. 3　The influence of initial defect on chaotic motive region

对图 1 ～ 图 3 进行分析可以得到以下结论：

(1) 当温度升高时，理想梁与有初始缺陷梁的 $f\eta^{-1} - T$ 曲线呈单调下降；即温度升高时，理想梁与非完善梁发生混沌运动区域将增大.

(2) 随着 r 值的增大，即梁的初始缺陷程度增加，非完善梁的 $f\eta^{-1} - T$ 比值升高，这说明非完善梁的混沌运动区域越来越小. 同时，在湿热状态下理想梁的混沌运动区域要比非完善梁的混沌运动区域大，这一点从图 3 中可以看出来.

(3) 当湿度升高时，有初始缺陷梁的 $f\eta^{-1} - T_0$ 曲线下移；即湿度升高时，非完善梁发生混沌运动区域将增大. 但从图 2 可以看出，$f\eta^{-1}$ 值改变不大，可以忽略，说明湿度对非完善梁的混沌运动区域不起控制作用.

(4) 当考虑剪切变形影响时曲线下移，说明混沌运动区域变大.

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THE CHAOTIC MOTION OF COMPOSITE BEAM WITH INITIAL DEFECT UNDER MOIST AND THERMAL STATE*

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Abstract The chaotic motion of composite beam with initial defect under moist and thermal state was studied considering the influence of shear deformation. And the factors that affect the chaotic motive region, such as shear deformation, initial defect, as well as temperature and humidity were discussed. The following conclusions were obtained: (1) the chaotic motive region of composite beam with initial defect increases when temperature ascends, or when the extent of the initial defect descends, or when the amount of the humidity rises, or when the influence of shear deformation is taken into account; (2) the chaotic motive region of composite beam without initial defect is larger than that with initial defect under moist and thermal state.

Key words initial defect, composite material, chaotic motion

Received 21 March 2005.

* The project supported by the National Natural Science Foundation of China(JJY-20114)